Nonlinear Dynamic Analysis of a Complex Rotor Bearing System

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The paper deals with the structural dynamic response of rotor-supported ball bearings. The mathematical model takes into account the sources of non-linearity such as the Hertzian contact force, surface waviness, varying compliance and internal radial clearance. Due to these sources there exists a transition from a non-contact to a contact state between the rolling elements and races. The effects of speed of the balanced rotor in which the ball bearings show nonlinear dynamic behaviour are analysed. The results are presented in the form of Fast Fourier Transformation (FFT), force time diagrams and phase plots. It is implied from the frequency spectra that the peak amplitude of vibrations appears at a varying compliance frequency.

Nomenclature

- c – equivalent viscous damping factor, Ns/m
- F_{θ_i} – local Hertzian contact force, N
- F_u – force due to unbalance rotor, N
- k – constant for Hertzian contact elastic deformation, N/m$^{3/2}$
- L – arc length of the wave of surface waviness, m
- N – number of wave lobes
- N_b – number of balls
- N_r – speed of the balanced rotor, rpm
- p – constant for Hertzian contact elastic deformation
- r – inner race radius, mm
- R – outer race radius, mm
- r_{θ_i} – radial displacement due to misalignment of races, mm
- R_{θ_i} – displacement at i-th ball
- t – time, s
- V_{cage} – translational velocity of the cage centre, mm/s
- V_{in} – translational velocity of the inner race, mm/s
- V_{out} – translational velocity of the outer race, mm/s
- W – radial load, N
- γ – internal radial clearance, mm
- λ_i – wavelength of surface waviness, m
- λ – Lyapunov exponent
- ω_{cage} – angular speed of the cage, rad/s
- ω_{in} – angular speed of the inner race, rad/s
- ω_{out} – angular speed of the outer race, rad/s
- Π_0 – initial wave amplitude of the wave of surface waviness, mm
- Π_p – maximum amplitude of the wave of surface waviness, mm
- θ_i – angular location of i-th rolling element, rad/s

1. INTRODUCTION

Accurate performance predictions are key to the design of rotor bearing systems. The critical role that the rolling element bearings play in the operation and performance of machine systems have rendered them vitally important. High-speed rotor bearing systems often show unpredictable dynamic responses due to manufacturing defects. As it is not possible to produce perfect surface or contours, even with the best available machine tools, so imperfection such as surface waviness in the rolling elements and races produced during the manufacturing process cannot be avoided. The radial and axial clearances provided in the design of bearings to compensate for thermal expansion, can also be a source of vibrations and introduce non-linearity in the dynamic system. When bearings are operated at high speed, they generate vibrations and noise. The behaviour of nonlinear systems often demonstrates extreme sensitivity to initial conditions. In the shaft bearing assembly supported by perfect ball bearings, the vibration spectrum is dominated by the vibrations at the natural frequency and the ball passage frequency (or the varying compliance frequency). The vibrations at this latter frequency are called ball passage vibrations (BPV).

Yamamoto[16] performed an analytical investigation on the vibratory behaviour of a vertical rotor-supported ball bearing with radial clearance. The conclusion of this work shows that the maximum amplitude at the critical speed decreases with increasing radial clearance. The subharmonic response in a simple rotor bearing system was detected experimentally by Bently.[3] Choi and Noah[5] who analysed the coherence of super- and sub-harmonics in a rotor bearing model using the harmonic balancing method along with a discrete Fourier transform procedure, which was originally used by Yamauchi.[17] The dynamic responses of high-speed rotors with bearing clearance have been studied by Ehrich.[6-9] These studies by Ehrich show the appearance of high sub-harmonic and chaotic responses in the rotor. Apart from super- and sub-harmonic responses, aperiodic whirling motions in a high-pressure oxygen turbo pump of the space shuttle main were also reported by Childs[4] as well as Kim and Noah[12].

Gustaffson et al.[10] studied the effect of waviness and pointed out that inner race waviness affects the amplitudes of vibrations at the ball passage frequency. Meyer et al.[13] presented a mathematical technique to predict the spectral components of vibrations emanating from effects like misaligned races, eccentric races, off-size rolling elements and outer race waviness. Wardle[14] showed that ball waviness produces vibrations in the axial and radial directions at different frequencies. Nizami Aktürk[1] presented a mathematical model consisting of inner, outer and ball waviness and showed that for an inner race most vibrations occur when the ball passage frequency and its harmonics coincide with the natural frequency.