Azimuthal duct mode detection in experiments are particularly beneficial for designing advanced fans and evaluating passive/active noise reduction methods. In this paper, we propose a new mode detection method based on compressed sensing, which can largely reduce the sensor numbers required by Shannon-Nyquist sampling theorem. The success of the compressed sensing methodology is based on the fact that incident waves are sparse in spinning modes and the sensors are located randomly. However, when the sensor number is small, the mode detection maybe not successful for many sensor displays. Therefore, besides introducing a more straightforward way of applying compressed sensing method to duct mode detection, this paper also studies properties of the sensor location in order to obtain the expected good mode detection results in the real experiment. The studies of sensor location properties are based on both Monte Carlo simulations and a real experiment test. This makes it quite promising to apply much less sensors in a real experiment and detect mode successfully with compressed sensing algorithm.

Keywords: sensor array, sensor location, compressed sensing, duct acoustic, mode detection

1. Introduction

Aero engines are the main cause of the noise problem of aircraft. To reduce the noise of aero engines, researchers are paying much attention to fan noise measurement techniques. Mode detection is used commonly to penetrate and analyse near-field sound propagation, scattering patterns from engine ducts [1-3] and far-field directivity predictions [4]. In mode detection experiments, microphone arrays are used widely [5]. In this paper, we focus on the azimuthal mode detection.
When performing azimuthal mode detection, the azimuthal pressure distribution is decomposed into azimuthal mode components. For the traditional method, the spatial Fourier series decomposition is conducted on the circular sensory array. Therefore, to obtain a sufficient spatial resolution and a low side lobe level, a large number of equally spaced microphones are required to satisfy Shannon-Nyquist sampling theorem. This makes the test quite expensive and hard to detect high-order modes.

To reduce the required number of sensors in the sensor array, or detect a much higher order modes with a reasonable small number of sensors, a couple of methods have been proposed in previous studies. For example, a sparse array is introduced and optimized to detect the mode of a model turbofan engine intake [6]. In [7] and [8], radial rakes are carefully positioned and traversed. Compressed sensing, also known as compressed sampling, is a new mode detection method that can reduce the required sensors and simplify the setups of sensor array. It has been developed in information technology and can relax the classical Nyquist-Shannon sampling theory significantly. The success of compressed sensing method for duct mode detection is based on the truth that the dominant modes at a certain blade-passing frequency are usually sparse [9]. There are literatures showing the promising application of compressed sensing to duct mode detection. In [10], Huang introduced the usage of compressed sensing by studying a linear-duct acoustic problem that is classical in aerospace. In [11], Yu and Huang applied compressed sensing method to detecting in-duct azimuthal modes. Following this, Yu et.al. conducted a further detailed study under different mode sparsity and mode ranges [9]. In [12], Bu et.al. studied the potential capability of compressed sensing method for engine tests in a practical testing facility. In [13], Behn et.al. applied compressed sensing method to flow ducts of turbomachines using circumferential sensor arrays, and investigated important features of compressed sensing such as stability and accuracy.

However, although the compressed sensing method demonstrated a very promising application to duct mode detection, the existing good results are based on a probabilistic condition. The recovery of signal from sub-sampled measurements requires the measurement matrix to satisfy the so-called restricted isometry property (RIP) for the s-sparsity signal [14]. Unfortunately, given a measurement matrix construction method, it is a hard task to prove the RIP. Therefore, a random measurement matrix, which usually satisfy the RIP with a high statistical probability [14], is usually adopted. This, however, makes the success of compressed sensing application to duct mode detection probabilistic, especially when the number of randomly chosen sensors are quite small. In a practical duct mode detection experiment, if we mount all the sensors on the sensor array, and randomly choose the sensors to find the best location, the superiority of the compressed sensing method cannot be brought into full play, as the actually used sensors and necessary space for mounted sensors are not reduced. This inspires us to find the location directly, not randomly, in a real experiment to detect the dominant modes with a good chance. In this paper, we explore a simulation-test strategy to find the actual location, and demonstrate that the mode detection with compressed sensing method can be invariant to signal to noise ratio (SNR) and location deviation at some sensor positions. This indicates that there may indeed exist positions where the reconstruction can be quite robust with compressed sensing method, and we can find them by a proper simulation and then apply the sensor display to the real experiment.

