OPTIMAL VIRTUAL MECHANICAL IMPEDANCES APPROACH FOR THE VIBROACOUSTIC ACTIVE CONTROL OF A PANEL

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In order to reduce the acoustic power radiated by a structure, co-located actuator and sensor pairs are used to virtually implement local mechanical impedance. For each pair, at each frequency of interest, the virtual mechanical impedance is a complex value which imposes a linear relationship between the dual local variables of the actuator input and the sensor output. The optimal virtual mechanical impedances are defined at each frequency as the ones which minimize the sound power radiated by the structure. From a theoretical point of view, they can be computed from the primary sound field and the transfer functions between the control actuators, structural sensors and far-field acoustic sensors. The proposed experimental approach is a two-step process: (1) measurement and identification of transfer functions to compute the optimal virtual mechanical impedances; (2) implementation of the computed virtual mechanical impedances with a real-time MIMO harmonic controller. Measured and computed results are presented for a simply supported thin plate (aluminum). Special focus is put on the discussion about such control approach versus a classical active damping where the virtual mechanical impedance is defined as real positive.

1. Introduction

As it combines high stiffness and low mass, the use of composite material in aeronautic structures leads to a decrease of the insulation of external noise, particularly at low frequencies. At these frequencies, active control appears as an effective way to reduce the acoustic power radiated inside the aircraft. Among different approaches, Active Structural Acoustic Control (ASAC) is widely studied. A challenging problem is then to estimate the acoustic radiation from local vibration measurements. For simple structures, radiation modes estimation and control [7] or wavenumber sensing [8] allows for attenuating the acoustic power radiated. In both cases, a centralized system with many control units is required. Decentralized control using distributed and independant devices reduces complexity, cost and increases the robustness to individual control unit failure. Most of previous work on decentralized control tends to achieve active damping using electrical shunts [3] or velocity feedback
control [4], assuming that most of the time, the reduction of structural kinetic energy leads to a radiated acoustic power attenuation. The use of dual and co-located actuator-sensor pairs guarantees a passive system which is stable in closed loop. Moreover, the dual co-located input-output allows to implement mechanical impedances [2]. The virtual mechanical impedance can be defined from a lumped parameter model (damper, mass, ...) or, as considered here, a suitable frequency response may be imposed as the target mechanical impedance [1]. This study presents a method to compute the optimal frequency dependence of the virtual mechanical impedance in view of sound power minimization. The practical implementation of the virtual impedance approach to simple structures is then detailed. Such an optimal control strategy is compared with classical active damping where the coupling between the vibration and the acoustic radiation of the structure is not taken into account.

2. Virtual mechanical impedance approach

A plate equipped with \( N_u \) control units, each composed of an actuator and a sensor is considered (Figure 1). At the angular frequency \( \omega \), the matrix \( H \) of transfer functions between actuator inputs and sensor outputs is fully populated. If \( u \) denotes the vector of actuator input signals, the resulting vector \( y \) of sensor output signals can be expressed as:

\[
y(\omega) = H(\omega)u(\omega) + d(\omega),
\]

where \( d \) is the vector of primary signals \( (y|_{u=0}) \). In the following, the primary disturbance is assumed to be an oblique acoustic plane wave \( P_{inc} \) impinging the panel. The choice of a force/velocity pair at the same point leads to a hamiltonian port where energy exchange can be controlled [2]. The transducer technology considered here is thus a unit composed of an inertial actuator and an accelerometer, the signal of which is time-integrated to provide local transverse velocity. The principle of the virtual impedance method is to locally control the ratio between the sensor output and actuator input of each pair. The control objective is thus expressed under the form of an imposed diagonal complex-valued matrix \( Z \) such that:

\[
Z_{ii}(\omega) = -\frac{u_i(\omega)}{y_i(\omega)} \quad \text{and} \quad Z_{ij}(\omega) = 0, \quad i \neq j
\]

