SIGNAL PROCESSING USING MODULAR STRUCTURED NEURAL NETWORK FOR NON-STATIONARY AND NONLINEAR ACOUSTIC SYSTEMS

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Paying attention to realistic systems in the actual engineering fields, we must very often treat their systems as stochastic systems with non-Gaussian, nonlinear and/or non-stationary properties. In this paper, a regression analysis method for such stochastic systems is proposed by introducing reasonably a modular structured neural network. The proposed modular structured neural network is constructed by the hierarchical combination of each expert neural network for analyzing the regression relationship between input and output signals in each local stationary section, and a neural network for the prediction of weights contained in the above expert neural network. The effectiveness of the proposed method is experimentally confirmed by applying it to the simulation and actual road traffic noise data.

1. Introduction

In most of actual stochastic systems, a specific signal fluctuates complicatedly in a non-Gaussian distribution form, owing to the compound effect of various kinds of factors. The nonlinearity of the systems makes it more difficult to evaluate the objective system from the viewpoint of internal physical mechanism. It becomes very important to analyze such a complicated stochastic system even from a functional viewpoint. From the engineering viewpoint, effective modeling methods for complicated stochastic systems are very important in various engineering fields. In these modeling methods, a regression analysis method between the input and output signals can be employed, especially from the viewpoint of a functional approach. Regression analysis methods are fundamentally practical and they have been used in various fields\(^1,2\). Here, it is noticeable that there originally exists linear and nonlinear correlation information between input and output stochastic signals. In this kind of standard model, the regression relationship is fundamentally constructed under the assumption of stationary system with time invariant inner parameters. However, the actual stochastic system, which we encounter in daily life, exhibits various types of non-stationary properties. It is very important to develop the regression analysis method for system identification by taking account of these non-stationary properties. They can be considered by the following two approaches:

- Estimation method by introducing the local stationary section,
- Estimation method by introducing the time varying parameter.

Furthermore, in order to treat more precisely the actual data, it is necessary to pay attention to the above both methods.
On the other hand, the theory and design of artificial neural networks have advanced significantly. Much of that progress has a direct bearing on signal processing. In particular, the nonlinear characteristics of neural networks as well as the universal approximation properties make them highly suited for solving difficult signal processing problems. Especially, the multilayered neural networks have been widely applied to various fields, which can approximate given nonlinear functions within any precision\(^4\). Several methods and models for stochastic system modeling by using the multilayered neural networks have been already proposed\(^5,7\). These methods and models have focused on the nonlinear mapping ability of multilayered neural networks. They are very effective to model nonlinear properties. However, there still remains the construction of an effective method for identifying these complicated stochastic systems with non-stationary property.

In this paper, a modeling method for non-stationary stochastic systems is proposed under the assumption of local stationary process. In order to evaluate reasonably the non-stationary property, we must consider the time varying statistics or changing property of some parameters reflecting the time varying statistics. In this paper, a regression analysis method for such stochastic systems is proposed by introducing a modular structured neural network. This modular structured neural network is constructed by the hierarchical combination of each neural network for analyzing the regression characteristics between input and output signals in the local stationary section, and a neural network for the prediction of weights contained in each neural network. Finally, the effectiveness of the proposed method is experimentally confirmed by applying it to the simulation and actual road traffic noise data.

2. Construction of modular structured neural network

Let us consider an arbitrary unknown system with time variant regression parameters. At this time, it is assumed that non-stationary input and output stochastic signals are observed in each local stationary section. In this case, let \( \mathbf{x}_i \ (i = 1, 2, \ldots, M) \) be the multidimensional non-Gaussian input signals in the \( i \)-th local stationary section. Moreover, let \( y_i \ (i = 1, 2, \ldots, M) \) be the one-dimensional output signal in the \( i \)-th local stationary section. From the statistical viewpoint, the input signal \( \mathbf{x}_i \) can be employed as the explanatory variable in this regression analysis. Furthermore, the output signal \( y_i \) can be employed as the objective variable. In general, the following nonlinear model can be expressed in the \( m \)-th local stationary section:

\[
y_m = f(\mathbf{x}_m) + e_m,
\]

where \( f(\bullet) \) denotes an unknown nonlinear function of the regression relationship between the input signal and output signal. Here, \( e_m \) denotes the modelling error in the \( m \)-th local stationary section. In this case, we can employ a multilayered neural network model as the system identification in the \( m \)-th local stationary section. We consider the prediction problem of finding the regression function in the forthcoming \((M+1)\)-th local stationary section. Therefore, this non-stationary system identification model reduces to a modular structured neural network, as shown in Fig. 1. The basic idea of this modular structured neural network has been already proposed for the prediction of non-stationary time series data with local stationary sections\(^8\). According to the basic idea of this modular structured neural network, let us consider the nonlinear and non-stationary regression analysis method for the stochastic system identification.

This modular structured neural network is constructed by the hierarchical combination of each neural network (ENN: Expert Neural Network for Regression Analysis) for system identification in the local stationary section, and a neural network (NNW: Neural Network for Prediction of Weights) for the prediction of weights contained in each ENN. To simplify the notation of mathe
mational expression, let us introduce the one-dimensional input signal $x_m$. So, the purpose of this regression analysis is to predict the single regression curve in the $(M+1)$-th local stationary section. Now, we introduce ENN with a three layered neural network model. For the $m$-th local section, the input-output relation of ENN can be represented by

$$
O^{3(m)}_n = \sum_{j=1}^{n_i^{ENN}} W^{32(m)}_{ij} O^{2(m)}_j + \Theta^{3(m)}_i = \sum_{k=1}^{n_i^{ENN}} W^{23(m)}_{jk} \left( \sum_{j=1}^{n_i^{ENN}} W^{22(m)}_{ij} x_m + \Theta^{2(m)}_j \right) + \Theta^{3(m)}_i,
$$

where $O^{3(m)}_n$ denotes the output of ENN and $O^{2(m)}_j$ denotes the output of the 2nd layer respectively. The superscript denotes the layer number, $n_i^{ENN}$ is the number of units in the $i$-th layer, and the number $n_i^{ENN}$ of input units is equal to 1. Here, $w$ denotes the weight vectors and $\theta$ denotes the bias term vectors after omitting the notation of their subscripts for simplification.

By taking account of the nonlinearity of stochastic system, each ENN has the following sigmoid function in the hidden layer:

$$
g(\xi) = \frac{1-e^{-\xi}}{1+e^{-\xi}},
$$

where each unit in input and output layers has $g(\xi) = \xi$ as its characteristic function. The learning object for ENN is to minimize the following error function $E^{(m)}$:

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**Figure 1.** A modular structured neural network for non-stationary regression analysis.
\[ E^{(m)} = \sum_{i=1}^{\Delta^{(m)}} \sum_{i=1}^{E^{(m)}} (y_i - o_i^{(m)})^2, \]  

(5)

where \( \Delta^{(m)} \) denotes the data size in the \( m \)-th local stationary section. Therefore, the regression analysis using ENN can be obtained by the minimization of the error function \( E^{(m)} \).

Next, we explain the learning algorithm and the structure of NNW, which can learn the relation between ENNs. It is reasonable that weights \( W^{(m)} \) in each local stationary section vary through the whole local stationary section. So, we introduce the following model in order to grasp the relation between weights \( W^{(m)} \):

\[ W^{(m)} = h(W^{(m-1)}, W^{(m-2)}, \ldots, W^{(m-n)}), \]  

(6)

where \( h(\bullet) \) denotes an arbitrary nonlinear function, and \( n \) denotes the order of a time series model with respect to weights. The learning object of NNW is to predict the weights of the next section by using time series data from the past weights \( W^{(m-i)} \) to the present weights \( W^{(m)} \). The learning object for NNW is to minimize the following error function \( E_W \):

\[ E_W = \sum (W^{(m)} - \hat{W}^{(m)})^2, \]  

(7)

where \( \hat{W}^{(m)} \) are outputs of NNW. Therefore, the weights \( W^{(m)} \) of ENN in the next section can be obtained as outputs of NNW. Here, BP algorithm \(^9\) is used as the learning algorithms for ENN and NNW.

