CONTROL PERFORMANCE OF SEMI-ACTIVE RAILWAY VEHICLE SUSPENSION USING MR DAMPER

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This paper presents control performance of a railway vehicle suspension system equipped with MR (magnetorheological) dampers. The dynamic model of the railway vehicle system is proposed and its governing equations are mathematically derived. MR dampers which can produce enough damping force to control railway vehicle are manufactured and evaluated experimentally. In order to reduce the vibration of railway vehicle effectively with damping force dynamic model, various controllers suitable for semi-active device system are designed. The vibration control performances of railway vehicle system including car body lateral motion and acceleration of MR damper are evaluated in frequency domain.

1. Introduction

To solve the huge traffic demand due to the improvement of a standard of living and rapid development of economic scale and also to reduce air pollution from the explosion of the road transport, the importance railway vehicle system is increasing as one of the mass transportation systems. In order to readjust the current transportation system converged on road traffic, transportation capability of railway vehicle must be improved. As one of solutions to resolve this issue, the high speed railway vehicle has been proposed and researched since 1960’s in France, Germany and Japan. The one of important facts for high speed technology is to maintain system stability. When railway vehicle is operated with high speed, whole system including car body, bogie and wheel-axle set is vibrated excessively. If this vibration is not controlled properly, the dynamic characteristic of the railway vehicle becomes unstable, and then the hunting phenomenon is occurred. The hunting phenomenon is defined as a self-excited lateral oscillation caused by the forward speed of the vehicle and wheel-axle set-rail interactive force, which comes from the creep-friction properties of the wheel-rail contact mechanism. There exist two critical velocities related with the hunting phenomenon. First one is associated with low velocity. This is caused by lateral oscillation of car body and can be found in railway vehicle with small damping force. Another one is occurred at high velocity. This case seems as severe oscillation of the wheel-axle set and truck connection [1, 2]. This abnormal dynamic behaviour aggravates riding quality and even cause derailment. Ride quality is
construed as the capability of the railway vehicle suspension system to retain the dynamic motion within the range of comfort or within the range to guarantee that there is no freight detriment. Ride quality relies on acceleration, rate of change of acceleration and displacement [1].

It is evident that suspension system has tremendous importance in railway vehicle transportation system. The suspension prevalently used in railway vehicle is passive suspension system. The conventional passive suspension is effective at low velocity and has cost effect with relative simple design. However, at high velocity it has performance limits and consequently can cause dynamic problems mentioned above. To overcome this, various types of active and semi-active suspension have been researched vigorously in the world. In this work, a semi-active suspension system is considered to enhance dynamic characteristic of railway vehicle with MR (magneto-rheological) fluid based damper (MR damper in short). MR fluid called smart material is used widely to control mechanical vibration with its application including MR damper for passenger car, military vehicle, MR mount for vessel diesel engine, etc. [3-7]. Consequently, the main contribution of this work is to improve stability of railway vehicle by reducing unwanted vibrations. Firstly, a governing equation of railway suspension system is derived by integrating MR dampers. Then, MR dampers are manufactured and their damping force capabilities are evaluated via experiment. Using these experiments result, a dynamic model of the MR damper is reconstructed and optimal controller is designed to reduce the vibrations. Finally, vibration control performances of the railway vehicle suspension system is evaluated in frequency domain.

2. Mathematical Model

In this work, 9-degree of freedom dynamic equations of railway vehicle having four wheel-axle sets, two bogies and one car body is adopted as shown in Figure 1. Wheel-sets are assumed that they have lateral displacements \( y_i (i = 1,2,3,4) \) and system disturbances are caused by displacement of wheel-sets. Primary suspension is located between wheel-axle set and bogie, secondary suspension is located between bogie and car body, two MR dampers are located in secondary suspension system laterally. The mechanical configuration of the railway vehicle is shown in Fig. 1. Table 1 shows each variable of car body and bogies. The variables in the following equations are defined as follows : \( 2l \) is bogie pivot distance, \( 2d \) is wheel-set pivot distance, \( 2b \) is lateral distance of secondary spring and \( h_1, h_2, h_3 \) are distances of centre, bogie and wheel-set pivot of secondary suspension respectively [8]. The governing equations of lateral, yaw and roll motion of bogie frame are derived as follow:

