A COMPARISON BETWEEN ADAPTIVE ANC ALGORITHMS WITH AND WITHOUT CANCELLATION PATH MODELLING

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The adaptive filters in active noise control (ANC) systems differ from other common adaptive filters in the existence of the cancellation path, which is the transfer function between the outputs of the adaptive control filters and the error sensors. Cancellation paths play a critical role in active noise control systems, and the corresponding adaptive algorithms usually require the information of the cancellation paths for updating the control filters. The most commonly used filtered-x LMS algorithm takes into account the cancellation paths by filtering the reference signal with an estimate of the cancellation path transfer functions. For many ANC applications, the cancellation path modelling must be carried out online to maintain the stability of the system, and one modelling method obtains the cancellation path information by injecting uncorrelated signal into the cancellation path. This paper will introduce the filtered-x LMS algorithm embedded with this online cancellation path modelling and the direction search LMS algorithm, which is one of the ANC algorithms that do not need an explicit model of the cancellation path. In the direction search LMS algorithm, the standard LMS algorithm is adopted to update the adaptive filter coefficients directly with the reference signal by automatically choosing a proper update direction based on the monitoring of the excess noise power. The performance of the two typical adaptive ANC algorithms, one with and one without cancellation path modelling, will be compared in terms of noise reduction level, tracking speed, computation load and robustness.

1. Introduction

It is well known that cancellation paths play a critical role in active noise control (ANC) systems because adaptive algorithms require the cancellation path response for updating the control filters.\textsuperscript{1-3} For example, the filtered-x LMS (FxLMS) algorithm takes into account the cancellation paths by filtering the reference signal with an estimate of the cancellation path transfer functions, while the filtered-e LMS algorithm involves filtering of the error signal with a time-reversed version of the cancellation path impulse responses. Although these algorithms do not need a very accurate model of the cancellation path to maintain the stability of the system, a fast and good estimation of the cancellation path transfer function does benefit convergence speed of the system because a larger convergence coefficient can be adopted with a better cancellation path modelling.

Usually the cancellation path transfer function varies with time, and the variations can be classified into three types. For the first type, where the variation of the cancellation path transfer
function is within a small range, the system does not need to do cancellation path modelling online. The model obtained offline can be used to update the control filter weights all of the time. For the second type, where the cancellation path transfer function varies rapidly by a large amount, the FxLMS algorithm might not be suitable if it cannot determine the correct cancellation path transfer function model in a short time. The last type is the most commonly met situation in practice, where the change is slow or irregular, but extensive over along period of time. Under this situation, the control system must implement the cancellation path modelling online or at regular intervals in order to maintain the stability of the system.

There are many methods for online cancellation modelling. For example, the modelling method that injects uncorrelated signal into the cancellation path obtains the cancellation path information by using a standard system identification approach, the modelling method that uses the control signal obtains the cancellation path information by using an extended system identification approach, while the simultaneous equation method-based modelling obtains the cancellation path information by observing the residual error signal change of the whole active control system caused by the change of the control filter coefficients. All these cancellation path modelling methods establish an explicit cancellation path model, which is used in the adaptive algorithms to generate the related multiplying signal for LMS update.

There are also some active control algorithms that do not need an explicit model of the cancellation path. Some of them are only applicable to certain conditions, some are quite complicated and have large computation load. Only one kind of algorithm that uses update direction searching strategy will be introduced in this paper due to its simplicity for implementation and reasonable good performance. In the so-called direction search LMS algorithm, instead of the FxLMS algorithm, the standard LMS algorithm is adopted to update the adaptive filter coefficients, where the reference signal does not need to pass through the cancellation path. The adaptive filter coefficients converge to the optimal values if the phase angle of the modelled cancellation path is within $90^\circ$ of that of the true cancellation path. If the phase angle of the cancellation path is outside of the range of $90^\circ$, the adaptive filter coefficients can still converge by changing the sign in front of the step size. The proper update direction of the adaptive filter coefficients (the proper sign in front of the step size) is chosen automatically by monitoring the excess noise power with the algorithm.

This paper will describe two typical adaptive ANC algorithms, one with and one without cancellation path modelling, and compare their performance in terms of noise reduction level, tracking speed, computation load and robustness.

