THE THICKNESS/RESONANCE PREDICTION OF AN ULTRA-THIN LAYER BY THE LOW-FREQUENCY ULTRASOUND

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A prediction method of the thickness, resonant space and frequency for the two cases of an ultra-thin layer and the narrow-band frequency spectra has been developed without the appearance of the resonance/anti-resonance. Using the characteristic of straight line and the period of $\pi$ of the layer phase shift obtained from the measured data, the relations of the line slope with time of flight and resonant space of the layer on the half-space substrate of infinite extent are analyzed. The resonant formulas are derived after the different impedance conditions are considered. Using the relations, the formulas and the curve fitting method, the thickness, resonant space and frequencies can be predicted from the less available measured data and no information of the resonance/anti-resonance. The method feasibility and anti-interference capability are analyzed and testified by the numerical simulation for the two cases of an ultra-thin layer and the narrow-band spectra, respectively.

1. Introduction

The thickness determination for the various materials has been a conventional and basic problem in the ultrasonic non-destructive evaluation for the long-term. Many publications or reviews have been devoted to this topic until now. In general, the extensively-used ultrasonic measurement/determination methods include: the separable pulse method, resonant method, other ultrasonic methods such as Lamb waves\textsuperscript{1,2} or Surface waves\textsuperscript{3}, the parameter inversions\textsuperscript{4,5} in the time or frequency domain have also played an important role on the thickness determination, however, for the two practical situations the problem of the thickness determination remains to be not well solved, one is an ultra-thin layer, in which not only the successive echoes from in the sample front and back interfaces are overlapped and unable to be separated, but also as go over to the frequency domain, no available resonance/anti-resonance in the frequency response zone. The other is the narrow frequency-band spectral, the experimental noises or disturbances may give rise to the problem and
narrow the useful frequency-width and leave the less available measured data. Besides no available resonance/anti-resonance that is similar with the case of the ultra-thin layer, the multi-value problem of phase shift/thickness also need to be considered and solved for the case of the narrow frequency-band. Due to the above factors, the separable pulse method and the resonant method break down for the above two cases, while other ultrasonic methods or the inversions involve the experimental complexity or other more parameters.

Recently, we had derived and expressed phase shift of the mid-layer in the multi-layered structure for the normal incidence as a sum form of the measured phase shift and the crossing phase shift after the further simplification of the recursive formula was performed. The characteristics of phase shifts of a linear-viscoelastic layer on the different elastic substrates of infinite extent are discussed. It is showed that phase shifts evaluated from the reflected coefficients consist of a group of the periodic parallel straight lines with the same slope for the layer on the elastic substrate of half-space of infinite extent. Using the property of straight line and the period of phase shift and the curve-fitting method, we had developed the prediction idea of the layer thickness in Ref.7, in which the layer thickness, resonant spacing and frequencies and corresponding order can be predicted from the determined slope after the impedance condition is considered. In the current paper, the proposed prediction idea is further detailed and specific, including the relations of the slope of straight line with time of flight and resonant space are analyzed. The resonance formulas are derived after the different impedance conditions are considered. Using the derived relations and formulas, the proposed prediction method of the layer parameters have the more physical and manipulative feasibility, even with the less available measure data. With the ‘noisy’ synthesized ultrasonic reflected waves, the prediction procedure of the layer parameters is numerically simulated and analyzed for the two cases of an ultra-thin layer and the narrow-band spectra, and the method feasibility and anti-interference capability are testified.

2. Theoretical model and formula

Using the recursive formula of the multi-layered structure for the normal incidence, the relation of input acoustical impedance and of the layer is written as

\[ Z_l(\omega) = \frac{Z_{l+1}(\omega) - jz_l \tan(\omega \psi_l)}{z_l - jZ_{l+1}(\omega) \tan(\omega \psi_l)} z_l, \]  

(1)

where is the discrete angular frequency and is the imaginary unit, and are acoustical impedance and phase-shift of the layer with time of fight of , and and are the thickness and compressional velocity, respectively.

We have

\[ \omega \psi_l = \text{Re[arctan} \frac{z_l}{jZ_{l+1}(\omega)} \text{]} - \text{Re[arctan} \frac{z_l}{jZ_l(\omega)} \text{]} + m \pi. \]  

(2)

Where \( \text{Re} \) represents real part. For the model of the three-layered structure of a linear-viscoelastic layer that adheres on an elastic substrate of half-space of infinite extent and immersed into water, the layer phase shift is simplified as

\[ \omega \psi_2 = \text{Re[arctan} \frac{z_2}{jZ_3(\omega)} - \text{arctan} \frac{z_2}{jZ_2(\omega)} \text{]} + m \pi. \]  

(3)

The layer thickness can be given as

\[ d_2 = \frac{c_2}{2 \omega} \{ A - \text{arctan} \frac{z_2}{jZ_2(\omega)} + m \pi \}. \]  

(4)
Where the constant $A$ reflects the phase-jump caused by the difference of acoustical impedance between the layer $z_2$ and the next substrate $z_3$, and $A = -\frac{\pi}{2}$, for the impedance condition $z_2 > z_3$, and for $z_2 < z_3$, $A = 0$. The term $\arctan[-\frac{z_2}{jZ_2(\omega i)}]$ in Eq.(3) and (4) is denoted as the measured phase shift that can be directly calculated from the reflected transfer functions.

