A METHOD FOR LIQUID LEVEL DETECTION USING THE AMPLITUDE OF LEAKY-RAYLEIGH WAVES

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Based on the attenuation characteristic of leaky-Rayleigh waves at the fluid-solid interface, we propose a method for liquid level detection using the amplitude of leaky-Rayleigh waves. A finite-difference modeling is performed to investigate the impact of different liquid level on the amplitude of leaky-Rayleigh wave generated by a near-surface impulsive plane source in the solid with vibration direction perpendicular to the fluid-solid interface, through which the influence of scattering brought by the gas-liquid interface itself and the energy leak in the propagation process is discussed. The numerical simulation result shows that the influence of the scattering effect of a gas-liquid interface itself very slightly contributes to the amplitude of the leaky-Rayleigh wave and the predominant cause of the attenuation is the energy leak; with the liquid level increases, the amplitude of leaky-Rayleigh wave pulse declines exponentially and its central frequency becomes lower. From this the correlation between the amplitude of leaky-Rayleigh wave and the liquid level is built. We further conduct an experiment, whose result is consistent with our modeling result despite some slight difference in the variation range of the amplitude and the frequency shift. Thus, the feasibility of our method is proved.

1. Introduction

Rayleigh waves are a type of surface wave that travel along the free surface of solid without attenuation while decreasing rapidly in amplitude as distance from the surface increases, which was predicted by Lord Rayleigh\(^1\). When the semi-infinite space outside the solid surface is filled with fluid, the Rayleigh-type surface wave propagates with a slightly higher velocity than true Rayleigh wave and attenuates during the propagation due to the continuous energy leak into the fluid. This kind of Rayleigh-type surface wave is known as leaky-Rayleigh wave or pseudo-Rayleigh wave.

The study of Rayleigh-type waves is related to a wide range of applications, such as surface acoustic wave sensor, the non-destructive evaluation, and surface wave tomography in seismic exploration, etc. Most of these applications focus on the wave velocity information for the detection of the solid half-space. However, there are very limited studies on utilizing the attenuation information and detecting the fluid half-space. Actually, the attenuation characteristic of leaky-Rayleigh wave is closely related to the effective properties of both solid and fluid. When the solid properties are constant, the attenuation of leaky-Rayleigh wave is only affected by the effective properties of fluid and in turn reflects them, which has the possibility of being applied to the detection of fluid properties or distribution. Therefore, a step-by-step study is needed and the simple case of a gas-liquid interface (perpendicular to the fluid-solid interface) is ideal for our preliminary study.
An early comprehensive study of leaky-Rayleigh waves came from Viktorov. He investigated the propagation and attenuation characteristic of pseudo-Rayleigh waves at the interfaces of a solid half-space with both a fluid layer and a fluid half-space. The character and existence conditions of leaky-Rayleigh wave were discussed theoretically and experimentally by Mozhaev and Weihnacht, and Glorieux et al., respectively. Unlike the initial view that leaky-Rayleigh wave exists only when the fluid wave velocity is lower than the leaky-Rayleigh wave velocity, their results showed that the former one can be a little higher. By developing a modified Cagniard method, de Hoop and van der Helden presented 2-D and 3-D semi-analytic solutions for fluid-solid configurations subjected to impulsive line and point sources. Zhu and Popovics derived the full analytical solution to the “Lamb” problem in a fluid–solid half-space system with a transient normal point loading at the interface and analyzed the acoustic waves generated by it. They also discussed the excitation and measurement of fluid-solid interface waves by investigating two types of impulsive source, the explosive action in fluid and the mechanical action on solid.

Here, our study involves a fluid-solid interface and a gas-liquid interface perpendicular to each other, which cannot be solved analytically and has to rely on numerical simulation. Finite-difference (FD) method with standard staggered grid scheme has been widely used to model acoustic/elastic wave propagation through complex structures due to the good compromise it offers between numerical accuracy and computational efficiency, and therefore is adopted by this paper. A key factor of our modeling is the accuracy of elastic FD solutions for the wavefield at the fluid-solid interface, where the contrasts in materials properties are large and the shear stress is discontinuous. The FD image method is introduced to implement the explicit treatment of the fluid-solid interface.

