MODELING AND OPTIMIZATION OF DUCT SILENCER WITH COMPLEX INTERNAL PARTITIONS

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A sub-structuring vibroacoustic approach is proposed to systematically deal with the difficulties and complexities involved in the modeling and optimization problem of duct silencers with complex internal partitions. The mixed separation of rigid/flexible structure with allowed air aperture between coupled acoustic domains is modeled as a compound structural interface, which greatly simplifies the coupling treatment and facilitates the original sub-structuring framework. Calculations using the proposed approach show excellent agreement with FEM analyses and experimental measurements. The flexibility of the method in performing possible system optimization is demonstrated through a number of numerical examples. It is concluded that a proper design of silencer internal configurations can significantly enhance the sound attenuation performance, which is desired for practical industrial applications.

1. Introduction

Expansion chamber silencers as passive noise control solutions are widely used in many engineering systems such as pipes, engines, ventilation ducts, etc. It has been shown that the well-known dome-like transmission loss characteristics of a simple expansion chamber can be altered by the internal configurations of silencer. In the literature, the effects of adding various internal partitions/baffles inside the expansion volume have been extensively investigated. Typical examples include horizontal extensions at the inlet/outlet\(^1,2\), axial baffles\(^3,4\), and flexible side-branch plates\(^5\). In these studies, the modeling of the internal coupled acoustic domains is mainly based on the analytical modal matching approach, which can successfully handle relatively simple silencer configurations. However, with increasing complexities, an effective modeling tool which can eventually allow an accurate estimation and flexible optimization of silencers performance is crucial.

Among a wide range of vibroacoustic modeling techniques, the so-called sub-structuring approaches offer appealing features owing to their modular nature. The strategy is to decouple the global system into uncoupled structural and acoustical subsystems, whose transfer functions can be calculated before the coupling, so that they can be assembled through the connecting interfaces. In such a manner, the coupling treatment is performed through writing rather simple continuity equations, and re-calculation is only required for subsystems with changing parameters for system optimization. In the literature, sub-structuring approaches have demonstrated their capability in modeling various structural-fluid coupling problems\(^6,7\), but their direct applicability for silencer applications involving complex partial partitions is yet to be explored. The challenge mainly comes from the mixed coupling interfaces among these systems, which may comprise both structural
(rigid/vibrating plates) and acoustical (air aperture) components. This forms parallel sound transmission between acoustic media which cannot be described using a unified structural interface.

In order to tackle the difficulty, this paper proposes a convenient treatment of mixed separations/interfaces for silencer applications under the sub-structuring principle, to investigate the effects of several typical silencer configurations and to provide guidelines for possible system optimization at the same time. In the paper, the proposed methodology is first formulated. Numerical examples are then discussed to demonstrate the validity and capability of the proposed formulation. Through visualizing the internal sound pressure field, the associated physical phenomena behind the simulated results will be explained as well.

2. Theoretical Formulation

The detailed formulation procedure of the proposed method, along with the necessary treatment of the subsystem transfer functions, is illustrated using an example of 3-D acoustic silencer with extensions at the inlet and outlet. As shown in Fig. 1, the rectangular silencer consists of an expansion volume separated by four pieces of partial baffles, allowing air apertures to directly connect the side-branch cavities and the main duct. Similar expansion chamber (but circular, concentric type) was studied by Selamet using modal based approach\(^2\), where the internal acoustic domain is divided into five sub-domains with eight sets of continuity equations to be established. With increasing numbers of internal partitions, it is obvious that that approach can hardly entertain.

![Figure 1. An example of duct silencer with inlet/outlet extensions and its sub-structuring treatment using the proposed PTF approach.](image)

As to the sub-structuring modeling, the so-called Patch Transfer Function (PTF) approach is employed in this study, which has been developed and validated through many vibro-acoustic problems\(^6-8\). The decoupling treatment of the global system is illustrated in Fig. 1, where the sub-divided acoustic domains, namely the inlet/outlet ducts and two side-branch cavities, are connected via four separation interfaces to the main chamber. In principle, as long as the pressure-velocity transfer functions (PTFs) for each structural and acoustical subsystem at each coupling interface can be determined, the whole system can be assembled using superposition principle of linear passive vibro-acoustic system. In other word, at the four interfaces numbered from 1 to 4, if a pure vibrating "structural" interface (even in the presence of aperture) can be used to characterize the vibro-acoustic coupling between the connected domains, the overall coupling equations can be established as follows:
where \( Y', V' \) represent the structural mobilities and vibrating velocities of the four vibrating "structural" interfaces, \( Z^d, V^d, Z^c, V^c, Z^\infty, V^\infty \) are the acoustic impedances and velocities of the inlet/outlet duct, main chamber, and two side-branch cavities at the corresponding surfaces. The general meanings and calculations of these quantities have been well-defined in the references\(^6\)-\(^8\).