The paper is organized as follows. Section 2 introduces the essential concepts of mode detection techniques, including traditional Fourier transform based method and compressed sensing based method. Section 3 demonstrates the influence of different SNR and sensor location deviation on mode detection results with Monte Carlo simulations. Section 4 shows the simulation-test results with a real experiment of a compressor. Section 5 presents the conclusions and discussion.
2. Models for azimuthal mode detection

2.1 Conventional method

The propagation of spinning modes inside cylindrical flow ducts can be approximately modeled by the linearized Euler equations. For a rigid, straight cylindrical duct with infinite length, the analytical solution of the linearized Euler equations can be derived as:

\[ p'(x, r, \theta, t) = \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} A_{mn} J_m(k_n r) e^{i(\omega t - k_x x - m\theta)} \]  

(1)

where \( m \) is the azimuthal mode order; \( J_m \) is the \( m \)th order Bessel function of the first kind; \( k_n \) is the \( n \)th radial wave number and can be derived by the hard-wall boundary condition; \( A_{mn} \) is the amplitude of the \((m, n)\)th order mode. Because we focus on the azimuthal mode in this paper, we sum all radial mode orders \( n \) for each azimuthal mode order \( m \), and denote the result as the complex azimuthal mode amplitude \( A_m \). A conventional azimuthal mode detection method adopts \( N \) equidistantly-distributed sensors in a sensor array, and the \( m \)-th azimuthal mode amplitude at certain fixed frequency can be calculated by [1]

\[ A_{mf} = \frac{1}{K} \sum_{k=1}^{K} p_f(\theta_k) e^{im\theta_k} \]  

(2)

where \( p_f(\theta_k) \) is the value of Discrete Fourier Transform (DFT) of the acoustic signal from the \( k \)th sensor at certain fixed frequency \( f \), such as the blade-passing frequency; \( \theta_k \) is the azimuthal location of the \( k \)th sensor. To suppress the background noise, RMS-averaging method can be used. RMS method averages over several FFT blocks and can be formulized as follows:

\[ A_{mf} = \frac{1}{K^2} \sum_{k=1}^{K} \sum_{l=1}^{K} e^{im\theta_k} C_{kl} e^{-im\theta_l} \]  

(3)

where \( C_{kl} = \frac{1}{2} \langle p(\theta_k)p(\theta_l)^* \rangle \) is the cross-power matrix.

2.2 Compressed sensing based method

Compressed sensing copes with the problem of solving underdetermined systems of linear equations when the solution vector is sparse. Based on the fact that the dominant modes at a certain blade-passing frequency are usually sparse, we can apply compressed sensing method to detect the dominant modes with a significantly smaller number of sensors required by Shannon-Nyquist sampling rate.

Given a sensor array with \( K \) equidistantly spaced microphones along the azimuthal direction, we get \( K \) time-domain acoustic signals. Performing DFT of the \( K \) signals, we obtain \( K \) complex sound pressure signals in frequency domain. Corresponding to a specific frequency, we have the matrix \( p_f = [p_f(\theta_1), p_f(\theta_2), \ldots, p_f(\theta_K)]^T \), where \( \theta_k = \frac{k}{K} \times 2\pi \) is the location of the \( k \)th sensor and \( k = 1, 2, 3, \ldots, K \). Note that equation (2) can be written as a matrix form:

\[ a_f = \Psi p_f \]  

(4)

where \( a_f = [a_{1f}, a_{2f}, \ldots, a_{Mf}] \) is the detected mode result with the highest mode order \( M \); \( \Psi \) is the basis set with elements \( \Psi_{m,k} = \frac{1}{K} e^{im\theta_k} \) and \( m = -\frac{M}{2}, -\frac{M}{2} + 1, \ldots, \frac{M}{2} - 1 \). From equation (4), we have

\[ p_f = \Psi^{-1} a_f \]  

(5)

When the measurement is sub-sampled, i.e., only some of the microphones are used for the entire sensor array. In this case, we can formulate equation (5) as follows:

\[ \tilde{p}_f = \Phi p_f = \Phi \Psi^{-1} a_f = B a_f \]  

(6)
where $\mathbf{B} = \Phi \Psi^{-1}$, and $\Phi$ is the measurement matrix. For example, if the 1st, 3th, 4th, 6th locations are chosen from the $K$ possible sensor locations, the measurement matrix

$$\Phi = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots \\
\end{bmatrix} \in \mathbb{Z}^{4 \times K}
$$