In this paper, a gas-liquid interface in fluid half-space is used to represent the changing liquid level, which is perpendicular to a fluid-solid interface that allows a propagating leaky-Rayleigh wave. Based on this model, a 2-D FD modeling is performed to simulate the wavefield subjected to an implosive plane source in solid near the fluid-solid interface, by which the impacts of the gas-liquid interface on the propagation and attenuation of leaky-Rayleigh wave are discussed. This also helps us build the correlation between the liquid level and the amplitude of leaky-Rayleigh wave. Finally, we conduct an experiment to demonstrate our prediction in numerical modeling.

2. Theoretical basis

Since many researchers have comprehensively studied the theory of surface waves at the fluid-solid interface, here we only introduce and discuss the main conclusions. A surface wave that propagates along a fluid-solid interface is a linear combination of all bulk waves with exponential decay in the vertical direction. Let \( c_{surf} \) be the complex surface wave velocity. By deriving the expressions of displacement and stress with scalar and vector potentials, and then substituting them into the boundary conditions at the interface, one may obtain the characteristic equation

\[
-\rho_s(2-G)^2 + 4\rho_s\sqrt{(1-QG)(1-G)} - \rho_sG^2\sqrt{(1-QG)(1-FG)} = 0.
\]

In the above equation, \( \rho_s \) and \( \rho_f \) are the solid and fluid densities, and notations \( G = (c_{surf} / c_s)^2, F = (c_s / c_p)^2 \) and \( Q = (c_s / c_f)^2 \), where \( c_f \) is the fluid wave velocity, and \( c_p \) and \( c_s \) are the compressional and shear wave velocities of the solid.

Viktorov indicated that, for any material properties, Eq. (1) always has one real root corresponding to the Scholte wave and almost all of its energy is in fluid. Its velocity \( c_{Sch} \) is less than any bulk wave velocities of the materials, which means it can propagate along the interface without attenuation. When the leaky Rayleigh wave exists, Eq. (1) has a pair of complex conjugate roots corresponding to it. Only one of the complex conjugate roots is acceptable since the other one represents a surface wave that travels with exponentially increased amplitude and thus doesn’t have physical meaning. The leaky-Rayleigh wave at the fluid-solid interface is similar with true Rayleigh wave in the same free surface but travels slightly faster. Its phase velocity can be obtained from \( c_{LR} \)
\[ \text{Re}(1/c_{\text{surf}}) \text{, which is usually larger than } c_f \text{ while smaller than } c_p \text{ and } c_s. \] These velocity relations imply that leaky Rayleigh wave radiates into the fluid bulk mode and therefore will attenuate during the propagation. The imaginary part of the reciprocal of \( c_{\text{surf}} \) accounts for the energy leaked into fluid. The larger the value of \( |\text{Im}(1/c_{\text{surf}})| \), the more energy leaks into fluid.

For fixed solid properties, the fluid properties hardly affect the velocity of leaky-Rayleigh wave but determine the attenuation coefficient of it. For example: when the fluid is air, the value of \( |\text{Im}(1/c_{\text{surf}})| \) is extremely small so that the leaky-Rayleigh wave can be approximately treated as the true Rayleigh wave that propagates along the interface without attenuation; when the fluid is water, the value of \( |\text{Im}(1/c_{\text{surf}})| \) is relatively large, and as a result more energy leaks into the fluid, which leads to an obvious attenuation during the propagation. Therefore, we may take the attenuation characteristic of leaky-Rayleigh wave as a response of the fluid half-space.

As far as the problem studied in this paper, for a fluid–solid half-space with a perpendicular gas-liquid, the attenuation of the leaky-Rayleigh wave is composed of three parts: the energy leaks into the gas, the energy leaks into the liquid, and the scattering effect brought by the gas-liquid interface. The first two parts of attenuation can be calculated by combining the roots of Eq. (1) and the propagation distance. However, the last one cannot be obtained analytically. Thus, we will solve our problem numerically using finite-difference method in the following section.