\[
m y_a'' + (2c_{py} + c_{cy}) y_a' + (2c_{py} h_a - c_{a_1} h_1) y_a + c_y (-y_a + l \phi_a - h_2 \phi_b) + (2k_{py} + k_{cy}) y_a + (2k_{py} - k_{cy}) \phi_a + k_{cy} (-y_a + l \phi_a - h_2 \phi_b) = c_y (y_{2i-1} + y_{1i}) + k_{py} (y_{2i-1} + y_{1i}) - f_{ai} \tag{1}
\]

\[
I_{xx} \phi_{a}'' + (2c_{px} b_1^2 + 2c_{px} d^2 + c_{px} b_3^2 + c_{px} b_1^2) \phi_a - (c_{px} b_3^2 + c_{px} b_1^2) \phi_a + (2k_{px} b_1^2 + 2k_{px} d^2 + k_{px} b_1^2) \phi_a - k_{px} b_1^2 \phi_a = c_{px} d (y_{2i-1} - y_{1i}) + k_{px} d (y_{2i-1} - y_{1i}) \tag{2}
\]

\[
J_{xx} \phi_{b}'' + (2c_{px} h_a - c_{a_1} h_1) \phi_b + (2c_{px} b_1^2 + 2c_{px} h_a^2 + c_{px} b_2^2 + c_{px} b_1^2) \phi_b + c_{px} h_a \dot{y}_b \mp c_{px} h_a l \dot{\phi}_a + (c_{px} h_a h_2 - c_{a_1} b_2^2) \phi_b + (2k_{px} h_a^2 - k_{px} h_a b_2^2 + 2k_{px} d^2 + k_{px} b_1^2) \phi_b + k_{px} d \dot{y}_b \dot{y}_b \pm k_{px} h_a \phi_b \pm (k_{px} h_a h_2 - c_{a_1} b_2^2) \phi_b = c_{px} h_a \phi_a (y_{2i-1} + \dot{y}_{1i}) + k_{px} h_a (y_{2i-1} + y_{1i}) + \dot{h}_a f_{ai} \tag{3}
\]

where the equation of front bogie \((i = 1)\) has upper sign, \( f_{ai} \ (i = 1,2) \) is MR damper force. Governing equations of lateral, yaw and roll motion of car body frame are derived as follows:
\[
m_b \ddot{y}_b - c_y \left( \dot{y}_{11} + \dot{y}_{12} \right) + c_y h \left( \dot{\phi}_{11} + \dot{\phi}_{12} \right) + 2c_y h \dot{y}_b + 2c_y h \ddot{\phi}_b - k_y \left( y_{11} + y_{12} \right) + k_y h \left( \phi_{11} + \phi_{12} \right) + 2k_y y_b + 2k_y h \ddot{\phi}_b = f_{a1} + f_{a2} \\
I_x \ddot{\phi}_b - c_{\phi} \left( \dot{\phi}_{11} + \dot{\phi}_{12} \right) + c_{\phi} h \left( \dot{\phi}_{11} - \dot{\phi}_{12} \right) + 2c_{\phi} h \dot{\phi}_b + 2(c_{a1} b_1^2 + c_{a2} b_2^2 + c_{a0} l^2) \ddot{\phi}_b \\
- k_{\phi} \left( \phi_{11} - \phi_{12} \right) + k_{\phi} h \left( \phi_{11} - \phi_{12} \right) + 2(k_{a1} b_1^2 + k_{a2} b_2^2) \ddot{\phi}_b = l(f_{a1} - f_{a2}) \\
J_{a1} \ddot{\phi}_b - c_{\phi} h \left( \dot{\phi}_{11} + \dot{\phi}_{12} \right) + c_{\phi} h h_1 c_{a1} b_1^2 \left( \phi_{11} - \phi_{12} \right) + 2c_{\phi} h h_2 \ddot{\phi}_b + 2(c_{a1} b_1^2 + c_{a2} b_2^2) \ddot{\phi}_b - k_{\phi} h \left( y_{11} + y_{12} \right) \\
+ \left( k_{a1} h h_1 - k_{a1} b_1^2 - k_{a2} b_2^2 \right) \left( \phi_{11} + \phi_{12} \right) + 2k_{a1} h h_2 \ddot{\phi}_b + 2(k_{a1} h b_1^2 + k_{a2} h b_2^2 + k_{a0} h l^2) \ddot{\phi}_b = h_z f_{a1} + f_{a2}
\]

