RECONSTRUCTION OF NON-STATIONARY SOUND FIELD USING THREE TIME-DOMAIN ACOUSTIC HOLOGRAPHY METHODS

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In this paper, three time-domain acoustic holography methods, including time domain holography, real-time near-field acoustic holography and time domain plane wave superposition method (TD-PWSM), are applied to reconstruct the non-stationary sound field radiated by an impacted plate. The reconstruction processes of these methods are briefly described first. And then, an experiment of a clamped rectangular steel plate impacted by a steel ball is investigated to testify the three methods. Experimental results demonstrate that the three methods not only can reconstruct the non-stationary signals from an impacted plate in the time domain, but also can be used to visualize the transient sound field in the space domain. Besides, it also proves that the TD-PWSM possesses the advantages of reducing some errors associated with the finite aperture effect and providing the potential ability of allowing an irregular microphone array.

1. Introduction

The transient noise radiated by impacted structures, e.g. riveting and hammering, not only disturbs the living of residents around, but also brings hearing impairment to workers, and therefore it has great engineering significance to study this type of noise. The transient sound radiation from an impacted plate has received attention\(^1\)\(^2\). Near-field acoustic holography (NAH)\(^3\)\(^4\) is an effective tool for the identification and visualization of sound source. Especially, time domain NAH is a very useful tool for studying transient sound radiation of an impacted plate both in time and space domains. Different time domain NAH methods have been proposed in order to investigate the backward propagation problem of transient sound field by pressure measurements on a microphone array nearby the source. The purpose of this paper is to compare three of them: time domain holography (TDH)\(^5\), real-time near-field acoustic holography (RT-NAH)\(^6\) and time domain plane wave superposition method (TD-PWSM)\(^7\). These methods are experimentally applied to reconstruct the non-stationary sound field radiated by an impacted rectangular steel plate in a semi anechoic chamber. In this paper, the reconstruction processes of these three methods are briefly recalled first. The reconstruction evaluation factors are also introduced to study the ability of the three methods to back-propagate efficiently and precisely the pressure field. Besides, we also use the experiment to testify that the TD-PWSM possesses the advantages of reducing some errors associated with the finite aperture effect and providing the potential ability of allowing an irregular microphone array while the DTH and RT-NAH don’t have.
2. Theories of TDH, RT-NAH and TDWSM

Figure 1. Synopses of TDH [path (a)], RT-NAH [path (b)] and TDWSM [path (c)].

Figure 1 synthetically presents the three time domain NAH methods used in this paper to back-propagate time-dependent pressure fields. \( p(x, y, z_h, t) \) denotes the pressure on the hologram plane located a distance \( z_h \) away from the source plane, \( p(x, y, z_r, t) \) corresponds to the pressure on the reconstruction plane located a distance \( z_r \) away from the source plane, \( p(k_x, k_y, z_h, t) \) and \( p(k_x, k_y, z_r, t) \) are the corresponding pressure time-wavenumber spectrums, \( p(k_x, k_y, z_h, w) \) and \( p(k_x, k_y, z_r, w) \) are the corresponding pressure frequency-wavenumber spectrums, and \( p(k_x, k_y, z_r, t) \) denotes the pressure time-wavenumber spectrums on the virtual source plane located a distance \( z_r \) away from the back of the source plane. \( F_{sy} \) and \( F_{sy}^{-1} \) denote two-dimensional spatial fast Fourier transform (SFFT) and the inverse 2-D SFFT with respect to \( x \) and \( y \), respectively, and \( F_t \) and \( F_t^{-1} \) denote Fourier transform and inverse Fourier transform with respect to time \( t \), respectively.

2.1 Time domain holography (TDH)

Hald
developed TDH method to realize the reconstruction of non-stationary sound field. The reconstruction process of TDH is depicted in the path (a) of Figure 1. In TDH, time and 2-D SFFT are first applied to the time-dependent pressure \( p(x, y, z_h, t) \) on the hologram plane, yielding the frequency-wavenumber spectrums \( p(k_x, k_y, z_h, w) \). And then, the backward propagation operator is applied to each spectral line, yielding the wavenumber spectrums \( p(k_x, k_y, z_r, w) \) on the reconstruction plane at all angular frequencies. Finally, the computation with inverse time and 2-D SFFT provides the time-space pressure \( p(x, y, z_r, t) \) on the reconstruction plane.