When realizing classical active damping, a real positive gain \( K \) is chosen at each frequency for each unit: \( Z_{ii}(\omega) = K \). In the following sections, \( Z \) is derived in order to minimize the acoustic power \( W > 0 \) radiated by the panel in the receiving half-space. The purpose is then to find \( Z^{opt} \) the diagonal matrix of optimal virtual mechanical impedances such that:

\[
Z^{opt}(\omega) = \arg \min_{Z(\omega)} W(Z(\omega))
\]
3. Numerical simulations

3.1 Approach

Numerical simulations of the virtual impedance approach for minimization of the acoustic power $W$ radiated by a plate are presented hereafter. A cost function $J$ is derived from $W$, involving uniform penalization of the control effort [5]:

$$J(u) = W + \beta u^H u,$$

where $\beta \in \mathbb{R}^+$ is the regularisation parameter and $(\cdot)^H$ denotes the hermitian. The optimal control input $u^{opt}$ is searched as the solution of $\nabla_u J(u^{opt}) = 0$. The $Z^{opt}$ diagonal matrix of optimal virtual impedances is thus derived as:

$$Z_{ii}^{opt} = -\frac{u_{i}^{opt}}{y_{i}^{opt}}$$

This approach allows defining a specific target for each control unit, assuming that the acoustic power radiated $W$ can be expressed as a function of the control input $u$ in order to calculate (4).

3.2 Vibroacoustic model

The simulated simply supported plate characteristics are presented in Table 1. The excitation is harmonic with angular frequency $\omega$. Under pure bending assumption, the actuator-plate-sensor transfer matrix $H$ is classically expanded over the structural modes:

$$H = \text{SMA}$$

where $M$ is the $N_m \times N_m$ diagonal matrix of modal admittances and $N_m$ is the modal truncation index. Also, $A$ is the $N_m \times N_u$ matrix that represents the coupling between inertial actuators and structural modes. At frequencies for which the bending wavelength is larger than the dimension of the inertial actuators, they can be modeled as punctual transverse force. The $N_u \times N_m$ matrix $S$ represents the coupling between structural modes and colocated sensors. The $(m, n)^{th}$ component of $A$ and $S$ are thus:

$$A_{m,n} = c^A \sin\left(\frac{m \pi}{L_x}x_{u,n}\right) \sin\left(\frac{m \pi}{L_y}y_{u,n}\right), \quad S_{n,m} = c^S \frac{1}{j\omega} \sin\left(\frac{m \pi}{L_x}x_{u,n}\right) \sin\left(\frac{m \pi}{L_y}y_{u,n}\right)$$

where $c^A$ is the actuator transduction coefficient (Newton / Volt), $c^S$ the sensor transduction coefficient (Volt / acceleration), $(x_{u,n}, y_{u,n})$ are the coordinates of the $n^{th}$ control unit and $m_x$, $m_y$ are strictly positive integers. Denoting $q$ the vector of modal amplitudes, (1) can be written under the form of the following system:

$$\begin{cases}
M^{-1}q = Au + b \\
y = Sq
\end{cases}$$

where $b$ is the coupling vector between the primary field and the structural modes ($d = Sb$). The radiated acoustic power $W$ can then be written as a quadratic function of $q$.

$$W = q^H \Omega q,$$

where $\Omega$ is the modal radiation resistance matrix [6]. Replacing (9) in (4), such that the optimal control input satisfies $\Delta_u J(u^{opt}) = 0$, one finds