where \( m_2, m_1, m_0 \) are mass of car body and bogie, \( J_{a1}, J_{a2} \) are X,Z-direction inertial moment of car body, \( J_{a1}, J_{a2} \) are X,Z-direction inertial moment of bogie. \( k_{a1}, k_{a2}, k_{a0} \) and \( k_{a0} \) are longitudinal, lateral, vertical and angular stiffness of secondary suspension, \( c_{a1}, c_{a2} \) and \( c_{a0} \) are lateral, vertical and yaw damping of secondary suspension. \( k_{p1}, k_{p2} \) and \( k_{p3} \) are X,Y and Z-direction stiffness of primary suspension. \( c_{p1}, c_{p2} \) and \( c_{p3} \) are X,Y and Z-direction damping of primary suspension, respectively.

<table>
<thead>
<tr>
<th>Table 1. Variables of railway vehicle system.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>Front bogie</td>
</tr>
<tr>
<td>Rear bogie</td>
</tr>
<tr>
<td>Car body</td>
</tr>
</tbody>
</table>

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**Figure 1.** Configuration of the railway vehicle.
3. MR Damper

In this work, two MR dampers are manufactured and their performances are tested experimentally. To manufacture these dampers, outer cylinder body, core, coil, etc. are modelled as shown in Fig. 2(a). Magnetic circuit is designed to produce enough magnetic field at 1.2A and its analysis is conducted through computer program. Based on the above process design parameters of the MR damper are determined; inner diameter is 80mm, outer diameter is 90mm, gap size 0.7mm. The double core is applied to produce large yield stress. It is noted here that MR dampers manufactured in this work are applicable to real railway vehicle suspension system. For making inner core, pure iron is used to produce maximum magnetic field. The outer cylinder and housing is manufactured with SM-30C due to the stiffness and weight. In order to create maximum yield stress, inner core is designed as double core. Total manufacture process is conducted by RMS Technology and manufactured MR dampers are shown in Fig. 2(b). For evaluation of them, hydraulic shaker is used and displacement measured by contact displacement sensor LVDT is differentiated to obtain input velocity. MR damping forces due to the magnetic field created by input current are measured by load cell located in upper part of MR damper. Figure. 4(a) shows F-v graphs of the MR damper and illustrates that at 1.2A it can be produce objective force that maximum yield stress 15kN when input velocity is 0.2m/s condition.

![Figure 2. MR damper model (a) and manufactured MR damper (b)](image)

![Figure 3. Mechanical model for the MR damper](image)
The MR damper damping force dynamic model as shown in Fig. 3. The phenomenological model of MR damper damping force is given by following equations. [9]

\[ F = c_i \dot{y} + k_i (x - x_o) \]  
(7)

\[ c_i \dot{y} = \alpha z + k_i (x - y) + c_i (\dot{x} - \dot{y}) \]  
(8)

\[ \ddot{z} = -\gamma |\dot{z} - \dot{y}|^{\alpha_{\nu}} - \beta (\dot{x} - \dot{y}) |\dot{y} | + A (\dot{x} - \dot{y}) \]  
(9)

\[ \dot{y} = \frac{1}{(c_o - c_i)} \{ \alpha z + c_o \dot{x} + k_i (x - y) \} \]  
(10)

\[
\begin{align*}
    c_i &= c_i(u) = c_{i_a} + c_{i_b} u \\
    \alpha &= \alpha(u) = \alpha_a + \alpha_{b0} u + \alpha_{b1} u^2 + \alpha_{b2} u^3 + \alpha_{b3} u^4 \\
    c_o &= c_o(u) = c_{o_a} + c_{o_b} u + c_{o0} u^2 + c_{o1} u^3 + c_{o2} u^4 + c_{o3} u^5 \\
    \dot{u} &= -\eta (u - v)
\end{align*}
\]  
(11)

where \( F \) is MR damper damping force and \( \alpha , c_0 \), terms are modified to adjust model to the characteristic of actual MR damper and \( v \) is input voltage sent to the current driver for the MR damper control. The parameters value determined following the damping force of manufactured MR damper are shown in Table 3. It can be seen that damping force from this phenomenological model for MR damper is well follow actual MR damper’s damping force in Fig. 5. The left F-v graph is experiment results and right F-v graph is model results with the parameters.