For conventional subsystems, PTFs such as patch impedances for acoustic cavity and semi-infinite duct, patch motilities for vibrating plate, etc, their calculations have been formulated in detail and are rather straightforward\(^7\). Thus, the remaining challenge mainly comes from the modeling of the mixed separation with the existence of air aperture, which creates parallel (both structural and acoustical) connection between the adjacent acoustic media. The proposed strategy is to find a possible means to consider the aperture as an equivalent or virtual vibrating structure, such that it can be integrated into the separation interface with the real rigid/flexible plate, forming a kind of "compound structure" to describe a mixed separation as a single piece.

The "structuralized" treatment of the air aperture is formulated as follows: consider an aperture connecting the two acoustic domains having a thin thickness of \( L_a \), its internal pressure field can be decomposed into two oppositely propagating acoustic modes:

\[
p_a(x, y, z) = \sum_n a_n^p \psi_n^p (e^{-jk_z z} + \hat{e}^n e^{jk_z z})
\]

where \( z \)-axis depicts the aperture thickness direction; \( a_n^p \) is the \( n \)-th modal amplitude of the aperture, \( \psi_n^p \) the corresponding planar eigenfunctions; \( \hat{e}^n \) the coefficients ratio between the acoustic waves traveling in the positive and negative \( z \) directions; \( k_z^n = \sqrt{(\omega \rho_0)^2 - k_x^2 - k_y^2} \) the corresponding wavenumber in \( z \)-axis, with \( k_x \) and \( k_y \) being subjected to the actual planar eigenfunctions.

The small aperture thickness compared to the acoustic wavelength of interest delimits its velocity fluctuation in \( z \) direction, which allows the pressures at the front and back aperture surfaces to be related using a Taylor's series expansion. Thus, the coefficients ratio \( \hat{e}^n \) for each acoustic mode can be calculated as:

\[
\hat{e}^n = \frac{jk_z^n L_a - 1 + e^{-jk_z L_a}}{jk_z^n L_a + 1 - e^{jk_z L_a}}
\]

By substituting the pressure gradient \( \frac{\partial p_a}{\partial z} = -j\rho_0 \hat{V}_z \) into Eq. (2) and making use of the modal orthogonality property, the modal amplitude response due to a normal pressure disturbance \( p \) at the aperture surface can be calculated as:

\[
jk_z^n L_a a_n^p (1 - \hat{e}^n) N_a^n = \int_{S_a} \psi_n^p \psi_n^p dS
\]

where \( N_a^n = \int_{S_a} \psi_n^p \psi_n^p dS \).

According to the definition of structural patch mobility \( Y' = \frac{\vec{V}'}{F'} \), the equivalent aperture mobility \( Y^n \) can be obtained as:
where \( i, j \) denote the receiving and exciting patches; the angular frequency \( \omega = 2\pi f \); \( s_i \) is the surface area of the sub-divided patches.

It can be seen that the aperture modeling in the present form can be fully viewed as a vibrating panel, whose equivalent structural properties including the panel thickness, density, rigidity, and introduced damping can be easily determined. As to the modeling of the real structures (either rigid or flexible), the structural mobility \( Y^p \) can be easily obtained without any technical challenge. Thus, the dynamics of a mixed separation can be described by combining the plate mobility \( Y^p \) and aperture mobility \( Y^a \) into a "compound structure":

\[
\begin{bmatrix}
  Y^p & 0 \\
  0 & Y^a
\end{bmatrix}
\begin{bmatrix}
  \bar{F}^p \\
  \bar{F}^a
\end{bmatrix} =
\begin{bmatrix}
  \bar{V}^p \\
  \bar{V}^a
\end{bmatrix}
\]

\[
Y^s \times \bar{F}^s = \bar{V}^s
\]

where \( \bar{F}^p \) and \( \bar{F}^a \) are the excitation forces impinging on the plate and aperture surfaces, \( \bar{V}^p, \bar{V}^a, \bar{V}^s \) are the vibrating velocities of the plate, aperture and compound structure. The brought benefit from the present formulation is obvious: different combinations of rigid/flexible structures and apertures among a mixed interface can be treated as a single compound subsystem, thus the modular sub-structuring framework can be maintained with the modeling effort being reduced.

