ANALYSIS OF LOW FREQUENCY RADIATION CHARACTERISTICS OF A FINITE CYLINDRICAL SHELL BASED ON VIBRATION AND RADIATION MODES

Shaohu Ding, Kean Chen, Xiyue Ma
School of Marine Science and Technology, Northwestern Polytechnical University, Xi’an, China 710072

e-mail: dingshaohu05@163.com

By means of the modal expansion approach, the coupled vibration and sound wave equation for a finite cylindrical shell is solved. The expressions of the radiated sound power and the radial squared velocity are investigated. By use of the solution of the equation, the contribution of the low order vibration modes to the radial squared velocity and the radiated sound power is discussed in different fluids. Then, the sound radiation for the finite cylindrical shells is investigated based on sound radiation modes at low frequencies in different fluids. The results show that more vibration modes contribute to the sound power of the finite cylindrical shells in air, so more sound radiation modes are required to calculate the sound power. However, in water, the influence of the fluid-structure coupling on the radial squared velocity of the shells is very remarkable, the performance of the sound power radiated from the finite cylindrical shells is decreased with the increase of the circumferential vibration modes, and only first several sound radiation modes are required to accurately calculate the sound power.

1. Introduction

The finite cylindrical shell is the basic structural element widely used in many industrial fields, such as aeronautical, space and marine structures. Sound radiation from such kind of shell plays a key part in the field of noise control. The vibro-acoustic characteristics of the shell excited by mechanical forces were extensively investigated.1–5 It is well known that the acoustic pressure acting on the structure becomes important when the structure is immersed in heavy fluid such as water, sound behaviour of the shell in water is very different from the case in air, and has been received much attention.3–5 On the other hand, many researchers have devoted to develop an efficient method to reduce the unwanted noise of cylindrical structures. Generally, those methods can be divided into two subgroups. The first is passive method, which is try to make a quiet structure by mounting additional mass or damping treatments through mechanical redesign. The second subgroup is active method,6–9, which reduce the structurally-radiated noise by using actuators, sensors, and control algorithms. The passive method is often impractical to reduce the low frequency noise; therefore, active control methods have emerged as practical alternatives to passive methods for reducing unwanted noise in the low frequency ranges.

There exist many literatures about vibro-acoustic characteristics of the cylindrical shell, however, in order to use active control methods to effectively suppress the noise radiated from a cylindrical shells, it is important to have a good understanding the modal characteristics of the
vibration and radiation of the structure in the low frequency ranges. Stepanishen\textsuperscript{2} investigated the radiation impedance of an infinite cylinder with finite length non-uniform velocity distribution. Chen et al.\textsuperscript{10} discussed the radiation power and radiation efficiency of the low order modes of finite stiffened double cylindrical shells. The results show the low order modes mainly affect sound radiation. Lin et al.\textsuperscript{11} studied the characteristics of modal sound radiation of finite cylindrical shells using finite element and boundary element methods and found the modal radiation efficiencies for each group of modes having the same circumferential modal index decrease as the axial modal index increases.

In this study, the contribution of the low order vibration modes to the radial squared velocity and the radiated sound power is discussed in different fluids. And then, the sound radiation for finite cylindrical shells in different fluids is investigated at low frequencies based on structural vibration modes and sound radiation modes.

### 2. Vibration and sound radiation from finite shells

Consider now a finite cylindrical shell which is terminated by two semi-infinite cylindrical rigid baffles as illustrated in Fig. 1, \( L \) denotes the length of the shell, \( a \) and \( h \) express the radius and the thickness of the shell, respectively. The shell is immersed in an unbounded fluid whose density is \( \rho \) and sound speed is \( c \). The fluid is assumed to be stationary and non-viscous.

![Figure 1. Baffled cylindrical shell and coordinate system.](image)

When simply supported boundary conditions are considered, the out-of-plane displacement of the shell can be written as

\[
w(z_0, \phi_0) = \sum_{m=0}^{n} \sum_{n=0}^{\infty} A_{mn}^\alpha \sin(n\phi_0 + \alpha \pi/2) \sin(m\pi z_0/L)
\]  

where subscripts \( m(m=1,2,3,...) \) and \( n(n=0,1,2,...) \) correspond to the axial mode order and circumferential mode order, respectively. \( A_{mn}^\alpha \) is the amplitude of the shell radial displacement response, \( \alpha = 0 \) represents anti-symmetric shell modes and \( \alpha = 1 \) represents symmetric shell modes.