**Table 2.** Parameters for the car model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_a )</td>
<td>850</td>
<td>( C_{a} )</td>
<td>8,000</td>
<td>( C_{0.5} )</td>
<td>-8.3333</td>
<td>( x_o )</td>
<td>-0.3</td>
</tr>
<tr>
<td>( \alpha_{b0} )</td>
<td>3233.3</td>
<td>( C_{0.0} )</td>
<td>73,000</td>
<td>( C_{1} )</td>
<td>70,000</td>
<td>( k_0 )</td>
<td>40,000</td>
</tr>
<tr>
<td>( \alpha_{b1} )</td>
<td>1478.3</td>
<td>( C_{0.1} )</td>
<td>-62,783</td>
<td>( C_{1b} )</td>
<td>50</td>
<td>( k_1 )</td>
<td>5,500</td>
</tr>
<tr>
<td>( \alpha_{b2} )</td>
<td>-852.78</td>
<td>( C_{0.2} )</td>
<td>25,125</td>
<td>( \gamma )</td>
<td>647.46</td>
<td>( n )</td>
<td>1</td>
</tr>
<tr>
<td>( \alpha_{b3} )</td>
<td>146.3</td>
<td>( C_{0.3} )</td>
<td>-4708.3</td>
<td>( \beta )</td>
<td>647.46</td>
<td>( \eta )</td>
<td>-190</td>
</tr>
<tr>
<td>( \alpha_{b4} )</td>
<td>-8.3333</td>
<td>( C_{0.4} )</td>
<td>375</td>
<td>( A )</td>
<td>3,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Figure 4. Damping force experiment and simulation results*
4. Controller Design

Equation. (1)-(6) can be rewritten in the matrix form as follow

\[ M\ddot{q} + C\dot{q} + Kq = F_u + F_d d \]  

(13)

where \( M, C, K, F_u \) and \( F_d \) are mass, damping, stiffness matrices and coefficient matrices of damping force and disturbance, \( u \) and \( d \) are vector of MR damper damping force and excitation, \( M \in \mathbb{R}^{8x8}, C \in \mathbb{R}^{8x8}, K \in \mathbb{R}^{8x8}, F_u \in \mathbb{R}^{8x2}, F_d \in \mathbb{R}^{8x4}, u = [f_{MR1} f_{MR2}]^T, d = [d_1 d_2]^T, d_2 = [\dot{y}_1 \dot{y}_2 \dot{y}_3 \dot{y}_4]^T \),. Equation. (13) can be rewritten in the state-space model as follow

\[
\begin{align*}
\dot{x} &= Ax + Bu(t) + Dd(t) \\
y &= Cx
\end{align*}
\]

(14)

where \( A \in \mathbb{R}^{18x18}, B \in \mathbb{R}^{18x2}, C_i \in \mathbb{R}^{18x8}, D \in \mathbb{R}^{18x8} \) and \( y \) is output vector. To control railway vehicle system using MR damper with proper control input, Sky-hook and LQR controller are used and each of the control results are compared. Following equation represent Skyhook algorithm.

\[ u_{sky-d} = C_{sky} V_{car} \]

(15)

where \( C_{sky} \) is skyhook gain and \( V_{car} \) is lateral velocity of car and \( u_{sky-d} \) is desired damping force from Skyhook controller. For LQR controller performance index is chosen as follow

\[ J = \lim_{t \to \infty} \int_{0}^{t} [x^T(t)Qx(t) + u_{LQR-d}^T R u_{LQR-d}] dt \]

(16)

\[ u_{LQR-d}(t) = -Kx(t) \]

