INTEGRAL COMPENSATION FOR DISTURBANCE REJECTION IN LONGITUDINAL FLIGHT CONTROL OF A FIXED-WING UNMANNED AERIAL SYSTEM

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Longitudinal flight control plays a high relevance role for conventional and unmanned aircraft due to its importance in take-off, climbing, cruise, descent, and landing phases of the flight cycle, where the elevator deflection works as the main control input. For that reason, an automatic controller is necessary for providing flight stability to these Unmanned Aerial Systems (UAS) against wind external disturbances. In the literature, several disturbance-estimation mechanisms have been proposed in control of diverse linear and nonlinear systems. In this work, an Integral Compensation Based-Control approach is introduced for pitch control of the fixed-wing UAS-S4 Écatl. The proposed robust scheme allows both suitable rejection of matched external disturbances and reference tracking tasks. Also, in order to deal with the effect of high-gain of the controller an anti-windup saturation is adequately implemented for the control input. It is worth to note the main advantage of the introduced control approach, is that implementation of an observer is unnecessary for disturbance estimation. The satisfactory performance of the proposed robust controller is assessed in several numerical simulations. Here, the results show that the controller is capable to properly carry out trajectory tracking tasks even in presence of matched disturbances.

Keywords: Longitudinal control, Unmanned Aerial System, Integral Compensation, Disturbance rejection, Flight stability.

1. Introduction

Considerable attention has been paid to Unmanned Aerial Systems (UAS) for civil and military applications. The challenges for making the UAS suitable for various mission especially for hostile envi-
Environment have led the focus of the researchers on the efficiency flight control development. Disturbance rejection remains one of the most important issues in flight control nowadays. Several methodologies that satisfy the disturbance rejection have been proposed and worked well in many practical applications. The authors in [1] implemented a nonlinear robust controller on an unmanned helicopter. The flight control system was based on a three-layer architecture control which includes an inner-loop for stability improvement and robustness; an outer-loop for position control and a flight scheduling layer for mission coordination. The inner-loop, in particular, was designed using a $H_{\infty}$ controller to ensure external disturbance rejection. The results obtained from flight tests were compared with the design specifications for military rotor-craft of the US army aviation, and showed the efficiency of the methodology for achieving the desired performance. In [2] a trajectory tracking controller using a Linear Parameter Varying (LPV) method is proposed. The controller architecture consists in an inner loop designed using a $\mu$ synthesis approach and an outer loop designed using the loop shaping technique. The gain scheduling problem initially is addressed using the LPV method, and thus is reformulated into a $\mu$ synthesis problem, where each variation from the reference model with the flight condition is considered as a disturbance. The results showed that the system could follow a random trajectory with a constant error. On the other hand, authors in [3] introduced a Linear Quadratic Gaussian (LQG) controller for longitudinal and lateral flight control. The LQG methodology was based on the Kalman filter and used as robust controller. The authors show that the proposed approach could achieve good performance against parameters variations. Even if the flight control methodologies present in the literature work well in many practical applications, their complexity makes them difficult to be directly implemented as they usually require an order reduction and induce much effort on the control inputs. This paper described an Integral Compensation Based-Control approach for pitch control. The robust controller was designed with the intent of external disturbance rejection and good performance especially for reference tracking. The effort of the control input was reduced using an anti-windup saturation. The methodology was applied on the UAS-S4 Éhecatl [4] designed and manufactured by Hydra Technologies, Fig. 1.

![Hydra Technologies UAS-S4 Ehecatl](image)

**Figure 1: Hydra Technologies UAS-S4 Ehecatl [4].**

## 2. Éhecatl UAS-S4 Flight Dynamic Equations

Linearized equations of motion considering small perturbations about an equilibrium flight condition (velocity and gravity vectors lie in the plane of symmetry of the vehicle) are used in this work [5], [6], with the state vector defined as follows

$$\mathbf{x} = \begin{bmatrix} u & w & q & \theta \end{bmatrix}^T$$

where $u$, $w$, $q$ and $\theta$ are the axial velocity (velocity component of roll axis), vertical velocity (velocity component of yaw axis), pitch rate, and pitch angle, respectively. On the other hand, the control input
vector is given by

$$\delta = \begin{bmatrix} \delta_e & \delta_T \end{bmatrix}^T$$

where $\delta_e$ and $\delta_T$ are the elevator and thrust (throttle) control variables, respectively. Then, an approximate form of the linearized state-space system equations for longitudinal dynamics using small-disturbance theory is the following:

$$A = \begin{bmatrix} X_u & X_w & 0 & -g_0 \cos \Theta \\ Z_u & Z_w & u_0 & -g_0 \sin \Theta \\ M_u + M_u Z_u & M_w + M_u Z_w & M_q + u_0 M_w & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(3)

$$B = \begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ Z_{\delta_e} & Z_{\delta_T} \\ M_{\delta_e} + M_{\delta_e} Z_{\delta_e} & M_{\delta_T} + M_{\delta_e} Z_{\delta_T} \\ 0 & 0 \end{bmatrix}$$