According to the classical theory of shell and considering the fluid-structure coupling, the cylindrical shell equation of motion is well known as\textsuperscript{1}

\[
M_{mn} \left[ \omega_{mn}^2 (1-i\eta) - \omega_n^2 \right] A_{mn}^\alpha = f_{mn}^\alpha - p_{mn}^\alpha
\]

where \( M_{mn} \) denotes the generalized mass of the shell mode, \( M_{mn} \left[ \omega_{mn}^2 (1-i\eta) - \omega_n^2 \right] \) denotes the mechanical impedance of the shell mode, \( \omega_{mn} \) is the in-vacuo natural angular frequencies, \( \eta \) is the
structural damping coefficient, \( f_{mn}^α \) is the generalized mode exciting force and \( p_{mn}^α \) denotes mode sound pressure induced by the fluid, respectively described as

\[
f_{mn}^α = \frac{E_a}{πL} \int_0^{2π} \int_0^L f(z_0, ϕ_0) \sin\left(\frac{mπz}{L}\right)\sin(απ/2)dzdϕ \tag{3}
\]

and

\[
p_{mn}^α = -iω \sum_{p=1}^{∞} Z_{pmn} A_{pn}^α \tag{4}
\]

where \( ε_n \) is the Neumann factor, \( ε_0 = 1 \) and \( ε_n = 2 \) for \( n > 0 \). \( f(z_0, ϕ_0)e^{iωt} \) is the harmonic point force exerted on cylindrical shell, the factor \( e^{iωt} \) has been omitted for brevity. \( Z_{pmn} \) is the radiation impedance of the shell which expresses the mode coupling between the axial shell modes (\( m \) and \( p \)) due to the fluids. The general expression for radiation impedance is

\[
Z_{pmn} = \frac{iρω}{ε_n} \int_{-∞}^{∞} \frac{H_n(\sqrt{k^2 - k_z^2} a)}{\sqrt{k^2 - k_z^2} H_n'(\sqrt{k^2 - k_z^2} a)} \int_0^L \sin\left(\frac{pπz}{L}\right)e^{-ik_zz}dz \int_0^L \sin\left(\frac{mπz}{L}\right)e^{ik_zz}dzdk_z,
\tag{5}
\]

where \( k = ω/c \) is the fluid wave-number, \( k_z \) is the structural axial wave-number. \( H_n() \) is the \( n \)-th order Hankel function of the second kind for radially outgoing wave. \( H_n'() \) is the derivative of the Hankel function with respect to its argument.

Substituting Eq. (3) and Eq. (4) into Eq. (2), the amplitude of the shell radial displacement response \( A_{mn}^α \) can be solved. And then, the radial squared velocity and the radiated sound power also can be calculated. The radial squared velocity is

\[
\left\langle \dot{w} \dot{w}^* \right\rangle = \frac{ω^2}{4ρ_a} \sum_{α=0}^{∞} \sum_{n=0}^{∞} \sum_{m=0}^{∞} A_{mn}^α \left( A_{mn}^α \right)^* \tag{6}
\]

where a dot above a variable denotes time derivative, the asterisk denotes the complex conjugate. Thus, the acoustic radiated power is

\[
W = \frac{1}{2} \text{Re} \left( \sum_{α=0}^{∞} \sum_{n=0}^{∞} \sum_{m=0}^{∞} V_{pn}^α Z_{pmn} V_{mn}^*α \right) \tag{7}
\]

### 3. Acoustic radiation modes

An alternative method to compute the modal contributions to the complex sound power can be derived using the eigenvectors of the acoustic impedance matrix \( Z \). In case of the radiated sound power, the eigenvectors of the resistive part of the acoustic impedance matrix \( \text{Re}(Z) \) correspond to the well-known acoustic radiation modes. Acoustic radiation modes are a set of orthonormal boundary velocity patterns that diagonalize the acoustic impedance matrix. At low frequencies, only a few acoustic radiation modes are found to be efficiently radiate, hence, the series expansion of the total radiated sound power can be virtually truncated without loss of accuracy.

The surface of finite cylindrical shell is uniformly divided into \( N \) elemental sources, the area of each elemental source is \( S \), the radiated sound power is given by using the near field method as

\[
W = \frac{S}{2} \text{Re} \left( \text{V}^H \text{ZV} \right) = \text{V}^H \text{R} \text{V}
\tag{8}
\]

where \( \text{V} \) is the vector of complex velocities of these elemental sources. The superscript \( ^H \) denotes the conjugate transpose. \( \text{R} = (S/2) \text{Re}(\text{Z}) \) is a real, symmetric, positive definite matrix, which is
proportional to the radiation resistance matrix for the elemental radiators. It has an eigenvector decomposition of the form,

\[ R = Q^\top \Lambda Q \]  

(9)

in which \( Q \) is an orthogonal matrix of eigenvectors, and \( \Lambda \) is a diagonal matrix of eigenvalues. Substituting Eq. (9) and Eq. (4) into Eq. (8), by define \( y = QV \), and Eq. (8) can be written as:

\[ W = y^\top \Lambda y = \sum_{k=1}^{K} \lambda_k |y_k|^2 \]  

(10)

where \( y_k = q_k^\top V \) denotes amplitude of the \( k \)th radiation mode, \( q_k \) denotes the \( k \)th radiation mode, and \( \lambda_k \) is termed as radiation efficiency coefficient. From Eq. (10), the radiation modes radiate independently, and the sound power becomes a summation of independent quantity \( y_k \) factored by eigenvalue \( \lambda_k \).