2. Description of the algorithms

A block diagram of the FxLMS algorithm with online cancellation path modelling is shown in Fig. 1, where the cancellation path is modelled by injecting uncorrelated random noise, $u(k)$, into the system. $x(k)$ is the reference signal from the primary disturbance and $P(z)$ is the primary path transfer function (structural/acoustic system) between the primary disturbance, $p(k)$, and $x(k)$. The actual control signal at the position of the error sensor results from filtering the output of the controller, $y(k)$, with the physical cancellation path transfer function, $C(z)$, which is modelled by an $M$ order FIR filter, $\hat{C}(z)$. The update equation for the cancellation path model is obtained by using,\(^3\)

$$\hat{c}_m(k+1) = \hat{c}_m(k) + 2\mu_c u(k-m) \varepsilon(k).$$  \hspace{1cm} (1)

where $\mu_c$ is the convergence coefficient for the cancellation path modelling, and

$$\varepsilon(k) = e(k) - \sum_{m=0}^{M-1} \hat{c}_m(k) u(k-m).$$  \hspace{1cm} (2)

The adaptation equation of the FxLMS algorithm for the $i^{th}$ control filter coefficient is: \(^3\)
where $\mu_w$ is the convergence coefficient for the control filter, and

$$x_i(k) = \sum_{m=0}^{M-1} \hat{c}_m(k) x(k-m).$$

One problem of the above algorithm is that an additional noise is added into the entire system, so it sets a lower limit of the disturbance reduction that the active control system can achieve. If online cancellation path modelling has to be carried out in the presence of the primary disturbance, the modelling signal generated at the error sensor usually should be larger than the primary noise to have a fast modelling. Although the modelling signal generated at the error sensor can be lower than the primary noise, the convergence coefficient for the cancellation path will be much smaller, resulting in a very slow modelling of the cancellation path.

Fig. 1. Block diagram of the FxLMS algorithm with cancellation path modelling.

Fig. 2 shows a block diagram of the active control algorithm without cancellation path modelling based on the direction search LMS algorithm.

Fig. 2. Block diagram of the direction search LMS algorithm for active control.

The adaptation equation for the control filter is written as:

$$w_i(k+1) = w_i(k) - 2\mu_w x_i(k-l) e(k).$$

(3)
\[ w_i(k+1) = w_i(k) - 2\mu_i x(k-l) e(k). \]  

(5)

where \( \mu_k \) is the convergence coefficient for the control filter and \( x(k) \) is the reference signal.

Unlike the FxLMS algorithm, the reference signal, \( x(k) \), is used directly to update the control filter coefficients without pre-filtering with a model of the cancellation path. Another difference with the FxLMS algorithm is that there is an extra module, referred to as the update direction search module, which is used to find the correct direction for the update of the LMS algorithm. The significance of the algorithm is its update direction search module, which changes the sign of the convergence coefficient by observing the amplitude change of the residual error signal. Fig. 3 shows the flowchart of the direction search LMS algorithm for active control, where the algorithm can be divided into four stages: the initialisation stage, the direction search stage, the control filter update stage and the performance monitoring stage.

![Flow chart of the direction search LMS algorithm for active control.](image)

The direction search LMS algorithm for active control is introduced with the following 8 steps, where step 1 is at the initialisation stage, steps 2 to 4 belong to the direction search stage, step 5 (and only step 5) belongs to the control filter update stage, and steps 6 to 8 are associated with the performance monitoring stage.

**Step 1:** Initialise the control filter, the number of the samples, \( N \), that are to be used for estimating the residual disturbance error power, the convergence coefficient, \( \mu \), the fluctuation factors, \( \delta_1 \), \( \delta_2 \), and the variation factor \( \gamma \).

**Step 2:** Freeze the control filter coefficient update for \( N \) samples and calculate the mean residual error power during the \( N \) sample period, \( \zeta_1 \), calculate the maximum residual error amplitude, \( e_{\text{max}} \), and calculate the mean reference signal power \( \chi_1 \).

**Step 3:** Start the control filter coefficient update using Eq. (5), and calculate the mean residual error power, \( \zeta_2 \), and the mean reference signal power, \( \chi_2 \) for another \( N \) samples. At the same time, monitor the amplitude of the residual error signal.

**Step 4:** Change the sign of the convergence coefficient, \( \mu \), if \( \zeta_2 / \chi_2 > \zeta_1 / \chi_1 \), or \( |e(k)| > (1+\delta_2) e_{\text{max}} \).

**Step 5:** Perform the control filter coefficient update using Eq. (5).

**Step 6:** Initialise \( n = 0 \), \( \zeta(0) = \zeta_2 \), \( \chi(0) = \chi_2 \).

**Step 7:** Calculate the mean residual error power, \( \zeta(n) \), and the reference signal power, \( \chi(n) \), using moving average.

**Step 8:** If \( \zeta(n)/\chi(n) > (1+\delta_1)\zeta(n-N)/\chi(n-N) \), or \( \zeta(n)/\chi(n) > \gamma \zeta_1/\chi_1 \), go to Step 2 and redo the direction search; otherwise, go to Step 5 and continue to update the control filter coefficients.

The algorithm begins by initialising the convergence coefficient with a sufficiently small positive value, and then the excess disturbance power is observed. If the disturbance power increases, indicating that the control filter coefficients are moving to increase the error, then the sign of the convergence coefficient is changed. After determining the correct direction, the control algo-
Algorithm has a structure similar to that of the LMS algorithm, but the reference signal does not need to be processed by the estimated cancellation path.

