EXPERIMENTAL VERIFICATION OF WAVE VIBRATION ANALYSIS OF A CLASSICAL MULTI-STORY MULTI-BAY PLANAR STEEL FRAME

C. Mei* and J.G. Cherng

Department of Mechanical Engineering, The University of Michigan - Dearborn,
4901 Evergreen Road, Dearborn, MI 48128, USA

*Phone: 01 313 593 5369, Fax: 01 313 593 3851, email: cmei@umich.edu

Wave vibration analysis approach is found a systematic and concise approach in solving complex vibration problems in built-up frame structures. From the wave vibration standpoint, vibrations propagate along a uniform waveguide, and are reflected and transmitted at discontinuities. In this study, a multi-story multi-bay steel frame is built. The wave vibration analysis approach is applied to study coupled bending and axial vibration in the planar frame. The analytical results are verified through real time vibration tests on the multi-story multi-bay steel frame. Comparisons are made between the analytical and experimental results.

1. Introduction

Vibrations are normally described in terms of modes. They can also be described in terms of waves. From the wave vibration standpoint, vibrations propagate along a uniform waveguide (that is, a structural element), and are reflected and transmitted at discontinuities (such as joints and boundaries) [1-3]. The propagation relations are governed by the equations of motion, the reflection, and transmission relations are determined by the equilibrium and continuity conditions at the discontinuities. With the availability of the propagation, reflection, and transmission relations, vibrations in a complex built-up frame can be analyzed by following a simple assembling procedure. Wave vibration analysis approach is proved to be a concise and effective approach in analyzing vibrations in complex built-up structures [4-8].

In this study, a three-story two-bay steel frame is built for the purpose of experimentally validating the wave vibration analysis of a built-up planar frame. This paper is arranged as follows. In the next section, the wave assembling approach is briefly introduced using the three-story two-bay steel frame as an example. Section 3 describes the experimental set up. In section 4, impact vibration test results as well as the wave analysis results of the three-story two-bay steel frame are presented. Comparisons are made between the analytical and experimental results. In section 5, conclusions are drawn based on the comparisons.

2. Wave Vibration Analysis

Figure 1 shows a planar frame of three stories and two bays. Such a planar frame consists of 15 uniform beam elements, consequently there exist 15 pairs of propagation relations. At the boundary of the planar frame, there exist three reflection relations. There are three types of joints, name-
ly, the “L”, “T”, and “+” joints. There is one “L” joint on each side of the top corner. There are two “T” joints on each side of the planar frame, and one “T” joint on the top story. There are three “+” joints, all located inside the planar frame. Each “L”, “T”, and “+” joint contains two, three, and four equations, respectively, in describing the relations among the incoming and outgoing waves from the beam elements joined at the corresponding joint. Details on the reflection and transmission relations can be found in [4-8].

**Figure 1 Wave Vibrations of a Three-story Two-Bay Planar Frame**

Consequently, there exist 60 matrix equations in describing the propagation, reflection, and transmission relations of vibration waves in the three-story two-bay planar frame. They are as follows:

- The propagation relations along the vertical and the horizontal beam elements:
  \[
  a_i^+ = f(L_V i)A_{i-1}^+, A_{i-1}^- = f(L_V i)a_i^-,
  c_i^+ = f(L_V i)c_{i-1}^+, c_{i-1}^- = f(L_V i)c_i^-,
  b_i^+ = f(L_V i)b_{i-1}^+, b_{i-1}^- = f(L_V i)b_i^-,
  C_{HIi}^+ = f(L_{HIi})A_{HIi}^+, A_{HIi}^- = f(L_{HIi})C_{HIi}^-, C_{HIi}^+ = f(L_{HIi})B_{HIi}^+, B_{HIi}^- = f(L_{HIi})C_{HIi}^-,
  \]
  where \( i = 1, 2, 3 \) and \( f \) denotes the propagation matrix.

- The reflection relations at the boundaries:
  \[
  A_0^+ = r_A A_0^-, C_0^+ = r_c C_0^-, B_0^+ = r_B B_0^- \quad (2)
  \]
  where \( r \) denotes the reflection matrices at the boundaries.

- The reflection and transmission relations at the L joints on the left and right corner of the top story are:
  \[
  A_{H3}^+ = r_{22} A_{H3}^+ + t_{12} a_3^+, a_3^- = r_{11} a_3^+ + t_{21} A_{H3}^-;
  B_{H3}^+ = r_{22} B_{H3}^+ + t_{12} b_3^+, b_3^- = r_{11} b_3^+ + t_{21} B_{H3}^- \quad (3)
  \]
  where \( r \) and \( t \) denote the reflection and transmission matrices at the L joints.
The reflection and transmission relations at the T joints on the left side, right side, and top story are as follows, respectively:

\[ a^-_i = r_{i1}a^+_i + t_{21}A^-_{hi} + t_{31}A^-_t, \]
\[ A^+_i = r_{i1}A^-_i + t_{12}a^+_i + t_{32}A^-_i, \]
\[ A^-_i = r_{33}A^-_i + t_{23}A^-_{hi} + t_{13}a^+_i, \]
\[ b^-_i = r_{i1}b^+_i + t_{21}B^-_{hi} + t_{31}B^-_t, \]
\[ B^+_i = r_{i1}B^-_i + t_{12}b^+_i + t_{32}B^-_i, \]
\[ B^-_i = r_{33}B^-_i + t_{23}B^-_{hi} + t_{13}b^+_i, \]

where \( i = 1, 2 \).

