Vibration Analysis of a Cracked Beam Subjected to a Moving Mass

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An analytical method is used to investigate the dynamic behaviour of a two-crack cantilever beam with a moving mass. In order to obtain the characteristic functions of a multi-cracked beam, the local stiffness matrices are taken into account. The Runge-Kutta numerical method has been used to solve the differential equations involved in analysing the dynamic deflection of a cracked cantilever beam with a moving mass. Comparisons are made between the dynamic deflection of a beam with a moving mass having no cracks and one with two-cracks, both of which are subjected to varying velocities and masses.

Nomenclature

A	– beam cross-sectional area
B	– width of the beam
E	– modulus of elasticity of beam material
g	– acceleration due to gravity
I	– section moment of inertia of the beam
L	– total length of the beam
m	– mass per unit length of the beam
M	– lumped mass of the moving body
n	– integer variable (varies from 1 to ∞)
q	– integer variable (varies from 1 to ∞)
t	– time
v	– velocity of the moving mass
W	– height of the beam
x	– distance of the point from the fixed end to the point of interest where deflection is desired
y	– transverse dynamic deflection of the beam
a_1, a_2	– crack depths
$T_{n,tt}(t) = d^2 T_n(t)/dt^2$	
$V_n = \int_0^L Y_n^2(x)\,dx$	
Y_n	– eigen functions of the beam
$a_1/W, a_2/W$	– relative crack depths
$\beta = L_1/L$	– relative crack position
$\gamma = L_2/L$	– relative crack position
γ_n	– eigenvalues
δ	– Dirac delta function
ρ	– mass density of the beam
ω_n	– natural circular frequency of the beam for the n-th mode

1. INTRODUCTION

Engineers have been investigating the potential hazard produced by the moving masses on structures for many several years. The dynamic response of structures carrying moving masses is a problem of widespread practical significance. In the early twentieth century, engineers such as Jeffcott[1] managed to calculate the vibration response of simple structures with a moving mass. The response characteristics of a beam subjected to a moving force were investigated by Florence[2], Steele[3], Kenney[4], and Smith[5]. Stanisic and Hardin[6], and Stanisic et al.[7] have developed a numerical-analytical method for determining the behaviour of beams with various boundary conditions and carrying a moving mass. Saigal[6] has developed expressions for beam structures with the help of the Stanisic et al. theory[7], which has a higher degree of practical significance. Later, Akin and Mofid[8] analysed such problems for finite beams with moving loads using a differential equation. Parhi et al.[9] have discussed the vibration analysis of a cantilever beam with a transverse crack using influence coefficients and stiffness elements at the crack section. None of these investigators have developed a theoretical approach for the dynamic deflection of a two-crack cantilever beam with a moving mass. In this paper, the local stiffness matrices at the cracked sections are taken into account for the dynamic deflection of a beam subjected to a moving mass.

2. EQUATION OF MOTION

The equation of motion for a uniform beam of mass m subjected to a moving mass M, as shown in Fig. 1, neglecting damping, can be written as

$$EI\frac{d^4y(x,t)}{dx^4} + m\frac{d^2y(x,t)}{dt^2} = \left[Mg - M\frac{d^2y(\eta,t)}{dt^2}\right]\delta(x-\eta), \quad (1)$$

where m is the mass per unit length of the beam, E is the modulus of elasticity of the beam, I is the moment of inertia of the beam cross-section, η is the distance considered from one end of the beam, t is the time taken by the moving mass to travel a distance η on the beam, δ is the Dirac delta function (see the Appendix), and x is the distance of any arbitrary point z on the beam.

The solution of Eq. (1) is assumed to be in a series form,

$$y(x,t) = \sum_{n=1}^{\infty} Y_n(x)T_n(t), \quad (2)$$