OPTIMAL PLACEMENT OF SECONDARY SOURCES IN
ACTIVE STRUCTURAL ACOUSTIC CONTROL OF BAFFLE
USING SIMULATED ANNEALING ALGORITHM

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Simulated annealing algorithm is considered as a relatively better way of obtaining optimal placement of secondary sources in active structural acoustic control (ASAC). In this work, a baffle which is attached to the pipe subject to a harmonically excited point force or plane wave is considered. There are more than $3\times10^7$ possible combinations in this case of choosing eight actuator positions from possible 36 positions on the baffle. The performances of the simulated annealing algorithm are compared in different situations to find the appropriate parameters and updating method for this application. A set of positions which gives an overall reduction on the surface of the hemisphere within 0.3 dB of the best achievable was found through simulated annealing algorithm. Results show that the optimally placed actuators can perform better sound radiation control than the specially selected ones. The use of optimally positioned actuators can efficiently achieve structural sound radiation control.

Keywords: ASAC, secondary sources, baffle, simulated annealing algorithm

1. Introduction

In active structural acoustic control, the placement of secondary sources has an important impact on the active control effect. There are two ways to solve the optimal placement problem: the analytical method and the optimization algorithm. The analytic method is to solve the optimal positions and number of secondary sources after obtaining the analytical expression of the objective function according to the sound field analysis under the condition of simple sound source and simple sound field [1-2]. The advantage of this method is that the physical concept is clear and the solving process is simple, while the disadvantage is that the analytical expression of the objective function is needed. The optimization algorithm solves the optimal positions and number of secondary sources under a certain optimization algorithm through the measured data. The optimization algorithm is suitable for complex sound source and complex sound field, and it is easy to obtain a closer global optimization solution.
The optimization algorithms that have been used include genetic algorithm [3], simulated annealing algorithm [4], subset selection method [5-6], etc. Simulated annealing algorithm is a typical optimization algorithm to solve the optimal placement problem, and shows better performance on such issues, even though the performance is somewhat dependent on the parameters used in the algorithm [7].

The earliest idea of simulated annealing (SA) was proposed by N. Metropolis and others in 1953. In 1983, S. Kirkpatrick [8] successfully introduced annealing ideas into the field of combinatorial optimization. It is a stochastic optimization algorithm based on the Monte-Carlo iterative solution strategy. The starting point is based on the similarity between the annealing process of solid matter in physical and the general combinatorial optimization problem. The simulated annealing algorithm starts from a certain higher initial temperature, with the continuous decrease of temperature parameters, and combines the probability to jump out from the local optimal solution to randomly find the global optimal solution of the objective function in the solution space and eventually tends to be globally optimal. Simulated annealing algorithm is a general optimization algorithm. In theory, the algorithm has the global optimization performance of probability.

This work is based on the model of a baffle structure attached to the pipe opening. The harmonic point force or plane wave excitation is considered. In this case, 36 possible positions on the baffle are selected for choosing eight actuator positions from them. There are more than $3 \times 10^7$ possible combinations in this situation. A set of positions which gives an overall reduction at the error points (hemispherical surface in this case) within 0.3 dB of the best achievable is found through simulated annealing algorithm. Results show that the optimally placed actuators can perform better sound radiation control than the specially selected ones.

2. Simulations

2.1 Piping system model with baffle

![Figure 1: Piping system model](image)

Figure 1 shows the piping system model. The inside of the pipeline and the front of the baffle are all water. The density of water is 1000 kg/m$^3$ and the speed of sound is 1500 m/s. The hemisphere behind the baffle in Fig. 1 is created to simulate spherical wave radiation as in open waters. The pipe material is carbon steel and the Poisson's ratio is 0.3. Its density is 7800 kg/m$^3$ and the Young's modulus is $E_c = 19.2 \times 10^{10}$ Pa. The straight pipe of the pipe structure has a nominal diameter of 200 mm and an outer diameter of 219 mm. The straight pipe near the elastic baffle is 2540 mm long, and the straight pipe far from the elastic baffle is 735 mm long. The outer diameter of the two elbows is 600 mm and the thickness is 9.5 mm. The square baffle has a side length of 2000 mm and a thickness of 32 mm. The elastic baffle is fixed on four sides, as shown in the blue line in Fig. 1. Coordinate origin is located in the coupling center between the pipeline and the baffle. The excitation is applied to the end face of the
pipe as primary source. The amplitude of the sound pressure excited by the plane wave is 630 Pa, and the point force is excited along the negative direction of the z-axis, which is 10N.