$$u^{opt} = -(A^HM^H\Omega MA + \beta I_d)^{-1}A^HM^H\Omega Mb,$$
Table 1. Mechanical data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, Width, Thickness</td>
<td>( L_x = 48 \times 10^{-2} \text{ m}, \ L_y = 42 \times 10^{-2} \text{ m}, \ h = 3.18 \times 10^{-2} \text{ m} )</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>( E = 68.5 \times 10^9 \text{ Pa} )</td>
</tr>
<tr>
<td>Poisson’s coefficient</td>
<td>( \mu = 0.33 )</td>
</tr>
<tr>
<td>Mass per unit area</td>
<td>( \nu = 8.586 \text{ kg.m}^{-2} )</td>
</tr>
<tr>
<td>Loss factor</td>
<td>( \eta = 5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Actuator trans. coef.</td>
<td>( c_A = 1.1 \text{ N.V}^{-1} )</td>
</tr>
<tr>
<td>Sensor trans. coef.</td>
<td>( c_S = 1.5 \times 10^{-3} \text{ V.m}^{-1}.s^2 )</td>
</tr>
<tr>
<td>([x_u \text{ (m)}, y_u \text{ (m)}])</td>
<td>([0.058, 0.337] \ [0.201, 0.293] \ [0.237, 0.213] )</td>
</tr>
</tbody>
</table>

where \( I_d \) is the \( N_u \times N_u \) identity matrix. The optimal mechanical impedance \( Z^{opt} \) is then calculated from (5) and enforced using with a controller [1], ensuring that for any \( b \) and \( y \), the control input \( u \) converges to an attractive point \( u^* \) such that:

\[
    u^* = -Z^{opt} y^* \tag{11}
\]

It is to be noted that the attractive point \( u^* \) may be different of \( u^{opt} \) because of possible variations of the primary disturbance \( d \).

3.3 Results

In this section, numerical results for two strategies are presented: classical active damping imposing \( Z^d \) and optimal virtual mechanical impedance imposing \( Z^{opt} \). In order to compare the two approaches, parameters \( K \) and \( \beta \) must lead to control inputs with the same order of magnitude.

In the following part, the simple case of an acoustic plane wave with normal incidence and magnitude \( P_{inc} = 10 \text{ Pa} \) (Figure 1) is considered as the primary disturbance. The value of \( b \) in (14) is then [1] :

\[
    b = \frac{8}{2\pi} |P_{inc}| \ |r| \tag{12}
\]

Considering a three control units system (Table 1) and replacing this value in (10), it is possible to calculate \( Z^{opt} \).

In control results shown in Figures 2 and 3, there is no penalization of the effort control (\( \beta = 0 \)). The optimal virtual impedance is compared with active damping when \( K \) is very large and consequently when the plate is locally pinned. On Figure 2, the optimal virtual impedance approach provides better sound power attenuation than classical active damping. It is interesting to note that in contrast to classical active damping the signal measured on sensor 1 is non null : the optimal control with \( Z^{opt} \) does not locally cancel the plate vibration. Results plotted on Figure 3 emphasize this difference: whereas active damping strategy tends to minimize the kinetic energy, optimal virtual impedance approach leads to minimize the volume velocity of the structure. In results shown in Figures 4 and 5, the penalization of the control input \( \beta \) is high and is compared to active damping with a low value for \( K \). This situation occurs when the control system is poorly designed regarding to the primary disturbance amplitude and both control strategies are limited by the maximum actuator input voltage. In such a situation, the two approaches provide almost the same results. It is thus interesting to note that when \( \beta \) is decreased, the optimal virtual impedance strategy differs from classical active damping.
Figure 2. Acoustic power radiated, input and output voltage on unit 1: control off (dashed), $Z^{opt}$ control for $\beta = 0$ (solid) and active damping for $K = 3000$ (dotted).

Figure 3. Kinetic energy and volume velocity of the plate: control off (dashed), $Z^{opt}$ control for $\beta = 0$ (solid) and active damping for $K = 3000$ (dotted).

Figure 4. Acoustic power radiated, input and output voltage on unit 1: control off (dashed), $Z^{opt}$ control for $\beta = 10^{-2}$ (solid) and active damping for $K = 10$ (dotted).

Figure 5. Kinetic energy and volume velocity of the plate: control off (dashed), $Z^{opt}$ control for $\beta = 10^{-2}$ (solid) and active damping for $K = 10$ (dotted).