3. Numerical simulation

3.1 Simulation method

Elastic wave propagation in a general isotropic elastic medium is governed by the equation of motion and the elastic constitutive relation. Using tensor notation, these equations can be written as the velocity-stress differential equations of the first order for the field variables \( v_i \) and \( \tau_{ij} \), where \( v_i \) is the particle velocity component and \( \tau_{ij} \) is the total stress component:

\[
\rho \ddot{v}_i = \tau_{ij} + f_i, \tag{2}
\]

\[
\ddot{\tau}_{ij} = \mu (v_{i,j} + v_{j,i}) + \lambda v_{k,k}\delta_{ij} + g_{ij}. \tag{3}
\]

In Eqs. (2) and (3), \( \rho \) is the medium density, and \( \mu \) and \( \lambda \) are the Lamé coefficients which can be obtained from the compressional velocity \( c_p \) and shear wave velocity \( c_s \); \( f_i \) is body force component per unit volume, and \( g_{ij} \) is the component of the external sources; \( \delta_{ij} \) is the Kronecker delta. The dot on top of the variables indicates their first derivative versus time \( t \). These two equations can also be used to describe the acoustic wave propagation in fluid by setting \( \mu = 0 \), and replacing \( \rho \) and \( \lambda \) as the fluid density \( \rho_f \) and the fluid bulk modulus \( K_f \), respectively.

![Figure 1. The stress/velocity components and the material properties in 2-D staggered grid](image)

Considering a 2-D Cartesian system with a horizontal axis \( x \) and a vertical axis \( z \) pointing downward, Eqs. (2) and (3) are discretized using staggered grid\textsuperscript{10, 11}, second order central finite differences in both space and time domain (hereafter referred to as the \( O(2,2) \) scheme). The locations of the stress and velocity components and the material properties on the staggered grid are shown in Fig. 1. During the time iteration, the velocity components and stress components are obtained by solving the FD equations and are used to update each other in a leapfrog fashion with time intervals \( \Delta t / 2 \), where \( \Delta t \) is the time step in one iteration.
When simulating wave propagation in heterogeneous media, effective media parameters are used to satisfy the boundary conditions implicitly. This is known as the parameter averaging technique.\(^{16-18}\) By assigning the interface between different materials to the shear stress grid point and averaging the spatial values of \(\mu\) and \(\rho\), effective media parameters in a staggered grid are given by

\[
\frac{\langle \rho \rangle_{\frac{j}{2}, \frac{i}{2}, \frac{1}{2}}}{\langle \rho \rangle_{\frac{j}{2}, \frac{i}{2}, \frac{1}{2}}} = \frac{\frac{\rho(j, i+1) + \rho(j, i)}{2}}{\frac{\rho(j+1, i) + \rho(j, i)}{2}}, \quad \frac{\langle \mu \rangle_{\frac{j}{2}, \frac{i}{2}, \frac{1}{2}}}{\langle \mu \rangle_{\frac{j}{2}, \frac{i}{2}, \frac{1}{2}}} = \frac{\frac{\mu(j, i) + \mu(j, i+1) + \mu(j+1, i) + \mu(j+1, i+1)}{4}}{1}. \quad (4)
\]

\[
\frac{\langle \rho \rangle_{\frac{j}{2}, \frac{i}{2}, \frac{1}{2}}}{\langle \rho \rangle_{\frac{j}{2}, \frac{i}{2}, \frac{1}{2}}} = \frac{\frac{\rho(j, i+1) + \rho(j, i)}{2}}{\frac{\rho(j+1, i) + \rho(j, i)}{2}}, \quad \frac{\langle \mu \rangle_{\frac{j}{2}, \frac{i}{2}, \frac{1}{2}}}{\langle \mu \rangle_{\frac{j}{2}, \frac{i}{2}, \frac{1}{2}}} = \frac{\frac{\mu(j, i) + \mu(j, i+1) + \mu(j+1, i) + \mu(j+1, i+1)}{4}}{1}. \quad (5)
\]