As mode detection is performed at fixed frequencies, the frequency index $f$ will not be denoted in the following for simplifying the presentation. With the fact that the mode vector $\mathbf{a}$ is sparse, we can solve the underdetermined system (6) using $l_1$ norm based compressed sensing method:

$$\min_{\mathbf{a}} \|\mathbf{a}\|_1 \quad \text{such that} \quad \mathbf{B} \mathbf{a} = \tilde{\mathbf{p}}$$

where $\|\mathbf{a}\|_1 = \sum_{i=1}^{K} |a_i|$ is the $l_1$ norm of vector $\mathbf{a}$. It is convex and can be solved by $l_1$ minimization algorithms [15]. Furthermore, to take into account the background noise, the optimization problem is converted to

$$\min_{\mathbf{a}} \|\mathbf{a}\|_1 \quad \text{such that} \quad \|\mathbf{B} \mathbf{a} - \tilde{\mathbf{p}}\|_2 < \epsilon$$

where $\epsilon$ is an empirical threshold related to the background noise. Once $\mathbf{a}$ is solved, the sparse mode is detected, and the complex signal of the whole array $\mathbf{p}_f$ can be reconstructed by calculating $\Psi^{-1} \mathbf{a}_f$.

3. Simulation study

In this section, we will demonstrate that the dominant mode detection can be invariant to signal to noise ratio (SNR) and location deviation at some special sensor positions. This indicates that there may indeed exist positions where the mode detection can be quite robust with compressed sensing method, and thus the simulation-test strategy can be effective as long as the simulation agrees with the experiment set-up, such as the sensor numbers.

To have an overall intuition of the effect of the sub-sampling, we performed a Monto Carlo simulation and process the spacially sub-sampled signals with compressed sensing. We simulate the acoustic signals with the model introduced in Section 2. The sensor array is set to be with 80 equidistantly displayed sensors. The dominant modes are set as $m = -5$ and $m = 28$. Because our aim is to detect the dominant modes, it is quite straightforward to use a “dominance” criteria: if the amplitudes of the modes at $m = -5$ and $m = 28$ are larger than modes at any other mode by 20dB, the detection is classified as successful, otherwise, as failed. At each case with a specific sensor number, we performed 1000 times to avoid the randomness of the sensor location choice. Therefore, we can define the successful rate as follows:

$$r = \frac{N_{\text{success}}}{N} \times 100\%$$

where $N_{\text{success}}$ is the number of successful detection runs; $N = 1000$ is the number of total runs. The result is demonstrated in Figure 1. From the figure we can see that only some of the detection are successful when the sensor number is small. In the following, we will try to find some properties of these successful locations.
Figure 1. Successful rates with respect to number of random selected sensors. The operation number at each fixed sensor number is 1000.

In a practical experiment, the signals are contaminated by various background noise and the sensor installation error may also exist. Therefore, we will explore the influence of the noise and the installation deviation on the sensor location that can induce a successful mode detection.

First, under each specific sensor number 4, 5 and 6, which is quite small, we randomly choose 200 sensor locations in a sensor array with 80 sensors. At each sensor location, we process the signals without noise with compressed sensing method. For the locations that lead to successful mode detection, compressed sensing is then performed for the signals with SNR ranging from -20dB to 20dB with an interval 2dB. Then, the results are evaluated by the dominance of target mode amplitudes as shown in equation (9), where \( N = 21 \) in this context. The successful ratio is shown in Figure 2 (a), where each point corresponds to a specific sensor display.

![Figure 2](image)

Figure 2. Effects of SNR (a) and sensor installation deviation (b) on the mode detection results when the sensor number is 4, 5 and 6, which is quite small. Each point corresponds to a specific sensor display.

From Figure 2 (a) we can see that the successful ratio of some sensor displays are 100%. This means that there indeed exist some location arrangements that are affected by the SNR to little extent. So it is very promising to acquire a good result in a real experiment by finding these location displays with simulations and then apply these arrangements to the sensor array of real experiment measurement.