It should be noted that in the above direction search LMS algorithm, there are only two choices, 180° and 0°, for the update direction, which is implemented by changing the sign in front of the step size. If the phase response of the secondary path is close to ±90°, the algorithm will converge very slowly. To solve the problem, a modified algorithm, called the Quad Direction Search LMS algorithm for active control has been proposed, where there are four choices 180°, 0°, and ±90° for the update direction. The modified algorithm is implemented in the frequency domain, and its steps are almost the same as those for the original direction search LMS algorithm except for steps 3 and 4 at the direction search stage.

The direction search algorithms mentioned above were first proposed for a single tonal primary disturbance cancellation. The multi-tonal primary disturbance control problem can be converted into several single tonal active control problems. For each single tonal active control problem, independent parameters are adopted, and then the update direction for each single tone is judged by monitoring its own error signal. For narrowband primary disturbances, the algorithm is similar to the algorithm for a single tonal primary disturbance. The broadband disturbance control problem can be converted into several narrowband active control problem where the bandwidth of each narrow band can be selected to be sufficiently narrow to use the algorithm. Fig. 4 shows the block diagram of the broadband ANC algorithm without the cancellation path modelling based on the subband structure.

![Figure 4](image)

**Figure 4.** Block diagram of the broadband ANC algorithm without cancelation path modelling.

The system consists of 5 parts, which are the modified reference signal generation, the subband signal generation, the subband LMS update direction search, the full-band adaptive coefficient update, and the full-band control signal generation (control filtering). The number of subbands depends on the properties of the cancellation path transfer function, which should be sufficient large to ensure the maximum phase change of the transfer function in each subband is less than 90°. To reduce the number of subbands, the group delay of the cancellation path can be roughly estimated first, which is used to generate the first modified reference signal by delaying the reference signal with the same amount of delay. The second modified reference signal is obtained by using the Hilbert transform to generate a signal which is 90° out of phase with the first modified reference signal.

The subband signal can be calculated by bandpass filtering, frequency shifting, and down sampling the full-band signal. For example, the qth subband signal \( e_q(k) \) can be calculated from \( e(n) \) as.
\begin{equation}
    e_q(k_s) = \sum_{k=0}^{K_s-1} a_k e^{-j2\pi \frac{q}{Q} k} e(Dk_s - k).
\end{equation}

where \( k_s \) is the subband index, \( D \) is the down sampling rate, \( a_k \) are the coefficients of a \( K_L \) point low pass prototype FIR filter and \( K_L \) usually is larger than the number of subbands \( Q \) to avoid aliasing. The calculation complexity for all subband signal generation can be reduced by using the polyphase FFT method.\(^{12}\)

In each subband, the update direction is searched by using the same method mentioned in Fig. 3, and the adaptation equation for the \( l \)th control filter coefficient of the control filter to minimise the summation of the squared error signal over all subbands can be written as:

\begin{equation}
    w_l(k_s + 1) = w_l(k_s) - 2 \sum_{q=0}^{Q-1} \mu_q \text{Re}\{e_q(k_s) R_q^*(k_s - l)\}.
\end{equation}

where \( \mu_q \) is the convergence coefficient for the \( q \)th subband, which can be \( \mu \) or \(-\mu\), \( R_q^*(\cdot) \) is the conjugate of \( R_q(\cdot) \), which can be the \( q \)th subband signal of the first or second modified reference signal, all depending on the direction search results at the \( q \)th subband. It should be noted that all subbands might be able to be processed in parallel to search its own update direction and modified reference signal. Once the update direction and modified reference signal are determined, the algorithm converges just like the common LMS algorithm, and no direction search is needed until the monitoring process detects a continuous increase of the residual noise level.

3. Comparisons

Table 1 shows the average number of real multiplications required per input sample to implement three typical ANC algorithms, where ‘FxLMS’ refers to the FxLMS algorithm mentioned in Fig. 1, ‘Subband FxLMS’ refers to the FxLMS algorithm based on the delayless subband filtering,\(^{12}\) and ‘Subband LMS’ refers to the broadband ANC algorithm without cancelation path modeling described in Fig. 4. The computational load for the online cancellation path modelling is not included in the table; however, it follows the same trends as for the FxLMS algorithm, and can be estimated by removing the contribution of the filtered reference signal generation part from that of the FxLMS algorithms.

