Analysis of Multiple-duct Variable Area Perforated Tube Resonators

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Conical concentric tube resonators are often used in commercial automotive mufflers. These are characterised by wave-coupling phenomenon across interacting ducts. Using a one-dimensional control volume approach, a mathematical model is presented for a generalised configuration of variable area perforated tube resonators. The analysis is applied to different configurations that account for waves in the incompressible mean flow and in the acoustic coupling between the interacting ducts due to the admittance of the intervening perforates. The problem has been solved by means of the Peano-Baker's series of matrix calculus, and the transmission loss results have been outlined. The predictions have been validated against the three-dimensional finite element analysis.

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Nomenclature

a_0 – speed of sound in air
f – frequency (Hz)
j – imaginary unit
k_0 – wavenumber
M_i – flow Mach number in the i-th duct
p_i – acoustic pressure
S_i – cross-sectional area of the i-th duct
U_i – flow velocity in the i-th duct
u_i – particle velocity fluctuation over U_i
u^*_{ik} – radial particle velocity at the interface of the i-th and k-th ducts
Y_0 – characteristic impedance of air

Greek Symbols

ζ_{ik} – perforate impedance at the interface of the i-th and k-th ducts
λ – latent root
μ – relative performance index
ρ_0 – air density
ρ_i – density perturbation over $\tilde{\rho}_i$ or ρ_0, in the i-th duct
σ_{ik} – porosity of the perforated interface

Subscripts

0 – air
i – i-th duct
ik – interface of the i-th and k-th ducts; the i-th row, k-th column element (of a matrix)

1. INTRODUCTION

A perforated tube surrounded by a uniform annular cavity forms a concentric tube resonator (CTR), which is used widely in the intake systems as well as the exhaust systems of automobiles.[1] A generalised algorithm has been presented for the coefficients of the system matrix of a muffler configuration with any number of uniform-area (parallel) interacting ducts.[2] The corresponding matrizant analysis, though limited to multiple coaxial ducts, has been reported elsewhere.[3] The perforated tube may combine with a variable area cavity to form a variable area concentric tube resonator (VCTR). The analytical model of the plane wave propagation and the transmission loss (TL) spectrum for one such primitive model has been reported recently.[4]

Sound transmission in a circular duct of continuously varying area was predicted by Alfredson[5] by the more primitive segmentation approach, which was in essence a segmentation of the duct into a number of subsections of uniform tubes with area discontinuity at the junction of each pair of subsections. The transmission matrix of a variable area duct carrying a compressible subsonic flow was then developed by Miles[6] with the assumption of a constant mean flow for each individual segment.

A solution of the wave equation with variable coefficients for a nonuniform duct with a moving medium was provided by Dokumaci,[7] making use of the matrizant method.[8] Later, he presented an exact transfer matrix formulation by making use of the Riccati equation of duct impedance for plane sound wave propagation, which applies to a single duct with inhomogeneities due to the axial gradient in the ambient conditions as well as the cross-sectional area variations.[9] It was later used to evaluate the performance of the acoustic elements used in the variable area CTRs. The conclusion drawn from the TL spectrum of the conical concentric tube resonators (CCTR)[4] is not entirely generic for all sorts of variable area CTRs, as an additional geometric parameter may play a role in shaping the generality into a different dimension.

Incidentally, several investigations have been reported in the literature on perforated tubes.[10,11] Modification to the multiple-duct variable area CTRs in the acoustic elements may open up new options for a designer. Inclusion of a number of variable area ducts in a certain arrangement may prove ineffectual when compared to other arrangements. So, to study the relative performance, one must analyse all possible structures or predict by extrapolating a common trend influenced by the physical factors.