2.2 Real-time near-field acoustic holography (RT-NAH)

Thomas et al. proposed a method called RT-NAH for studying transient sound fields. The reconstruction process of RT-NAH shown in the path (b) of Figure 1 can be described as follows: The time-wavenumber spectrums of the pressure are first acquired by applying the 2-D SFFT to the measured time dependent pressure signals, then a time convolution between the obtained time-wavenumber spectrums and an inverse impulse response is performed, yielding the time-
wavenumber spectrum of the pressure on the reconstruction plane; Lastly, the application of inverse 2-D SFFT to the obtained spectrums yields the time-space pressure on the reconstruction plane.

2.3 Time domain plane wave superposition method (TD-PWSM)

Similarly to RT-NAH, TD-PWSM presented by Zhang et al.\(^7\) performs the reconstruction directly in the time domain. The path (c) of Figure 1 briefly shows the reconstruction process. In TD-PWSM, the time-dependent pressure \(p(x, y, z_h, t)\) on the hologram plane can be expressed as a superposition of time convolutions between the time-wavenumber spectrum \(p(k_x, k_y, z_v, t)\) of the pressure on a virtual source plane and the corresponding time-domain propagation kernel at each wavenumber. By discretizing the time convolutions directly and the inverse solving process, the time-wavenumber spectrums \(p(k_x, k_y, z_v, t)\) at all wavenumbers are solved. The solved spectrums \(p(k_x, k_y, z_v, t)\) are used as the input of another superposition of time convolutions, yielding the time-dependent pressure \(p(x, y, z_v, t)\) on the reconstruction plane. Comparing to TDH and RT-NAH, the double infinite integral of the 2-D spatial Fourier transform is discretized directly in the wavenumber domain in TD-PWSM, which makes it avoid some errors associated with the finite measurement aperture in theory and possible to use an irregular microphone array.

3. Experiment and analysis

An experiment was carried out in a semi-anechoic chamber to testify these three methods. A clamped steel plate with the size of 0.45 m×0.45 m was impacted by a steel ball to generate a transient sound field as shown in Figure 2. An accelerometer fixed on the back of the plate was set as the trigger to activate the microphone array recording the pressure information.

![Image](image)

**Figure 2.** Experiment: microphone array, steel plate, pendulum, electromagnet and trigger accelerometer.

Figure 3 shows the position relationships between the plate plane \(S\), the virtual source plane \(V\), the reconstruction plane \(R\), and the hologram plane \(H\). 9×9 points are distributed on the planes \(R, H\) and \(V\), and the grid spacing is 0.05 m in both \(x\) and \(y\) directions. The signal was sampled at a frequency \(f_c = 102.4\) kHz providing 1024 sampling points. In the experiment, the virtual source plane \(V\) was located at \(z_v = -0.008\) m. The array with 5×5 microphones was used to acquire the pressures on the hologram plane \(H\) with \(z_h = 0.03\) m. The pressure on the reconstruction plane \(R\) with \(z_r = 0.01\) m was also measured by the array to compare with the pressure reconstructed by the three methods for assessing the relevance and precision of them. In this experiment, zero-padding and exponential filtering were used in TDH, and zero-padding and regularization were used in RT-NAH. While, only regularization was used in TD-PWSM.
3.1 Reconstruction with TDH, RT-NAH and TD-PWSM

Figure 3. Geometric description of plate plane \( S \), the virtual source plane \( V \), the reconstruction plane \( R \), and the hologram plane \( H \). Two points \( R_1 \) and \( R_2 \) are selected for comparison, marked with the symbol “+”.

Figure 4. Time domain waveform comparisons between the measured pressures (line with plus sign) and the reconstructed pressures (solid line) by the three methods at two points (a) TDH \( R_1 \), (b) TDH \( R_2 \), (c) RT-NAH \( R_1 \), (d) RT-NAH \( R_2 \), (e) TD-PWSM \( R_1 \), (f) TD-PWSM \( R_2 \).
The reconstructed pressures at two points \( R_1 \) and \( R_2 \) identified in Figure 3 were calculated by these three methods and compared with the measured pressures at the same positions. In Figure 4, the reconstructed pressures by these three methods at the point \( R_1 \) are all in good agreement with the measured pressure. While at the point \( R_2 \), TD-PWSM method seems to give the best results in terms of amplitude, and another two methods present a little amplitude difference.