<table>
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<th>Table 1. The average number of real multiplications required per input sample for 3 ANC algorithms.</th>
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<td>FxLMS</td>
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<td>Subband FxLMS</td>
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<td>Subband LMS</td>
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In Table 1, the lengths of the control filter and the cancellation path model are both assumed to be \( L \). For the subband algorithms, it is assumed that the length of the prototype filter is \( K_L \), the down sampling rate is \( D \), and the number of subband is \( Q \). For each subband signal (reference and error signals) generation, \( 2(K_L + Q \log_2 Q)/D \) real multiplications are needed per input sample, where it is assumed that \( 2Q \log_2 2Q \) real multiplications are required for a \( 2Q \) point FFT. For each subband, the complex filtered reference signal generation and the complex LMS update each needs \( 4L/D \) multiplications per input sample. Because the input signals are real, only \((Q/2+1)\) complex subbands need to be processed per \( D \) samples. For the subband to full-band weight transformation in the subband FxLMS algorithm, \( K_L + Q \log_2 Q \) multiplications are needed per \( D \) samples.
It can be seen from Table 1 that significant computation load saving can be obtained with the broadband ANC algorithm without cancelation path modelling. For example, for an ANC system with \( L = 4096, K_L = 128, D = 16, \) and \( Q = 32, \) the average number of real multiplications required per input sample for control signal generation (control filter filtering) for all 3 ANC algorithms are the same number of 4096, but the number of real multiplications for the control filter update (including the filtered-x signal generation, control filter update and the subband/full-band transformation) are 8192, 2102 and 1060, respectively. The saving will become more significant with multi-channel ANC systems. The main reasons for the computational complexity reduction for the broadband ANC algorithm without cancelation path modelling are the use of block processing via FFT, removal of the filtered-x signal generation and update of the control filter at a lower rate.

Because no explicit cancelation path transfer function is involved in the broadband ANC algorithm without cancelation path modelling, there is also a significantly saving of on-chip memory in Digital Signal Processing (DSP) systems while implementing the algorithm. Furthermore, because there is no need to program for cancellation path modelling and the commonly used LMS algorithm can be used in the implementations, the ANC algorithm described in Fig. 4 is relatively simple and compact for implementation and is especially suitable for low cost mass production applications.

Although the broadband ANC algorithm without cancelation path modelling requires considerably fewer computations and offers greater configuration simplicity, it does not converge toward the optimum value in the quickest manner as no cancellation path transfer function is used.\(^\text{10}\) Fig. 5 shows the experimental results of the two algorithms carried out in an active noise control system in a duct where the terminal condition at one end of the duct changes from absorption, open, close, open to absorption again.\(^\text{11}\) For the FxLMS algorithm, the cancellation path model used in the control filter update is the one with the absorption end, so the noise reduces quickly in the first and the fifth stages (absorption end), but does not attenuate anymore from the second to the fourth stages (open, close and open ends). For the broadband ANC algorithm without cancelation path modelling shown in Fig. 4 (named ‘Subband’ in Fig. 5), 4 subbands are used to control the band limited noise from 100 Hz to 500 Hz. It is obvious that the algorithm can converge under all terminal conditions. Although its convergence speed may be slower than the FxLMS algorithm with the correctly estimated cancellation path model, the noise reduction it achieved is almost the same as the converged FxLMS algorithm.

![Figure 5. The mean square error of an ANC system in a duct with the FxLMS algorithm (FxLMS) and the broadband ANC algorithm without cancelation path modelling (Subband) under different terminal conditions from absorption, open, close, open to absorption again, where for the FxLMS algorithm, the cancellation path model used in the control filter update is the one with the absorption end.](image-url)
Applying subband techniques might bring faster convergence to the ANC system because the dynamic range is greatly reduced in each subband and each subband can have its own convergence coefficients. The disadvantage is that although there is no delay in the control signal generation path, there is a delay at the control filter update path. Unlike the FxLMS algorithm which updates every sample, the subband LMS algorithm updates every $D$ samples. In some applications where the primary or secondary path changes rapidly, the subband FxLMS may exhibit inferior performance. The robustness of the two algorithms should be similar, but depends on the specific properties of the cancellation path. An analytical analysis on the stability of the algorithms is still not available at present.

4. Conclusions

This paper describes the filtered-x LMS algorithm embedded with an online cancellation path modelling method which obtains the cancellation path transfer function by injecting uncorrelated signal into the system, and then compares it with the direction search LMS algorithm, in which the standard LMS algorithm is adopted to update the adaptive filter coefficients directly with the reference signal by automatically choosing a proper update direction based on the monitoring of the excess noise power. It is found that the broadband ANC algorithm without cancelation path modelling described in the paper (the direction search LMS algorithm) has much simpler configuration and lower computation load than the FxLMS algorithm. Experiment results confirm that although the convergence speed of the direction search LMS algorithm may be slower than that of the FxLMS algorithm with the correctly estimated cancellation path model, the noise reduction achieved is almost the same as the converged FxLMS algorithm. The direction search LMS algorithm is relatively simple and compact for implementation and is especially suitable for low cost mass production applications.

REFERENCES