From Eq.(3), it is derived that the measured phase shift is the linear function of the frequency $f$, so we have

$$\arctan[-\frac{z_2}{jZ_2(\omega i)}] = kf.$$  \hspace{1cm} (5)

Where $k$ is the slope. In Eq.(4) the periodical characteristic or the ambiguity of the layer phase shift $\omega i\psi_2$ is reflected by the parameter $m_i$, however, $m_i$ is only produced by the measured phase shift in the right side of Eq.(4), so the measured phase shift also is the periodical function with the same period of the layer phase shift $\omega i\psi_2$. It is concluded that the measured phase shift consists of a group of the periodic parallel straight lines with the same slope in the frequency spectra. Since resonant space $\Delta f$ satisfies $2\pi f = \pi$ and $k\Delta f = \pi$, so we have

$$k = 2\pi f = \frac{\pi}{\Delta f}.$$  \hspace{1cm} (6)

Eq.(6) is the relation of the slope of the measured phase shift with time of flight $\psi_2$ and resonant space $\Delta f$. Since the measured phase shift is the linear function of the frequency, the slope of the straight line can be easily estimated by using the curve-fitting method, the layer thickness and resonant space can be predicted using Eq.(6), even using the more less measured data and without the appearance of the resonance in the useful frequency zone.

Resonant space depends on the slope, while the impedance condition decides the origin of the layer phase-shift and resonant frequency, if the relation of the resonance with the difference between acoustical impedance of the layer and the next substrate is known, then each order resonant frequencies can be predicted from the reflected transfer functions. For the impedance condition $z_2 < z_3$, resonant frequency $f_n$ satisfies

$$2\pi f_n = \frac{2n+1}{2}, \hspace{1cm} n = 0, 1, \ldots.$$  \hspace{1cm} (7)

Then the resonant formula related to $k$ is given as

$$f_n = \frac{2n+1}{4\psi_2} = \frac{(2n+1)\pi}{2k} \hspace{1cm} n = 0, 1, \ldots.$$  \hspace{1cm} (8)

With the similar analysis, for the impedance condition $z_2 > z_3$, the resonant frequency satisfies

$$2\pi f_n = n\pi, \hspace{1cm} n = 0, 1, \ldots.$$  \hspace{1cm} (9)

We have

$$f_n = \frac{n}{2\psi_2} = \frac{n\pi}{k} \hspace{1cm} n = 0, 1, \ldots.$$  \hspace{1cm} (10)

Therefore after the line slope $k$ is estimated by using the curve fitting and considering the impedance condition, each resonant frequency can be predicted only with the comparison of $z_2$ and $z_3$ and the accurate value of $z_3$ is unnecessary to be given.
3. Numerical analysis

In this section, numerical simulation and analysis of the thickness calculation for two cases of an ultra-thin linear-viscoelastic layer and the narrow-band frequency spectra has been performed. Acoustical properties of the constituent materials are listed in Table 1.

Table 1. Acoustical properties of the constituent materials.

<table>
<thead>
<tr>
<th>Materials</th>
<th>$\rho_i$ (g/cm$^3$)</th>
<th>$c_i$ (mm/μs)</th>
<th>$d_i$ (μm)</th>
<th>$\beta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immersed Liquid 1(water)</td>
<td>0.997</td>
<td>1.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thin layer 2(Plexiglas)</td>
<td>1.175</td>
<td>2.630</td>
<td>30, 70, 150</td>
<td>0.126</td>
</tr>
<tr>
<td>The substrate 3(Al)</td>
<td>2.790</td>
<td>6.35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig.1 shows the amplitude of the reflected coefficients and the normalized layer phase shifts with the two different thicknesses ($70\mu$m and $30\mu$m) for the non-noise case. Since the first resonant frequency of the ultra-thin layer far exceeds the useful frequency band, here the ambiguity problem of the resonant order is unnecessary to be considered and then let $m_i=0$. As shown in Fig.1, the layer phase shifts calculated from the measured phase shifts in Eq.(3) consist of the two straight lines from the origin with the different slopes for two thicknesses. Using the curve fitting method, the slopes are easily estimated by Eq.(5). The slopes are estimated as $k_1=0.0534$ for the thickness of $70\mu$m and $k_2=0.023$ for $30\mu$m. Since resonant space satisfies Eq.(6), resonant space can be predicted with the known slope. In Fig.(1), resonant space are predicted as 18.7 MHz and 43.48 MHz for the thicknesses of $70\mu$m and $30\mu$m, respectively.

In Fig.1, for Plexiglas-Al structure, the impedance condition is $z_2 < z_1$, using the layer resonance formula in Eq.(8), the first two resonances $f_n(n=0,1)$ can be calculated as 21.7 MHz and 65.2 MHz with the layer thickness of $30\mu$m, which all far exceed the useful frequency region.