In order to evaluate the reconstructed results with the three methods quantitatively, two indicators \( Ep \) and \( Ea \) related to the phase and the amplitude were used (the definitions of \( Ep \) and \( Ea \) can be found in Ref. 8). Here, it is noted that when \( Ep \) is close to one, there exists a high phase similarity; when \( Ea \) is near zero, the amplitude difference is small.

![Spatial maps for the indicator Ep obtained by (a) TDH, (b) RT-NAH, (c) TD-PWSM and for the indicator Ea with a contour line at the value 0.1 obtained by (d) TDH, (e) RT-NAH, (f) TD-PWSM.](image)

Figure 5 showed the mapping results of \( Ep \) and \( Ea \) for the three methods. It can be found that all the three methods achieve the high phase similarity and little amplitude difference at most space points. The values of \( Ep \) and \( Ea \) at the two points \( R_1 \) and \( R_2 \) are shown in Table 1.

<table>
<thead>
<tr>
<th>Point</th>
<th>( Ep )</th>
<th>( Ea )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TDH</td>
<td>RT-NAH</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>0.9859</td>
<td>0.9835</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>0.9134</td>
<td>0.9013</td>
</tr>
</tbody>
</table>

Similarly, for showing the reconstructed results of the three methods in the space domain, two time instants 1.46 ms and 5.05 ms were selected. The comparisons were given in Figure 6. It can be found that the spatial distributions of pressure field reconstructed by the three methods are almost the same as the measured pressure fields.
3.2 Finite aperture effect and irregular microphone array

TD-PWSM performs the reconstruction by replacing the 2-D SFFT that is generally used in TDH and RTNAH with the direct discretization of double infinite integral in the wavenumber domain, theoretically avoiding some wrap-around errors associated with the SFFT. Since the point located at the edge of the reconstruction plane is easily influenced by the wrap-around error, the reconstructed qualities with TDH and RT-NAH at the point $R_2$ are weaker than that of TD-PWSM in Figure 4. In order to accurately estimate the influence of finite aperture on the three methods, Figure 7 shows the comparisons of the values of $E_p$ and $E_a$ for all edge points on the plane $R$. It is obvious from Figure 7 that although the values of $E_p$ calculated by TD-PWSM at several edge points are equal to those of TDH and RT-NAH and the values of $E_a$ at several edge points are larger than those of TDH and RT-NAH, on the whole the best reconstruction of phase and amplitude is still provided by TD-PWSM.

Besides, irregular microphone array can be used by TD-PWSM to realize the reconstruction of sound field while TDH and RT-NAH cannot in theory. Here, an irregular microphone array depicted in Figure 8(a) is applied in TD-PWSM.
Figure 8. (a) Irregular microphone array, (b) Time domain waveform comparisons between the measured pressures (line with plus sign) and the reconstructed pressures (solid line) at point $R_1$.

Figure 8(b) showed the time-domain waveform comparisons between the measured pressure and the reconstructed pressure at the point $R_1$. Similarly, the same two time instants were used to assess the reconstructed results in the space domain, as shown in Figure 9. The reconstructed results demonstrate that TD-PWSM with irregular microphone array still work well.

Figure 9. Spatial distributions of the measured pressures at (a) 1.46 ms and (b) 5.05 ms versus the reconstructed pressures obtained by TD-PSWM with irregular array at (c) 1.46 ms and (d) 5.05 ms

4. Conclusions

An impacted steel plate as the non-stationary source was carried out to test the reconstruction ability of the studied three time-domain acoustic holography methods. The experimental results proved that these different methods could provide the satisfying back-propagated pressure in both
time and space domains by using the measurement data on the hologram plane. Comparing to the reconstructed results with TDH and RT-NAH at all edge points, the errors of TD-PWSM at these points were smaller, which illustrated that the TD-PWSM really could avoid some errors associated with 2-D SFFT. Moreover, TD-PWSM with irregular array could also have the ability to reconstruct the non-stationary sound field. However, the problem of which kind of array used in TD-PWSM can provide the best reconstruction still needs to be further investigated in the future.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant Nos. 51322505, and 11274087).

REFERENCES