We also conduct the simulation of signals with sensor installation errors. With respect to the standard location of each sensor, the installation error is assumed to obey a Gaussian distribution with zero-mean, \( \sigma \)-standard deviation. Here, \( \sigma \) is normalized between two adjacent sensors of the equidistantly spaced sensor array, i.e.

\[
\sigma = \frac{\sigma'}{\Delta \theta}
\] (10)
where $\Delta \theta = \frac{2\pi}{N}$ is the normalized angle, $\sigma'$ is the un-normalized value of standard deviation.

The procedure is similar to the process of the noise contaminated signals. We first search the simulated sensor displays where compressed sensing can detect the dominant modes successfully. There is no installation deviation of the simulated signals and the search is conducted by randomly choosing 200 sensor locations from a simulated sensor array with 80 equidistantly spaced sensors. The number of randomly chosen sensors are also set as 4, 5 and 6 separately. For successful locations, compressed sensing is then performed with the signals with installation error from 0.1% to 2.8% with a logarithmic interval $\log_{10} 2$.

Then, the results are evaluated by the dominance of target mode amplitudes as shown in equation (9), where $N = 15$ in this context. The successful ratio is shown in Figure 2 (b), where each point corresponds to a specific sensor display. It demonstrates that similar to the case under various SNR, the successful ratio of some sensor displays are 100%. Although some location arrangements are quite sensitive to the installation error, there still exist many sensor displays that are not affected by the mounting deviation. This, quite naturally, also shows that it is promising to acquire a good result in a real experiment by finding these location displays with simulations and then apply these arrangements to the actual experiment measurement.

4. Experiment study

In this section, we explore the correlation of the successful sensor display between the simulated signals and experimental signals to demonstrate the potential of the simulation-test strategy introduced in Section 1. The model used for simulating signals is the one described in Section 2, and the sensor array is simulated with 32 equidistantly spaced sensors to be consistent with the setup of real experiment. The setup of the experiment with a compressor test rig is shown in Figure 3. It consists of an anechoic chamber, a compressor test section, an inlet casing, a power transmission module and an exhaust module. Figure 4 (a) demonstrates the 32 equidistantly spaced sensor array along the azimuthal direction. The signal acquisition software is programmed with LabVIEW, and the equipment is from National Instruments. Figure 4 (b) (c) and (d) demonstrate the mode detection results at the first blade passing frequency at shaft speed 10000rpm. We detect mode with the conventional method using data collected by the entire sensor array, and the result is displayed in Figure 4 (b). We can see that the mode at $m = 3$ is dominant in this working condition. To find a specific sensor display to apply compressed sensing method to the real experimental data, we randomly choose 5 sensors in the simulated sensor array without noise and installation deviation. The selected sensor arrangement can detect mode successfully and the result is shown in Figure 4 (c). It has a successful ratio 100% for SNR range from -20dB to 20dB with interval 2dB, and a successful ratio 100% for installation error range from 0.1% to 14.87% with logarithmic interval $\log_{10} 2$. The result of mode detection for experimental data with these selected 5 sensors is demonstrated in Figure 4 (d), which shows that the dominant modes are successfully detected. We should note that the conventional method can only detect mode order lower than 3 with sensor number 5.
Figure 3. Schematic of compressor test rig.

Figure 4. Equidistantly spaced sensor array (a) and mode detection results at the first blade passing frequency at shaft speed 10000 rpm (b, c, d). (a) is derived by the conventional method with 32 equidistantly deployed sensor array; (b) is acquired by compressed sensing method with the 5 chosen sensors, in which case the conventional Fourier Transform can only detect the mode order lower than 3.

5. Conclusion

In this paper, we introduced a more straightforward way of applying compressed sensing method to duct mode detection. Besides, this paper also studied properties of the sensor location under various SNR and sensor installation errors with a Monte Carlo numerical study. The simulations reveal that the mode detection with compressed sensing can be invariant to signal to noise ratio (SNR) and location deviation at some sensor positions. This inspires us that we can find the robust sensor positions by a proper simulation and then apply the sensor display to the real experiment. Thus, we proposed the simulation-test strategy and demonstrated its promising potential in the application of a real experiment. However, we should also note that the results in this paper are still a preliminary. A more detailed study, such as the influence of the simulation model and sparsity of the dominant modes, needs to be done in the future.
The more in-depth compressed sensing theory should also be studied in future to guide the choice of the sensor displays when we only use a small number of sensors in a sensor array.

REFERENCES