**Figure 1.** The amplitude of reflected coefficients and the normalized phase shifts with the two different thicknesses.

**Figure 2.** The amplitude of the ‘noisy’ reflected coefficients (a) and the fitting curve of phase shift (b). Fig.2(a) corresponds to the signals with the added 2.0% noises.
To investigate the effect of the noises on the thickness prediction, here the numerical procedure has been introduced. Fig 2 shows the amplitude of reflected coefficients of the synthetic ultrasonic signals with 2% random noises and the corresponding layer phase shift calculated from the measured phase shift. To improve the predicted accuracy, the used reflected coefficients are symmetrically selected around the main frequency $4.0 \text{MHz}$. Using the curve fitting method, the slope is estimated as $k = 0.024$, the layer thickness and resonant spacing are predicted as $31.3 \mu m$ and $41.6 \text{MHz}$, the first two resonances are calculated as $20.8 \text{MHz}$ and $62.4 \text{MHz}$.

The thickness prediction for the case of the narrow-band frequency spectra that may arise from the narrow-band transducer or the noises and disturbances has been a conventional problem in ultrasonic evaluation. Different from the case of the ultra-thin layer without the prediction of the parameter $m_i$, the ambiguity problem of the layer phase-shift makes $m_i$ uncertain in Eq.(4) and produces the multi-value problem of the thickness prediction in the existing inversions. Utilizing the characteristic of straight line and the period of $\pi$ of the layer phase-shift, here the layer thickness can be predicted without considering the ambiguity problem. The resonant order $m_i$ can be also determined for any frequencies in the case of the narrow-band spectra, even with the less available measured data.

Fig.3 shows the amplitude of reflected coefficients and the normalized phase shift of the layer with the thickness of $150 \mu m$. For the purpose of visualization and comparison, the amplitude and the layer phase-shift in the large region of $24 \text{MHz}$ are also plotted in Fig.3. Using the curve fitting, the slope is estimated as $k = 0.0115$, then $d_z$ is evaluated as $151.2 \mu m$. Using the relation $\Delta f = \pi k^{-1}$, resonant space $\Delta f$ is calculated as $8.69 \text{MHz}$, and considering the impedance condition, the resonant frequency can be predicted using Eq.(8) or (10), here the first two resonances $f_i (i = 0,1)$ are calculated as $4.35 \text{MHz}$ and $13.05 \text{MHz}$, respectively, for $z_2 < z_1$. Although the small errors happens between the true values and the estimated values of the thickness, resonant space and the resonant frequencies, which arises from the less number of significant digits of the estimated slope. The validity of the prediction method of the layer thickness is testified by the numerical analysis for the case of the narrow-band spectra with the less available measured data.
Fig. 4 shows the reflected coefficients for the case of the narrow-band spectra with the synthetic signals with the added 2% random noises. The discrete layer phase-shift is calculated from the ‘noisy’ reflected coefficients, and the corresponding fitting curve with the estimated slope \( k = 0.112 \) are plotted in Fig. 5, for the comparison of resonant space, only the fitting line corresponds to one period of the layer phase shift and the straight line calculated from the true reflected coefficients are also plotted. It is shown that the two straight lines coincide well with the slope and the length. The large deviation between the ‘noisy’ and true reflected coefficients is displayed in two ends of the real and imaginary parts of reflected coefficients in Fig. 4, however, the data points of the discrete layer phase-shift calculated from the ‘noisy’ reflected are far smooth, especially in the upper part of the useful frequency region. The different observations in Fig. 4 and 5 arise from arc tangent operation for the measured data in Eq. (3), which narrows the deviation amplitude of measured data, especially as the layer phase shift is around \( (2n+1)\frac{\pi}{2} \) \((n = 0, 1, \ldots)\). It is shown that the proposed prediction method has the stronger anti-interference capability as arc tangent operation is introduced in Eq. (3). After the slope of the fitting curve become known in Fig. 5, and considering the impedance condition, the thickness, resonant space, frequencies and corresponding order can be successively predicted from the measured data with the less available measured data.

![Figure 5. The ‘noisy’ phase shift and the corresponding fitting curve.](image)

4. Conclusions

A prediction method of the thickness, resonant space and frequency for the two cases of an ultra-thin layer and the narrow-band frequency spectra has been proposed. It is shown that the layer phase shift evaluated from the measured data of the layer on an elastic half-space substrate with infinite extent consists of a group of the periodic parallel straight lines with the same slope. Due to the characteristic of straight line and the period of \( \pi \) of the layer phase shift, it is proved that the slope of straight line is related with 2 times time of flight and the inverse of resonant space. Using the curve fitting method, the thickness and resonant space can be evaluated from the slope with the less available measured data and no information of the resonance. The resonant frequency can be further predicted with the impedance condition and the known slope. The method feasibility and anti-interference capability is analyzed and testified by numerical simulation for the two cases of an ultra-thin layer and the narrow-band spectra, respectively.
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