The 36 possible positions of actuators on the baffle are shown as Fig. 2. The positions are numbered from one to thirty-six for the convenience of subsequent research. The reason why there are only 36 positions selected is according to the first few modes of the model, we do not need to select more points in low-frequency range below 200Hz that we are interested in. Now the problem is to choose the best several positions for placing the actuators among the 36 positions.

![Figure 2: Possible positions of actuators](image)

2.2 Active control

Every transfer function of the point force to far field (hemispherical surface in this case) sound pressure is calculated, as well as the transfer function of primary source to far field sound pressure. The error function can be expressed by

\[
p = H_p F_p + H_s F_s.
\]

(1)

where \( F_p \) and \( F_s \) represent the excitation force of primary source and secondary source, respectively, while \( H_p \) and \( H_s \) represent the transfer functions of the primary source and the secondary source to the various points on the surface of the hemisphere, respectively. The objective function of the control can be written as

\[
J = p^H p + F_s^H R F_s.
\]

(2)

The matrix \( R \) is a weight matrix of the secondary source \( F_s \), which is used to limit the output of the optimal control force to ensure the robust stability of the controller. Since the number of points on the surface of the hemisphere is much larger than the number of primary excitation source and secondary control force, which means that the dimension of \( p \) is much larger than the dimension of \( F_p \) and \( F_s \), the control system is overdetermined. Therefore, the optimal control force is [9]

\[
F_{so} = -(H_s^H H_s + R)^{-1} H_s^H H_p F_p.
\]

(3)
Figure 3 shows the noise reduction effect of these thirty-six points when each point controls individually, and the excitation frequencies are 50Hz, 100Hz and 150Hz, respectively. The single point control effects under sound excitation and force excitation are given. It shows that the noise reduction effects of the two excitation ways are similar among these frequencies. Therefore, the following is mainly for the optimization of the situation of force excitation.

3. Optimal placement

3.1 Simulated annealing method

According to the Metropolis criterion, the probability that a particle tends to equilibrate at temperature $T$ is $\exp(-\Delta E/(kT))$, where $E$ is the internal energy at temperature $T$, $\Delta E$ is the amount of change, and $k$ is the Boltzmann constant. Using solid annealing to simulate the combination optimization problem, the internal energy $E$ is simulated as the objective function value, in this work $E$ stands for the reduction in the sum of the mean square pressures on the surface of the hemisphere, and the temperature $T$ is evolved into the control parameter related to acceptance rate for a solution. So the simulated annealing algorithm for the solution optimization problem is as follows: starting from the initial solution $i$ and the initial value $T$ of the control parameter, repeating the iteration of "generate new solution $\rightarrow$ calculate objective function difference $\rightarrow$ accept or discard" for the current solution, and gradually attenuating the $T$ value. The solution at the end of the algorithm is the approximate solution obtained, which is based on the Monte Carlo iterative solution method, a heuristic random search process. The annealing process is controlled by a cooling schedule, including the initial value $T$ of the control parameter and its attenuation factor $\Delta t$, the number of iterations $L$, and the stop condition $S$, etc.

3.2 Application of SA

The placement of the actuators is optimized in the case where the number of channels is determined to be eight. Therefore, the problem is simplified after eliminating the influence of the channel number on the optimization result. The key now is to find the most suitable initial temperature $T$, the attenuation factor $\Delta t$ and the updating method used. The following section takes 150 Hz as an example to find the most suitable parameters.

Generate a vector $A$ with one to thirty-six as its elements and sort them randomly. The first eight elements are selected to generate a new vector $B$ as the initial solution. The update of the new solution is to completely update all elements of vector $B$, in this work we call it overall change method. The effect of initial temperature on the performance of the simulated annealing algorithm for the problem of finding optimal 8 secondary source positions from 36 is shown in Fig. 4 and Fig. 5. The results shown in Fig. 4 are all when the attenuation factor is 0.999, while the results shown in Fig 5 are all when the
initial temperature is 1. As can be seen from Fig. 4 and Fig.5, the simulated annealing algorithm has the best performance in this application when the initial value of $T$ is set to 1, and attenuation factor $\Delta t$ is set to 0.999.