However, for a fluid-solid interface, the shear modulus of fluid which is zero cannot be substituted into Eq. (5) so that the shear modulus at the interface should be set to zero separately. In order to obtain a more accurate result, we introduce the FD image method\(^{12}\) proposed by van Vossen et al. to our simulation. An interesting conclusion they stated is that the fluid-solid interface located at the normal stress grid points of the solid brings more accuracy. By moving the fluid-solid interface to the normal stress grid points of the solid and imaging \(\tau_{xz}\) components as odd functions around the interface, the FD image method assures that \(\tau_{xz}\) vanishes at this boundary and thus allows the boundary conditions to be satisfied explicitly at the normal stress gridpoints. Further, according to their numerical example\(^{12}\), the result of the \(O(2,2)\) scheme is better than that of the \(O(2,4)\) scheme for modeling of interface waves when the grid size is sufficiently small.

![Figure 2. Schematic diagram of the gas-liquid-solid interfaces model](image)

In staggered grid FD modeling, the \(O(2,2)\) scheme requires sampling at least 10 to 15 gridpoints per minimum wavelength to limit the numerical dispersion. When the grid size is fixed, stability of the \(O(2,2)\) scheme is ensured by taking time steps \(\Delta t < \min(\Delta x, \Delta z) / (1.414 V_{\text{max}})\), which is known as the Courant stability condition\(^{10, 11}\), where \(\min(\Delta x, \Delta z)\) is the smallest grid size and \(V_{\text{max}}\) is the largest wave velocity. An unsplit convolutional perfect matched layer\(^{19}\) is implemented outside the boundary of the wavefield to mitigate spurious reflections.

The gas-liquid-solid interfaces model to be simulated is illustrated in Fig. 2. The size of the wavefield is 12cm×36cm in the x-direction and z-direction, respectively. A vertical fluid-solid interface divides the whole space into a fluid region with a size of 3cm×36cm and a solid one with a size of 9cm×36cm. In the fluid region, a horizontal gas-liquid interface is applied to represent the liquid surface and its location is determined by the variants of our model. The absorbing boundary layer surrounds the wavefield is 2cm thick. The black asterisk indicates the source position and the white ones the positions of the ten receivers with a spacing of 2cm, where the shortest vertical distance between source and receiver is 12cm and the longest one is 30cm. They are all assigned in solid and 1.5cm away from the fluid-solid interface. The source is a Ricker wavelet with a central frequency of 100kHz and is placed at the horizontal velocity gridpoint to approximately simulate a shear wave transducer (setting \(g_{xx}, g_{zz}, g_{xz}\) and \(f_z\) in Eqs. (2) and (3) to zero, and \(f_z\) as a function of
time defined by a Ricker wavelet). The materials parameters are listed in Table 1. The grid size is chosen as 0.1mm and the time step is 0.01μs. Here, we define the vertical distance between the gas-liquid interface and the furthest receiver as the liquid level \( h \).

### Table 1. Parameters of the materials

<table>
<thead>
<tr>
<th></th>
<th>Gas (air)</th>
<th>Liquid (water)</th>
<th>Solid (aluminum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressional wave velocity (m/s)</td>
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<td>1500</td>
<td>6260</td>
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<tr>
<td>Shear wave velocity (m/s)</td>
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<td>0</td>
<td>3080</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>1.21</td>
<td>1000</td>
<td>2700</td>
</tr>
</tbody>
</table>

3.2 Simulation result

![Figure 3](image_url)

Figure 3. The snapshots of the fluid/bulk pressure at three instants of (a) 40μs, (b) 70μs and (c) 100μs with a liquid level of 12cm; (d) the waveforms (horizontal particle velocity) of different source-receiver distance.