(4)

where matrix elements depend on dimensional derivatives and their dimensionless aerodynamic coefficients [5], [6], then let introduce the longitudinal model as a set of first-order differential equations as

$$\dot{x} = Ax + B\delta$$

(5)

The numerical values of parameters given in eqs. (3) and (4) were extracted from [4]. Here, the linearized model for a certain trim flight condition of the UAS-S4 (velocity: 41.3040 m/s, altitude: 6,097.6 m and mass: 53.11 kg) is given by the next state space model, where parameters from Table 1 are considered:

$$A = \begin{bmatrix} -0.0726 & 0.2346 & -0.9547 & -9.7830 \\ -0.3729 & -4.5992 & 43.3325 & -0.2240 \\ -0.1308 & -1.3599 & 0.4664 & -0.0118 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(6)

$$B = \begin{bmatrix} -0.0133 \\ 0.0631 \\ -0.1525 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

(7)

Table 1: General characteristics of the UAS-S4.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing span</td>
<td>4.2 m</td>
<td>Maximum take-off weight</td>
<td>80 kg</td>
</tr>
<tr>
<td>Wing area</td>
<td>2.3 m²</td>
<td>Loitering airspeed</td>
<td>35 kn</td>
</tr>
<tr>
<td>Total length</td>
<td>2.5 m</td>
<td>Maximum speed</td>
<td>135 kn</td>
</tr>
<tr>
<td>Mean aerodynamic chord</td>
<td>0.57 m</td>
<td>Service ceiling</td>
<td>15,000 ft</td>
</tr>
<tr>
<td>Empty weight</td>
<td>50 kg</td>
<td>Operational range</td>
<td>120 km</td>
</tr>
</tbody>
</table>
3. An Integral Compensation based control for pitch motion control

In this work we proposed and extension for improving the disturbance rejection capabilities of the control methodology introduced by authors in [7], where a type 1 servo system [8] is designed for flight stability of an UAS. Consider the system given by eq. (6) and eq. (7) to be augmented, in order to design a tracking mechanism for the pitch angle, and is given as follows:

\[
\dot{x}_a = A_a x_a + B_a \delta_e + Gr
\]  

(8)

or in its expanded form:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\xi}
\end{bmatrix} =
\begin{bmatrix}
A & 0_{n \times 1} \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\xi
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix} \delta_e +
\begin{bmatrix}
0_{n \times 1} \\
1
\end{bmatrix} r
\]  

(9)

where \( r \) is the reference to be followed, with \( n = 4 \). Then, the output must follow the reference in steady-state as \( y(\infty) = r \), so that:

\[
\begin{bmatrix}
\dot{x}(\infty) \\
\dot{\xi}(\infty)
\end{bmatrix} =
\begin{bmatrix}
A & 0_{n \times 1} \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
x(\infty) \\
\xi(\infty)
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix} \delta_e(\infty) +
\begin{bmatrix}
0_{n \times 1} \\
1
\end{bmatrix} r(\infty)
\]  

(10)

by substracting eq. (10) from eq. (9), we obtain:

\[
\begin{bmatrix}
\dot{x} - \dot{x}(\infty) \\
\dot{\xi} - \dot{\xi}(\infty)
\end{bmatrix} =
\begin{bmatrix}
A & 0_{n \times 1} \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
x(\infty) - x \\
\xi(\infty) - \xi
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix} [\delta_e - \delta_e(\infty)]
\]  

(11)

that can be expressed in terms of system error

\[
\begin{bmatrix}
\dot{x}_e \\
\dot{\xi}_e
\end{bmatrix} =
\begin{bmatrix}
A & 0_{n \times 1} \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
x_e \\
\xi_e
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix} \delta_{ee}
\]  

(12)

Due to \( r(\infty) = r \), the reference term is canceled, and the control law is established as follows:

\[
\delta_{ee} = -K_x x + k I \xi
\]  

(13)

It is important to note that in eq. (12), \( \delta_{ee} \) is not exactly the same as the elevator control input in eq. (2). Then, now we can express the system given by eq. (12) in its compact form as

\[
\dot{e} = A_a e + B_a \delta_{ee}
\]  

(14)

and the closed-loop system dynamics is given by replacing \( \delta_{ee} \) from eq. (13) into eq. (14), as follows:

\[
\dot{e} = (A_a - B_a K_{x_a}) e
\]  

(15)

In order to ensure closed-loop stability, the LQR theory is used for finding optimal value for \( K_{x_a} = [K_x - k_I] \), where the cost function is given as follows:

\[
J = \frac{1}{2} \int_0^\infty (e^T Q e + \delta_{ee}^T R \delta_{ee}) dt
\]  

(16)

The Ricatti’s equation in eq. (17) is solved with the function \textit{care} of MATLAB. The values of the weighted matrices were calculated with Particle Swarm Optimization (PSO) optimization algorithm,
while the maximum overshoot and the ITAE index performance were considered in the fitness function design.

$$A_a P + PA_a + Q - PB_a R^{-1}B_a^T P = 0$$

(17)

It will be seen that this servo-tracking system is able to deal properly with regulation and tracking tasks. However, the design and analysis were made regardless of disturbances, whereby a proper performance is not guaranteed. Thus, for disturbance attenuation, the above introduced feedback controller is used as the base controller plus multiple iterated error integrals.