![Figure 4: Effect of initial temperature at 150Hz](image1)

![Figure 5: Effect of attenuation factor at 150Hz](image2)

![Figure 6: Effect of updating methods at 150Hz](image3)

Another updating method is used, in which the eighth to thirty-sixth elements of vector $A$ are placed in another vector $C$, each time an element between vector $B$ and vector $C$ is swapped, the regenerated vector $B$ is a new solution, in this work we call it single element change method. Figure 6 shows the
comparison of this updating method with the previous updating method. The results shown in Fig. 6 are all when the attenuation factor is 0.999 and the initial temperature is 1. As can be seen from Fig. 6, the method of completely updating the vector B can achieve better results in this application. But the method to update only one element converges faster.

The comparison of the maximum noise reduction that can be achieved by the exhaust algorithm and the simulated annealing algorithm is shown in Fig. 7. The global optimal noise reduction of the exhaust algorithm at 50 Hz, 100 Hz, and 150 Hz is 30.4dB, 15.7dB, and 17.9dB, respectively, but needs to search all 30,260,340 possible combinations. The simulated annealing algorithm only needs to search 100,000 combinations to obtain the noise reduction of 30.1dB, 15.5dB, and 17.7dB at 50 Hz, 100 Hz, and 150 Hz, respectively, which is within 0.3 dB of the best. In fact, we don’t need to search so many combinations to get the results we want, as long as one tenth of the combination used here is enough. It can be seen from Fig. 3 (b) that the eight points with the largest single point noise reduction for 150Hz are 2, 4, 6, 8, 26, 28, 30, 32, respectively. If these eight positions are used as a solution, the calculated noise reduction is 16.6dB, inferior to the optimal solution obtained by the simulated annealing algorithm 17.7dB. It shows that optimally placed actuators can perform better sound radiation control than the specially selected ones.

Simulated annealing algorithm has also been used to find the optimal eight actuator positions which minimizes the objective function equal to the sum of the mean square pressures on the surface of the hemisphere at the three harmonic frequencies 50Hz, 100Hz and 150Hz. The result is shown in Fig. 8. The eight actuator positions found by the simulated annealing algorithm can achieve noise reduction of 28.1dB, 12.9dB, and 17.0dB at 50Hz, 100Hz, and 150Hz, respectively.
It should be noted that the results from Fig. 4 to Fig. 8 are all averaged results of 20 runs. Since the simulated annealing algorithm is a random search algorithm, there are some differences in the results each time. The actuator positions found in the results of 20 runs are shown in Fig. 9. Figure 9 (a) shows the optimized positions of the actuators found by simulated annealing algorithm at 150 Hz, and Fig. 9 (b) shows positions at three frequencies of 50Hz, 100Hz and 150 Hz. The depth of the colour of each position in the picture represents the number of times this position is selected, the darker the colour, the more times this position is selected. As can be seen from Fig. 9 (a) that the optimized actuator positions found at 150 Hz are concentrated on the diagonal, the most frequently found locations are 3, 7, 11, 15, 19, 23, 27, 31, 34, 36, while the optimized actuator positions found at three frequencies of 50Hz, 100Hz and 150 Hz, which are shown in Fig. 9 (b), are more dispersed as the effects of three frequencies must be considered at the same time.

4. Conclusions

The simulated annealing algorithm shows good performances for the problems of finding the optimal eight actuator positions from 36 positions. The performance is somewhat dependent on the parameters and the updating method used in the algorithm. In this work, the updating method and parameters suitable for this application are found by comparing the performance of the algorithm under different conditions. The simulated annealing program is reliably able to find a set of positions, which gives an overall reduction on the surface of the hemisphere within 0.3 dB of the best achievable. Subsequent experiments will be performed with actuators placed at these positions, which are selected by the simulated annealing algorithm to verify the noise reduction effect.

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REFERENCES