Taking the simulation result when \( h \) is 12cm as an example, we firstly investigate the propagation of surface waves at the fluid-solid interface with a gas-liquid interface perpendicular to it and discuss the attenuation of the leaky-Rayleigh wave. Fig. 3(a)-(c) shows the snapshots of the fluid/bulk pressure at three different instants. The solid and dash lines represent the fluid-solid and gas-liquid interfaces, respectively. At early instants (as shown in Figs. 3(a) and 3(b)), almost no energy transmits into the fluid due to the extremely large contrasts in material properties between the air and the aluminum, and the grey level of the leaky-Rayleigh wave in the snapshots hardly decreases. Later in Fig. 3(c), with the leaky-Rayleigh wave propagates across the horizontal location of the gas-liquid interface, the majority of its energy transmits through the gas-liquid interface, and it begins to leak energy into the fluid resulting from the reduced contrasts in material properties. Also, we may observe a compressional wave, a lateral wave, a reflected leaky-Rayleigh wave and a Scholte wave in these three snapshots. The existence of the gas-liquid interface leads to a slight scattering of the leaky-Rayleigh wave, which contains three parts: the part that transforms into the Scholte wave, that becomes the reflected wave, and that propagates to the deep side of the solid.

The corresponding waveforms are shown in Fig. 3(d). For brevity, we use P wave to represent the compressional wave and S wave the shear wave. The receivers corresponding to the first four waveforms at the distances of 12~18cm have the same horizontal locations of the gas region, and the other six receivers have the same horizontal locations of the liquid region. After enlarging the displayed amplitude of some specific time ranges in Fig. 3(d), it is clear that the waveforms consist of four different wave groups: a P and S-P, a S and leaky-Rayleigh, a Scholte, and a reflected leaky-Rayleigh wave group, where the S-P wave refers to the lateral wave between the S and P waves (the amplitude of the Scholte and reflected leaky-Rayleigh waves are magnified 100 times and 40 times, respectively). The leaky-Rayleigh wave which dominates its wave group has much larger amplitude than any other wave modes but cannot be separated from the wave group in the time domain since it has a similar velocity with the S wave. The amplitude of the Scholte wave and reflected leaky-Rayleigh wave groups which are both generated by the scattering effect of the gas-liquid interface are only 0.92% and 0.14% of the amplitude of the S and leaky-Rayleigh wave group. Also, we ob-
serve that the amplitude of the S and leaky-Rayleigh wave group remains unchanged in the waveforms recorded by the nearest four receivers and then declines markedly with the distance increases. These phenomena show that the scattering effect of a gas-liquid interface only slightly contributes to the attenuation of the leaky-Rayleigh wave, and the predominant cause of the attenuation is the leak during the propagation at the liquid-solid interface.

![Figure 4.](image)

**Figure 4.** (a) The waveforms (horizontal particle velocity) and (b) the normalized frequency spectrums of the leaky-Rayleigh wave received by the furthest receiver for the cases of different liquid level

To investigate the correlation between the amplitude of the leaky-Rayleigh wave and the liquid level, we then select the waveforms of different $h$ received by the furthest receiver and put them together, which is given by Fig. 4(a). The result is consistent with what we have learned from Fig. 3: the S and leaky-Rayleigh wave group has the largest amplitude throughout the recording time; the other two wave groups have very limited amplitude and they cannot be observed without rescaling the displayed range of the vertical coordinate. Obviously, with $h$ increases, the amplitude of the S and leaky-Rayleigh wave group declines notably. Its arrival time stays the same while the Scholte wave’s arrival time varies according to $h$. As shown in Fig. 4(b), the frequency spectrums of the S and leaky-Rayleigh wave group provide a more detailed description. Besides the notable frequency peak corresponding to the leaky-Rayleigh wave at low frequency, there is also a frequency peak at about 200kHz corresponding to the S wave, whose amplitude is small and does not rely on the $h$. Moreover, since the attenuation coefficient of the leaky-Rayleigh wave is proportional to the frequency, its high frequency components attenuate more severely than the low frequency components, and as a result the frequency peak of leaky-Rayleigh wave shifts to low frequency with $h$ increases.