3. Experimental consideration

3.1 Simulation results

In order to confirm the validity of the proposed method, let us apply this method to a prediction problem of non-stationary stochastic system identification through the following simulation experiment. To simplify the calculation procedures, we adopt the single regression model with one-dimensional input-output regression relationship. Here, we consider a case when the regression coefficient is varying in a form of sinusoidal curve with the time \( t \). The relationship between the input signal \( x_t \) and the output signal \( y_t \) is given as follows:

\[ y_t = \sin \frac{2\pi}{1000} x_t + 1 + \xi_t. \]  

(8)

The stochastic input signal \( x_t \) has been generated from Gaussian random numbers with zero mean and unit variance. Moreover, the noise component \( \xi_t \) has been also generated from Gaussian random numbers with zero mean and unit variance. The numbers of non-stationary input and output data \( x_t \) and \( y_t \) are respectively 2100. As one example of prediction problems, these data are divided into 21 blocks in order to get each local stationary section. Therefore, the number of data in each individual section is 100. The learning data up to the 20-th local stationary section are used. The final goal of this problem is to predict the regression curve in the 21st local stationary section. On the other hand, as another example, the same data up to 2050 are divided into 41 blocks in order to get each local stationary section. The number of data in each individual section is 50. In this case,
learning data up to the 40-th local stationary section are used. The final goal is to predict the regression curve in the 41st local stationary section. The former case is defined as Case A. Moreover, the latter case is defined as Case B. The numbers of layers in ENN are set as $n_1^{ENN}=1, n_2^{ENN}=3$ and $n_3^{ENN}=1$ without its bias terms. The numbers of layers in NNW are set as $n_1^{NNW}=20$, $n_2^{NNW}=3$ and $n_3^{NNW}=20$ for Case A. The numbers of layers in NNW are set as $n_1^{NNW}=40$, $n_2^{NNW}=3$ and $n_3^{NNW}=40$ for Case B. These numbers are selected reasonably according to the nonlinear properties and the periodic and/or cyclic phenomena in these simulation data.

After grasping reasonably the periodic and/or cyclic property of each weight, as one example, the predicted value of $w_{3,2}^{1,4}$ using NNW is shown in Fig. 2. The predicted weight agrees well with the actual one to be predicted in the final local stationary section. Based on the predicted results of weights, the predicted result of regression curve in the 21st local stationary section is shown in Fig. 3. The objective regression curve to be predicted is directly obtained by applying ENN to the 21st data. After regarding this system as a conventional stationary one, the conventional linear regression curve using the usual regression analysis is also shown in this figure. The regression curve using ENN in the previous 20-th local section is added in this figure. Moreover, the predicted result using the proposed method for Case B has been also obtained. According to these results, the predicted results using the proposed method are much better than those using the conventional method. This is because of the reasonable consideration of non-stationary and nonlinear properties in this stochastic system identification.

![Figure 2. Predicted result of the weight $w_{4,1}^{2,3}$ using NNW.](image-url)
3.2 Application to road traffic noise