The reflection and transmission relations at the “+” joints located inside the planar frame:

\[ c^-_i = r_{i1}c^+_i + t_{21}C^-_{hi3} + t_{31}C^-_{hr3}, \]
\[ C^-_{hi3} = r_{22}C^-_{hi3} + t_{12}c^+_i + t_{32}C^-_{hi3}, \]
\[ C^-_{hr3} = r_{33}C^-_{hr3} + t_{23}C^-_{hi3} + t_{13}c^+_i, \]

Free vibration responses can be obtained by writing equations (1) to (5) in the following matrix form

\[ Az = 0; \]

where \( A \) is a square coefficient matrix of size 180 by 180, and \( z \) a wave component vector of size 180. The natural frequencies of the frame are obtained by setting the determinant of the coefficient matrix \( A \) to zero.

Forced vibrations can be analyzed by following a similar procedure. Waves generated by externally applied force/moment can be found by considering the continuity and equilibrium conditions at the external force/moment applied point [9].

3. Experiments

A planar three-story two-bay steel frame is built by welding uniform metallic beam elements together. The material and geometric properties of the steel frame are: Young’s modulus \( E = 198.87 \text{GN/m}^2 \), Poisson’s ratio \( \nu = 0.30 \), and mass density \( \rho = 7664.2 \text{kg/m}^3 \). The width and thickness of the beam elements are 0.50 in and 1.00 in, respectively. The measured lengths of the legs of the frame are: \( L_{v1} = 20.75 \text{in} \), \( L_{v2} = 24.50 \text{in} \), and \( L_{v3} = 25.00 \text{in} \). The horizontal distances between two adjacent legs are measured as: \( L_{h1} = L_{h2} = 18.00 \text{in} \). The boundaries are all free.

In the experimental set up, the frame is suspended by fishing lines as shown in Figure 2. Impulse vibration tests are conducted on the planar frame. The measurement devices include: Brüel & Kjær impact hammer 8202, Brüel & Kjær accelerometers 4397, and LMS SCADAS 16-channel spectrum analyzer. Planar vibrations are excited by using the impact hammer. Accelerometers are mounted at various locations to capture vibrations of the frame.

4. Comparisons of Analytical and Experimental Results

Impact tests are conducted on the planar frame to study the in-plane vibrations. Accelerations measured at six different locations. The locations of impact force and accelerometers are illustrated in Figure 3.
**Figure 2** The Experimental Set-up

**Figure 3** Locations of impact hammer and accelerometers
Wave vibration analysis is based on the classical bending and axial vibration theories. The calculated and measured natural frequencies are recorded in Table 1. The calculated and measured inertance frequency responses (acceleration over force) are presented in Figure 4. It can be seen that good agreements have been reached.

Table 1 Calculated and measured natural frequencies

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Natural Frequencies (Hz)</th>
<th>Calculated</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.8</td>
<td>26.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>28.8</td>
<td>28.5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30.0</td>
<td>30.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>33.3</td>
<td>33.0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>48.6</td>
<td>48.5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>54.2</td>
<td>54.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>93.3</td>
<td>94.0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>104.2</td>
<td>104.0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>124.8</td>
<td>126.0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>134.5</td>
<td>135.5</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>178.8</td>
<td>179.0</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>186.5</td>
<td>185.0</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>203.8</td>
<td>202.5</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>211.1</td>
<td>209.0</td>
<td></td>
</tr>
</tbody>
</table>
5. Conclusions

A three-story two-bay steel frame is built for the purpose of experimentally validating the wave vibration analysis approach. In-plane vibrations are excited using an impact hammer with accelerations picked up by accelerometers mounted at various locations on the frame. The measured and calculated inertance frequency responses agree with each other reasonably well. The predicted and measured natural frequencies also show good agreement. This study validates that the wave analysis approach is an accurate approach in analyzing vibrations in complex built-up frame structures.

Acknowledgments

The first author gratefully acknowledges the support on this project from the Civil, Mechanical and Manufacturing Innovation Division of the National Science Foundation through Grant #0825761.

The authors would like to thank M. Brown and M. Solstad, Senior Engineering Technicians of the Mechanical Engineering Department, for their assistance with constructing the planar frame structure for experiments. The authors also thank graduate students F. Dang for deriving the reflection and transmission matrices of the cross joint, and S. Wu and X. Chen for their assistance with the experiment.

References