4. Experimental measurement

![Figure 5.](image)

**Figure 5.** Schematic diagrams of (a) the measurement system and (b) the experimental setup

The measurement system depicted in Fig. 5(a) consists of a pulse generator (HP 214B), a digital oscilloscope (Tektronix DPO3014) and a pair of transducers (Panametrics V1548). We use the inner space of a half-filled water tank and the side surface of an aluminum cylinder to approximately represent the gas-liquid-solid interfaces model, and thus the location of the liquid surface
can be easily adjusted through controlling the water volume in the tank. As shown in Fig. 5(b), the height of the aluminum cylinder is 30cm and its bottom diameter is 20cm. Two identical piezoelectric 3-cm-diam 100-kHz shear wave transducers on the edges of the top and bottom surfaces of the aluminum cylinder are used to generate and detect leaky-Rayleigh wave pulses. Their polarization directions are perpendicular to the bottom edge while their centers are 1.5cm away from the edge. The transmitting signal is a 5-μs-width square pulse which corresponds to the central frequency of 100kHz. The vertical distance between the receiver and the liquid varies between 0 and 30cm during the measurement, which is in accordance with our numerical simulation.

Figure 6. (a) The waveforms and (b) normalized frequency spectrums of the leaky-Rayleigh wave

The experimental result is shown in Fig. 6. The time domain waveforms obtained from the experimental measurements (as shown in Fig. 6(a)) agree with our numerical simulations except that there are additional reflected arrivals due to the boundaries of the aluminum cylinder. Since the Scholte wave is overwhelmed by the reflected arrivals, we may only observe a P wave group and an S and leaky-Rayleigh wave group. The frequency spectrums of the S and leaky-Rayleigh wave group are shown in Fig. 6(b). Overall, two phenomena are consistent with what we have discussed in the numerical simulations: with the liquid level increases, (1) the amplitude of the S and leaky-Rayleigh wave group declines and (2) its central frequency becomes lower.

Figure 7. Comparison of the simulation result and the experimental result

We then extract the amplitude and the frequency shift in Fig. 4 and Fig. 6, and compare the simulation result with the experimental result. The relation curves of the normalized amplitude with the liquid level are plotted in Fig. 7(a). They indicate that the amplitude of leaky-Rayleigh wave decreases exponentially when the liquid level rises. However, the curves obtained from the numerical result have a larger variation range. The normalized frequency shift in Fig. 7(b) is defined as the relative central frequency change to the case of the 0cm liquid level. It is obvious that the higher the liquid level, the larger the frequency shift, and there is an approximately linear correlation between them. Still, the variation range of the experimental result is smaller than the simulation result.

The differences between our simulation result and experimental result can be attributed to the following two aspects. (1) The excitation of leaky-Rayleigh waves is closely related to the pulse source. A Ricker wavelet used in our numerical simulation is not sufficient to describe the frequency response of a transducer. This would lead to the difference in the frequency spectrum of the
leaky-Rayleigh wave. Also, the contact area between the transducers and the aluminum cylinder cannot be modeled in a 2-D finite difference algorithm. (2) The water used in our experiment is not strictly treated and might contain micro-bubbles. According to the study result given by Kieffer\textsuperscript{20}, even trace amount of air in the water can bring about a notable influence to the velocity of the fluid.

5. Conclusions

Leaky-Rayleigh waves have been widely used in many areas related to the detection and exploration, but the application of their attenuation characteristic is not fully investigated. Here we propose a method for liquid level detection using the amplitude of leaky-Rayleigh waves. The theoretical basis of this method is the notable difference of the attenuation characteristic of leaky-Rayleigh waves at the gas-solid and liquid-solid interfaces. The simulation result shows that the scattering effect of a gas-liquid interface is weak, and the predominant cause of the attenuation is the energy leak into the fluid; the amplitude and frequency shift of the leaky-Rayleigh wave can well reflect the variation of the liquid level. Despite some slight differences, our experimental result is consistent with the simulation result and thus demonstrates the feasibility of the method.

REFERENCES