Next, let us consider a prediction problem of road traffic noise. The $L_{eq}$ noise evaluation index for the time interval of 10 seconds has been measured successively. At the same time, the vehicle number per 10 seconds passing through the observation point has been counted. The measurement has been started at the same time of traffic signal change because of the achievement of periodically stationary process. We adopt the above traffic flow as the input signal $x$ (i.e., the explanatory variable). Moreover, we adopt the normalized value of $L_{eq}$ with zero mean and unit variance as the output signal $y$ (i.e., the objective variable). The prediction of $L_{eq}$ is essential in the field of noise evaluation and/or regulation problems. Many researchers pointed out the nonlinear property between the traffic flow and the $L_{eq}$ noise evaluation index. So, the saturation phenomena of $L_{eq}$ with the increase of traffic flow have been confirmed. The numbers of input and output signals are respectively 720. These measured data have been divided into 24 data blocks. The size of single block is 30 in each local stationary section. The data up to the 23rd section are used for learning. The numbers of layers in ENN are set as $n^{ENN}_1=1$, $n^{ENN}_2=3$ and $n^{ENN}_3=1$. The numbers of layers in NNW are set as $n^{NNW}_1=10$, $n^{NNW}_2=3$ and $n^{NNW}_3=10$.

Base on the predicted results of weights by using NNW, the predicted result of regression curve in the 24-th local stationary section is shown in Fig. 4. The objective regression curve to be predicted is directly obtained by applying ENN to the 24-th data. In the case when assuming artificially this system as a conventional stationary one, the conventional linear regression curve is calculated and shown in this figure. The regression curve using ENN in the previous 23rd local section is added in this figure. As shown in this figure, this system is apparently one of the non-stationary
systems. The predicted result using the proposed method can grasp reasonably the nonlinearity of saturation phenomena between the traffic flow and the $L_{eq}$ noise evaluation index. That is, we can find out the nonlinear and saturation phenomenon between the input and output signals in this experiment.

![Figure 4](image-url)  
**Figure 4.** Predicted result of the regression curve by using the proposed method.

4. Conclusion

In most of actual stochastic systems, a specific signal fluctuates complicatedly in a non-Gaussian distribution form, owing to the compound effect of various kinds of factors. The nonlinearity of the systems makes it more difficult to evaluate the objective system from the viewpoint of internal physical mechanism. It becomes very important to analyze such a complicated stochastic system even from a functional viewpoint. From the engineering viewpoint, effective modeling methods for complicated stochastic systems are very important in various engineering fields. In these modeling methods, a regression analysis method between the input and output signals can be employed, especially from the viewpoint of a functional approach. Regression analysis methods are fundamentally practical and they have been used in various fields. Here, it is noticeable that there exists linear and nonlinear correlation information between input and output stochastic signals. In this kind of standard model, the regression relationship is fundamentally constructed under the assumption of stationary system with time invariant inner parameters. However, the actual stochastic system, which we encounter in daily life, exhibits various types of non-stationary properties. It is very important to develop the regression analysis method for system modeling by taking account of these non-stationary properties.

Several methods and models for stochastic system modeling by using multilayered neural networks have been already proposed. These methods and models have focused on the nonlinear mapping ability of multilayered neural networks. They are very effective to model nonlinear properties. There still remains the construction of an effective method for complicated stochastic systems with non-stationary property.
In this paper, a modeling method for non-stationary stochastic systems has been proposed under the assumption of local stationary process. In order to evaluate the non-stationary property, we must consider the time varying statistics or changing property of some parameters reflecting the time varying statistics. That is, a regression analysis method for such stochastic systems has been proposed by introducing a modular structured neural network. This modular structured neural network has been constructed by the hierarchical combination of each neural network for finding the regression characteristics between input and output signals in the local stationary section, and a neural network for the prediction of weights contained in each neural network. Finally, the effectiveness of the proposed method has been experimentally confirmed by applying it to the simulation and actual road traffic noise data.

The advantages of the proposed method can be summarized as follows:

- The input-output relation of NNW can grasp the changing property of time varying characteristics.
- The model parameters in the local stationary section to be predicted can be obtained by outputs of NNW.
- Even if the objective system is complicated, the size of the local model ENN can be reduced.

However, there still remain future problems to be considered as follows:

- Relations between the prediction ability and the sizes of ENN and NNW,
- Comparative study with statistical prediction methods for non-stationary data,
- Adaptability to other engineering fields, and so on.

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**REFERENCES**