After the unknown subsystem velocities at the four interfaces are solved, the sound transmission loss for assessing the silencer's noise attenuation performance can be calculated as

\[
\text{TL} = 10 \log_{10} \left( \frac{1}{\tau} \right)
\]

where \( \tau \) is the ratio between the transmitted and incident sound power. The incident sound power \( \Pi_i \) corresponding to a normal plane wave excitation with pressure amplitude \( p_0 \) is:

\[
\Pi_i = \frac{|p_0|^2}{2 \rho_s c_0} \sum_{i=1}^{N} s_i
\]

where \( N \) is the number of total patches among the incident surface, and the transmitted power \( \Pi_t \) is:

\[
\Pi_t = \frac{1}{2} \sum_{i=1}^{N} \left( \Re \left( Z^d V_i^d \right) \right) s_i
\]

The asterisk denotes the complex conjugate of the patch velocities.

3. Numerical Simulations

The proposed sub-structuring formulation with the flexible treatment of mixed interface allows a systematic investigation on the influences of different internal configurations. This section presents numerical simulations based on several typical silencer examples. As modeled in Section 2, acoustic silencer with side-branch partitions, or referred as extended inlet/outlet\(^2\) when the partitions are short, involves many important physical phenomena that should be recognized in the initial stage of the silencer design.

In Fig. 2 (a), the main chamber of an expansion silencer is partially separated from its side-branch cavities by four pieces of extension baffles. Due to the introduced horizontal partitions, the affected internal acoustic field will lead to a completely different Transmission Loss (TL) behavior. For three different cases, the calculated TL curves using the proposed approach are compared to that of an empty chamber. Note that the accuracy of the calculations have been fully verified against
FEM analyses, which are not presented here for the sake of clear comparison. It is observed that the effect of the extensions generally increases the TL performance by adding strong attenuation peaks to the original dome-like TL. The generating mechanism of these peaks is most likely due to the resonator effect of the side-branch cavities, which yields resonances at $f_r = n c_0 / 4 L_{sc} \ (n = 1, 3, 5 \ldots)$ under 1-D plane wave assumption ($L_{sc}$ is the horizontal length of the side-branch). As seen from the simulated TLs, this prediction is valid when the frequency range is low enough compared to the cut-off frequency, where the 1-D model can possibly be applied to tune these resonance positions for strong narrow-band noise control performance. While at higher frequencies, the propagation of the higher-order acoustic modes causes failure to the plane wave model, thus requiring three-dimensional modeling tools for more accurate prediction.

![Figure 2](image1.png)

**Figure 2.** Numerical simulations: (a) TLs of silencers with side-branch partitions; (b) TLs of silencers with axial baffles.

Figure 2 (b) considers another example where the added partial partitions are axially located inside the main chamber, leaving an air passage for ventilation purpose. The predicted TL for a pair of baffles in the middle ($d=L/2$) shows that the influences on the periodic domes mainly come “in pairs”: the first TL dome in each pair is narrowed and weakened, while the second dome is widened and increased. To explain the shifted first TL trough from 550Hz to 470Hz, the internal pressure field is visualized in Fig. 3. It is seen that the air volume inside the chamber forms a slightly bended conduit around the baffle and the chamber wall. As a result, the characteristic wavelength of the first non-zero chamber mode is lengthened to around 0.7m, as opposed to 0.6m for the empty chamber. While for the second trough remained at 1150Hz, the baffles are located at the original maximum pressure (negative) line, thus exerting no influence on the pressure field.

![Figure 3](image2.png)

**Figure 3.** Visualized internal pressure field for silencer with a pair of axial baffles at its first TL trough, $f=470$Hz.

The above two cases focus on relatively simple configurations, while the capability of the proposed formulation is to be demonstrated through more challenging examples. Figure 4 presents
several complex silencers with different combinations of partial partitions inside the expansion chamber: Case.1 as the benchmark is the same as the one with two pairs of axial baffles as presented in Fig. 2; Case. 2 adds extensions (0.08m) at the inlet; Case.3 adds two more horizontal partitions inside the second and the third sub-chamber. It is obvious that the three-dimensional modeling of such systems using traditional modal approach is very tedious, while the present modular treatment provides a more convenient tool.