Then, the augmented system (9) is rewritten as

$$\begin{bmatrix} \dot{x} \\ \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} A & 0_{n \times 1} & 0_{n \times 1} & 0_{n \times 1} \\ -C & 0 & 0 & 0 \\ 0_{1 \times n} & 1 & 0 & 0 \\ 0_{1 \times n} & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \end{bmatrix} \delta_e + \begin{bmatrix} 0_{n \times 1} \end{bmatrix} r$$

(18)

where it is easy to see that $\xi_1$, $\xi_2$, $\xi_3$ are the iterated error integrals expressed as extended states, which contribute for attenuating disturbances affecting the second order pitch dynamics. Moreover, the LQR theory is used for finding the optimal control gains. Then, the control law is as follows:

$$\delta_{ee} = -K_x x + k_{I_1} \xi_1 + k_{I_2} \xi_2 + k_{I_3} \xi_3$$

(19)

In this way, the stabilization and controllability problems are solved within the same control scheme, where an acceptable level of disturbance attenuation is achieved.

4. Simulation results

Several numerical simulations were performed to highlight the effectiveness of the introduced approach for attenuating disturbance effects by simple addition of the error integrals into the augmented system. Thus, in order to assess the effectiveness of the introduced compensation, a disturbance profile affecting the pitch tracking is injected to the system, simulating the wind effects over the UAS. The graphic representation of this profile is presented in Fig. 2 and is mathematically defined by

$$d_{\theta} = 0.8 \sin \left( \pi \frac{t - 30}{31} \right) + 0.4 \sin \left( \pi \frac{t - 30}{7} \right) + 0.08 \sin \left( \pi \frac{t - 30}{2} \right) + 0.056 \sin \left( \pi \frac{t - 30}{11} \right)$$

(20)

Fig. 3 shows the pitch tracking by employing the controller given by eq. (13), where the perturbed and unperturbed motion of the Éhecatl UAS-S4 is portrayed.
Figure 2: Disturbance affecting the Éhecatl UAS-S4 pitch dynamics.

Figure 3: Pitch motion control with basic integral compensation.

Here, the feedback controller allows a proper tracking (solid line) of the reference profile (dashed line). Nevertheless, when the disturbance is affecting the UAS motion, the quality for tracking the planned reference is decreased. On the other hand, in Fig. 4 the closed-loop system response is shown, where
the integral compensation allows a better attenuation of disturbances, meanwhile a proper reference tracking is performed. In the same sense, due to the high gain properties of the compensation, a basic anti-windup mechanism is suitably employed. Notice, that only by including the error integrals to the augmented system and then calculating the feedback control gains by LQR and PSO theories, the control design process is simplified, and at the same time, an acceptable disturbance attenuation level is obtained.

![Graphs](image)

(a) Unperturbed longitudinal motion.
(b) Control input for unperturbed motion.
(c) Perturbed longitudinal motion.
(d) Control input for perturbed motion.

**Figure 4:** Pitch motion control with full integral compensation.

Finally, in order to highlight quantitatively the effectiveness of the proposed motion controller, the Integral of Time multiply by Absolute Error (ITAE) index was calculated in both with and without disturbance compensation. So, we can see in table 2 the index values for pitch tracking, where is evident a proper performance of the disturbance compensation based on tracking error information.

<table>
<thead>
<tr>
<th>Compensation</th>
<th>Controller 1</th>
<th>Controller 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>10.7478</td>
<td>6.8132</td>
</tr>
<tr>
<td>Yes</td>
<td>17.4267</td>
<td>11.2581</td>
</tr>
</tbody>
</table>

**Table 2:** ITAE index performance
5. Conclusions

In this paper, the longitudinal control problem for the Écatl UAS-S4 is addressed by means of feedback control and integral compensation. Here, the introduced approach requires reduced model information for control, where closed-loop stability is attained. Moreover, simulation results show a quick and smooth tracking performance even in the presence of disturbances, which verify that the proposed method is effective. Notice, a good disturbance attenuation is achieved without using a disturbance observer mechanism. Future work will be directed for applying the proposed control scheme for controlling lateral dynamics of the Écatl UAS-S4.

REFERENCES