(17)

where \( Q \) and \( R \) are symmetric semi-positive and positive-definite matrices and \( u_{LQR-d} \) is desired damping force from LQR controller \( u_d = [f_{MR1}^d f_{MR2}^d]^T \) and \( K = R^{-1}B^T P \), where \( P \) is obtained by \( P^T A - AP - PBR^{-1}B^T P = -Q \) called as ARE (Algebraic Riccati equation). Desired damping force cannot be applied to MR damper which is semi-active device, so input voltage must be commanded. To calculate input voltage which make MR damper follow desired force, following equation is used.

\[ v = \begin{cases} 
V_{max}, & \text{if } (u_d - u) \times u > 0 \\
0, & \text{if } (u_d - u) \times u < 0
\end{cases} \]

(18)

where \( V_{max} \) is maximum input voltage, and if the signs of relative damping force and actual damping force are same, then input voltage is \( V_{max} \) else zero which is semi-active condition.[10]

5. Results and Discussions

In order to compare each of the control results, Korean ride quality of railway vehicle evaluation method, KS R 9216, is used.

\[ W_0(f) = \frac{f^4}{f^4 + 0.0256}, \sqrt{\frac{100000000}{f^4 + 100000000}}, f = \text{frequency} \]

(19)
\[ W_s(f) = \frac{(f^2 + 256)(2601.123)}{(0.3969f^4 + 52.787f^2 + 2601.123)256} \cdot \frac{0.8281f^4 - 3.6858f^2 + 26.126}{0.8281f^4 - 7.364f^2 + 104.294} \cdot W_s(f) \quad (20) \]

Riding quality evaluation index = \(\text{RMS}(\text{Acceleration} \times W_s(f))\) \quad (21)

In the simulation, for Sky-hook controller, the value of skyhook gain \(C_{sky} = 500\) and for LQR controller, \(Q_{i(7,7)} = 10^3\), \(Q_{i(9,9)} = 10^4\), \(Q_{i(16,16)} = 10^5\), \(Q_{i(18,18)} = 10^6\) and \(R = 0.01 \times I_{22}\), \(V_{m} = 1.2\text{Volt}\). Figure 5 and 6 show car body riding quality results of passive, skyhook and LQR controller simulation in frequency domain and results of acceleration in frequency domain multiply ride quality weighting function. It is illustrated that amplitude of 0.4Hz and 2Hz which are main peak frequencies of acceleration are effectively reduced, but in range from 4 Hz to 8Hz, relatively high frequency, amplitudes of acceleration are larger than passive damper result. However, frequency range having a decisive effect on ride quality is from 0.4Hz to 3Hz, thus larger amplitude in relative high frequency range has minor significance in terms of ride quality. Each of ride quality results are shown in Table 4

### Table 4. Ride quality simulation results

<table>
<thead>
<tr>
<th>Riding quality index</th>
<th>Passive Damper</th>
<th>MRD Passive</th>
<th>MRD with Skyhook</th>
<th>MRD with LQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.4672E-4</td>
<td>5.6420E-4</td>
<td>4.5797E-4</td>
<td>3.9094E-4</td>
<td></td>
</tr>
<tr>
<td>Effective</td>
<td>-12.76%</td>
<td>-29.18%</td>
<td>-39.28%</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5.** Simulation results of railway vehicle using MR damper with skyhook controller

**Figure 6.** Simulation results of railway vehicle using MR damper with LQR controller
In this table, LQR controller produce the best result of ride quality. MR damper passive mode has 12.76% effect because the MR dampers are manufactured much larger than passive damper’s specification.

6. Conclusion

In this work, a 9-DOF railway vehicle suspension system featuring MR dampers was proposed and its vibration control performance was evaluated. Before deriving the governing equation of the suspension system, a real-sized MR damper which can be applicable to railway vehicle was designed and manufactured. The field-dependent damping force of the MR damper was then investigated and utilized for the suspension simulation. In order to control railway vehicle effectively, a semi-active control algorithms including skyhook and LQR controller are properly designed. It has been demonstrated that the proposed semi-active suspension system can reduce the unwanted vibrations of the railway vehicle.

REFERENCES