A Coupled FEM-BEM Formulation in Structural Acoustics for Imaging a Material Inclusion

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A rectangular plate, point supported at four corners, is irradiated with a normally incident acoustic wave. The plate has an inclusion embedded in it supposedly at an unknown location. A structural-acoustic coupled formulation using FEM and BEM is developed and the inclusion is imaged (located) from the surface displacement response of the plate. The efficacy of the formulation is demonstrated by varying the location, size, geometry, and material properties. For variations in location, size, and geometry of the inclusion, the method is seen to give satisfactory results. The frequency of the incident wave is limited to 4775 Hz due to the available computing power. This limits the smallest size and clarity of the image for certain cases. For some cases where the material properties are widely different, and where one would expect the results to distinguish the two materials clearly, the image clarity is less than satisfactory. A list of factors is presented for possible future work.

1. INTRODUCTION

The Finite Element Method (FEM) has evolved into a standardised technique for simulating structural dynamic systems. However, it can also be used in solving a large class of partial differential equations. This is well demonstrated in standard textbooks of FEM such as Zienkowicz,[1] Cook,[2] etc. The advent of the Boundary Element Method (BEM) as a generalised numerical procedure for partial differential equations is of comparatively recent origin. BEM has been used extensively in the areas of electromagnetics, electrostatics, geophysics, and acoustics, to name a few. Each of the above methods (FEM and BEM) has its own unique advantages. For example, FEM generates sparse matrices which are computationally efficient. However, since the entire domain is discretised, the size of the problem is potentially large. In problems where the domain of interest extends to infinity, (exterior problem) the standard FEM formulation cannot be used. On the other hand, in BEM, only the boundary of the domain needs to be discretised and hence the size of the problem is smaller. Thus, the exterior problem becomes tractable. However, the matrices are full and evaluation of the matrix requires significant computational effort.

In the area of acoustics, of the several problems which can be tackled using BEM, we will be focussing on solving steady state radiation/scattering problems. In this context, Shenck[3] was one of the earliest to present a formulation of boundary integral equations in acoustic radiation problems. He presented the Surface Helmholtz Integral Formulation and the Interior Helmholtz Integral Formulation. However, he showed that for each of these formulations there occur frequencies where the solution becomes non-unique. The frequencies at which this non-uniqueness occurs are different for the two formulations. Thus, he recommended the use of both these formulations in tandem to arrive at an overdetermined system of equations which may be solved in a least squared sense. Burton and Miller[4] also studied the non-uniqueness problem associated with boundary integrals in acoustics and suggested the use of a linear combination of the Surface Helmholtz Equation (SHE) along with its differentiated form to circumvent this problem. Most later researchers have used either Shenck's or Burton-Miller's method in their work and have had their reasons to offer for doing so. Meyer et al.[5] used the boundary integral equation to solve for the acoustic radiation field from vibrating bodies of arbitrary shape. They developed an accurate and efficient numerical procedure to evaluate the singular oscillatory kernels appearing in the integral equations presented by previous workers. Seybert et al.[7] showed that for problems involving axisymmetric geometry and boundary conditions, it is possible to have a simplified model representation which eases the computational effort. Seybert and Soenarko[8] solved the problem of radiation and scattering of acoustic waves in a half space. For the case of the infinite boundary problem, a modified Green's function of the Helmholtz equation was developed by placing an image point for each source point symmetrically behind the half plane. Extending notions of FEM, Seybert et al.[6] formulated an isoparametric boundary element to solve the Helmholtz equation and used this method in predicting sound radiated from vibrating bodies. Jiang et al.[9] carried some studies on error and convergence with relation to wavelength of the acoustic wave and characteristic dimensions of the boundary element chosen to represent the geometry of the vibrating structure. All these works did not consider the effect of elasticity of the structure in affecting its acoustic radiation.

As is well known (see Fahy,[10] Cremer and Heckl,[11] Junger and Feit[12]) the finite elasticity of the radiating structure affects its radiation specially in cases such as underwater acoustics. The structure, in the presence of acoustic loading, not only radiates, it also scatters its own radiated field, leading to structural acoustic coupling. This phenomenon was studied by many researchers by coupling the FEM based structural model and the BEM based acoustic model. One of the earliest works found on this topic was by Wilton.[13] The method proposed finding the acoustic loading force on the structure from the acoustic pressures and also enforcing con-