Comparing to Case.1, the two cases with added horizontal partitions exhibit extra TL peaks at different frequencies, and the zero-attenuation troughs at several positions are lifted-up with widened attenuation bandwidth. These two effects generally contribute to a better silencing effect. On the other hand, the pressure drop related to the flow performance of such silencer can also be improved by introducing side-branch partitions. It is noted that the parameters used in the present simulations are randomly selected, while properly tuning these parameters will further improve the performance of such silencers to satisfy the actual working requirement. As to the calculation efficiency, it is worth mentioning that the calculation time for each case is less than one minute for frequency range below 3000Hz, with the calculation convergence and accuracy can be fully guaranteed. Therefore, the computational cost for optimal silencer design using the proposed method is rather low.

Figure 4. TL calculations for several complex silencer configurations with combined internal partitions.

4. Experimental Validation

In order to further validate the proposed modeling framework, the TL of a rectangular silencer, consisting of dual expansion chambers with partially covered side-branch cavities, is measured experimentally and compared to the numerical predictions using the proposed PTF approach. As illustrated in Fig. 5, the measurement is conducted using the four-microphone method, where two pairs of 1/2-in. phase-matched microphones (B&K 4187) are used at the upstream and downstream, together with a conditioning amplifier (B&K Nexus 2691), to measure the acoustic pressures inside the duct. The cross-section of the inlet/outlet duct is 100mm x 100mm, and two independent experiments with different downstream loading conditions are used to simulate physical anechoic termination. For the expansion chamber, two side-branch cavities with a height of 100mm and length of 300mm are connected to the mainstream, where a pair of axial baffles is placed in the middle with 160mm long horizontal partitions. The axial baffles made of 5mm-thick steel plates are sealed with rubber, and the four edges of the 1mm-thick horizontal partitions are inserted into the
thin gap between the duct and side-cavity walls. The data acquisition system is controlled by an NI Labview program, running from 50Hz to 2000Hz with an increment of 10Hz. The measured TL curve is presented in Fig. 5 (b).

During the simulation, all the partition walls are assumed to be acoustically rigid at the first stage, and the damping loss factors for acoustic domains are taken as zero. In Fig. 5 (b), the predicted TL is plotted in comparison to the experimental result, where the two curves show excellent agreement along the frequency range, except for some noticeable discrepancies around the first TL peak. In the presence of the internal partitions, the TL level is significantly increased compared to that of an empty expansion chamber (maximum TL ≈ 5dB), while a strong stop-band from 300Hz to 700Hz is observed from the computational result with maximum TL=90dB at 600Hz, which could be particularly useful in performing narrow-band noise control.

As to the discrepancy between the two curves, the strong TL peak in the simulation is decreased and separated into two sub peaks in the experiment, which is possibly due to the vibration of the thin horizontal partitions being used. Since the present approach is capable of dealing with panel vibration among a mixed separation, the effect of adding flexural vibration to the horizontal partitions is also studied. The partitions are assumed as simply supported along the edges for the sake of simplicity, while the actual clamped boundary conditions can be simulated but considered to be beyond the major scope of the study. When the panel vibrational effect is accounted, the calculated TL as presented in Fig. 5(b) shows better agreement with the experimental curve, while the remaining small discrepancies may be explained by the neglected fluid and structural damping in the simulation, and geometric imperfection in the experiment. The first TL peak is split into two smaller ones due to the coupling of the structural resonances, which could deteriorate the maximum attenuation performance that can be achieved when the narrow but strong stop-band is required. This suggests that not only the internal layout should be considered, but also the structural vibration should be noticed at some critical frequencies during practical silencer design.

5. Conclusions

A systematic sub-structuring formulation based on the Patch Transfer Function (PTF) approach has been proposed to deal with reactive expansion chamber silencer involving complex internal partitions. The method is shown to be more flexible and versatile compared with other analytical modeling tools such as modal based approach. The coupling between the subdivided acoustic domains through mixed interfaces is facilitated using a compound structural subsystem, with an equivalent structuralized treatment for air aperture. The proposed formulation has been applied to investigate the effects of several typical silencer configurations, including complex cases with
combined internal partitions, where the periodic TL pattern of an empty expansion chamber is shown to be greatly affected. The predictions are always shown to be consistent with FEM results throughout the analyses, and the computational cost using the proposed formulation is greatly reduced due to the modular treatment. The numerical simulations also suggest some basic guidelines for possible improvement of a silencer's performance, where considerable space exists for performance optimization while the design cost using the present formulation is very low.

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References