4. Experiments

4.1 Optimal virtual mechanical impedance computation

In practice, the terms $\Omega, S, A, M$ in (10) are unknown. It is thus necessary to formulate the problem with measurable physical quantities. An experimental identification of the primary trans-
mitted acoustic disturbance and transfer functions between control inputs and sound radiation in the receiving space are required in order to calculate $J$, $u^{opt}$ and $Z^{opt}$. This is accomplished by measuring active nearfield sound intensity over a surface $S$ discretized in $N_S$ points enclosing the plate in the receiving space. An estimation of the radiated power is calculated from the sum of active intensities measured and the cost function of equation (4) then takes the form:

$$J = \frac{1}{2} \text{Re}\{ (v)^H p \} + \beta u^H u$$

(13)

The $N_S \times 1$ vectors $p$ of acoustic pressure and $v$ of normal acoustic velocity measured over $S$ are given by:

$$\begin{align*}
  p &= H_p u + p_p \\
  v &= H_v u + v_p
\end{align*}$$

(14)

where $H_p$ is the $N_S \times N_a$ transfer function matrix between actuator inputs and acoustic pressures measured over $S$, $H_v$ is the $N_S \times N_a$ transfer function matrix between actuator inputs and normal acoustic velocities measured over $S$, $p_p$ is the $N_S \times 1$ vector of primary acoustic pressures measured over $S$, and $v_p$ is the $N_S \times 1$ vector of primary normal acoustic velocities measured over $S$. The optimal control inputs are then given by:

$$u^{opt} = - \left[ (H_v^n)^H H_p^n + (H_p^n)^H H_v^n + \beta I_{N_a} \right]^{-1} \left[ (H_v^n)^H p_p^{(n)} + (H_p^n)^H v_p^{(n)} \right]$$

(15)

from which it is possible to calculate $Z^{opt}$ using relation (5).

### 4.2 Simply supported aluminum plate

The structure is an aluminum plate with approximate simply-supported boundary conditions. Its mechanical parameters and the position of the 3 control units are shown in Table 1. Each unit is composed of a Visaton®EX 60 S inertial actuator and a Bruel & Kjaer®4397 accelerometer. An acoustic enclosure with twelve Monacor®SP-60 loudspeakers is used to create the primary disturbance (Figure 6). An acoustic waveguide was mounted on the receiving side of the plate in order to estimate the radiated acoustic power via intensity measurements performed using a $(p,p)$ probe over a plane surface located 10 cm from the plate with a spatial sampling of $44\text{mm} \times 40\text{mm}$. These measurements were performed using a single probe mounted on a displacement unit, which also carried a Doppler laser vibrometer to measure the transverse velocity of the plate in front of each intensity scan point. The signal generation, signal acquisition and control implementation were done using a real-time Opal-RT(R) controller whose sampling frequency was set to 40 kHz. As in the theoretical case, two situations were considered. Measurement noise did not however allow to consider $\beta = 0$. Nevertheless the results shown in Figures 7 and 8 were obtained with a small penalization of the control input and a large active damping gain ($\beta = 10^{-4}, K = 100$). Even if the experimental conditions are quite different to the theoretical model (especially in terms of the primary disturbance), major aspects of the results are similar. While the two strategies lead to different results when beta is small, both approaches provide almost the same results when the penalization of the control input becomes large. (Figures 9 and 10).
5. Conclusions

This study presents the optimal virtual mechanical impedance approach and its comparison with a classical active damping strategy. With a properly designed control system, the penalization of the control effort can be tuned to a very low value. By minimizing structural volume velocity, this optimal virtual mechanical impedance strategy performs better sound power attenuation than classical active...
damping strategy. It is important to recall that in the study, an harmonic excitation is considered as primary disturbance. In contrast to classical active damping, the optimal frequency response of the virtual impedance approach is not limited by parametric models. On the other hand, future work will be required to achieve this active control in the case of a non harmonic disturbance. This method can be applied to more complexe structures. Therefore, on going work is to implement this new strategy on a composite curved aeronautic panel featuring a window.

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