4. Results and discussion

Modal characteristics of vibration and sound radiation of finite cylindrical shells are studied based on structural vibration modes and sound radiation modes at low frequencies in different fluids. A finite cylindrical shell submerged in fluid is schematically shown in Fig.1. The geometry parameters and of the cylindrical shell is \( L = 1.2 \text{m}, a = 0.4 \text{m} \) and \( h = 0.003 \text{m} \). The structures are made of steel (density of \( \rho_s = 7850 \text{kg/m}^3 \), Young’s modulus \( E = 2.1 \times 10^9 \text{N/m}^2 \), Poisson’s ratio \( \sigma = 0.3 \)). The density and speed of the fluid is \( \rho_f = 1.21 \text{kg/m}^3 \) and \( c_e = 344 \text{m/s} \) for air, and \( \rho_w = 1000 \text{kg/m}^3 \) and \( c_w = 1500 \text{m/s} \) for water. The structural damping is introduced by means of a complex elastic modulus, i.e. \( E(1-i\eta) \), where \( \eta = 0.01 \) is the damping loss factor of shell. It is assumed that the radial excitation force is centred at a point \( z_0 = 0.44, \phi_0 = 0 \), its magnitude is 1N, and along the radius direction of the shell.

4.1 The modal vibro-acoustic characteristics of a finite cylindrical shell

The modal contribution of the low order vibration modes to the radial squared velocity in different fluids are plotted for first six circumferential modes \( n = 0-5 \) in Figure 2 and 3. From Figure 2, it can be found that the peaks for all circumferential modes have almost same amplitude except \( n = 0 \). In contrast, when cylindrical shell is submerged into water, there are more vibration modes than that in air, all vibration modes shift to low frequency due to fluid-structure coupling, and the peaks for \( n = 0 \) and \( n = 1 \) are much lower than other modes, this is because the effect of fluid-solid coupling is the largest on the low order circumferential modes.

Figure 4 and 5 present the modal contribution of the low order vibration modes to the radiated sound power in different fluids. The total radiated sound power is calculated from Eq. (7) by truncating \( m = 1-10 \) and \( n = 0-10 \). Figure 4 shown that in air, each circumferential mode contributes almost 100% to total radiated sound power at discrete frequencies below 1000Hz, for example, the \( n = 1 \) modes contribute nearly 100% at approximately 784Hz, the \( n = 2 \) mode contributes nearly 100% at approximately 383Hz, and \( n = 2, 3, 4 \) modes dominate the peak of the corresponding resonance frequencies. Compared to the case in air, the results in water exhibit very different behaviour. In Figure 5, it is shown that the structural-dominated modes are \( n = 1 \) and \( n = 2 \), the \( n = 0 \) mode only has a peak corresponding to approximately 862Hz, and structural vibration modes with \( n = 3, 4, 5 \) have a little contribution to total sound power. This indicates that the performance of the sound power radiated from the finite cylindrical shells in water is sharply
decreased with the increase of the circumferential vibration mode order, the vibration energy stored in the high order circumferential modes and does not effectively radiate to outward.

**Figure 2.** Modal contributions to the radial squared velocity from a cylindrical shell in air.

**Figure 3.** Modal contributions to the radial squared velocity from a cylindrical shell in water.

**Figure 4.** Total radiated sound power and modal contributions to the sound power from a cylindrical shell in air.
4.2 The radiated sound power using sound radiation modes

The radiation modes contribution to the sound power from a cylindrical shell in air and water are illustrated in Figure 6 and 7, respectively. The radiated sound power is calculated by Eq. (7) and Eq. (10). When ambient fluid is air, the different truncation of $K$ is compared. The results show that for $K=12$, there is a great error, three peaks of resonance frequency can not be calculated. When truncate $K=20$, there are two peaks can be obtained. Until $K=30$, the radiated sound power can be accurately calculated without any error in range of 500Hz. In water, due to higher circumferential modes ($n \geq 3$) contribute a little to the sound power below 1000Hz, it can be seen that only first ten order radiation modes are needed to calculate the sound power. The difference between the results in air with that in water indicates that higher order sound radiation modes correspond to the structural vibration modes with higher circumferential mode order.
5. Conclusion

This study provides greater physical insight into modal contribution of the low order vibration modes to the radial squared velocity and the radiated sound power in different fluids. The modal vibro-acoustic characteristics of the cylindrical shell show a total different situation in air and water. For the purpose of controlling sound power radiated from a finite cylindrical shell, the sound radiation for finite cylindrical shells is further investigated based on sound radiation modes at low frequencies, thus provide a tool to enable the active control of the vibro-acoustic responses of finite cylindrical shells.

REFERENCES