On the Mode-Coupling Instability of Mechanical Systems Due to Sliding Friction Constraint

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A general lumped parameter model with two modes of vibration coupled through sliding friction constraint is considered. Stability analysis is carried out considering perfectly rigid and compliant contact models. The linear viscoelastic model and the non-linear Hunt-Crossley model of compliant contact are used to study the influence of the model of contact damping on stability. Influences of the contact parameters and the ratio of the modal parameters on the stability of sliding have been discussed in details. Mainly, three types of instabilities specifically, low-frequency flutter, high-frequency flutter, and divergence are found to exist in the class of systems considered. The condition of the existence of high-frequency flutter is shown to coincide with the condition of the Painleve paradox. Perfectly rigid contact consideration fails to provide qualitative as well as quantitative description of sliding instability in a specific parameter zone, where only a compliant contact model can properly describe the instability phenomenon. For low contact damping, stability boundary obtained from the perfectly rigid contact consideration is not the limit of that obtained from a compliant contact model, even with very high contact stiffness. Contact damping plays an important role in ascertaining the shape, size, and characteristics of the stability boundary.

1. INTRODUCTION

Friction induced instability in elastic systems has received serious attention from a large number of researchers. Several seminal review articles on the topic are available.[1-3] Depending upon the mechanisms involved, friction induced sliding instability may be broadly classified into two categories: 1) instability induced by the Stribeck effect and 2) mode-coupling instability. The central focus of the present article is on a particular type of mode-coupling instability, which goes by the name of 'kinematic constraint instability' in early literature. The term 'kinematic constraint instability' is relevant only for perfectly rigid contact assumption, where the contact boundaries are viewed as the rigid constraints to the motion. In reality, any contact surface involves elastic deformation, however small this may be. Thus, contact force emerges as the coupling force among the different modes of vibration of a sliding elastic structure. Several early studies, assuming perfectly rigid contact, are available on the topic.[1]

Mode-coupling instability is caused due to the coupling of several modes of vibration of one or more structures sliding along frictional constraint. Mode-coupling instability may or may not involve structural coupling. Hoffman and coworkers[4,5] consider mode-coupling instability in presence of structural coupling terms. Even though contact is modelled as compliant, the effects of contact parameters, such as contact stiffness and damping are not discussed. Duffour and Woodhouse[6,7] consider the transfer function approach to discuss the stability of sliding frictional contact at a single point. Discussion on the two-mode interaction is quite comprehensive and the effect of a third mode is also considered. The effects of contact compliance in normal and tangential directions are also addressed. However, the role of contact damping is missing in their discussion. Roles of normal, tangential, and angular mode coupling in friction-induced vibration have been addressed several times in reference.[8-10]

Besides mechanical systems with sliding components, sliding instabilities of the tectonic plates during earthquakes are also studied using the same class of models discussed above.[11,12] Other important classes of problems, similar to the problems of sliding instabilities, are discussed under the heading of the 'dynamics of multibody systems with unilateral constraints.'[13-15] These classes of problems deal with the dynamics of the contact bounce and the hopping phenomena of rigid or elastic multibody systems sliding along frictional boundaries. These studies find applications in robotic devices used for the sliding manipulation of objects.

Kinematic constraint instability is the central theme of the present article. Though in reference[1] this kind of instability is discussed in the context of rigid contact, it can be viewed as the special case of mode-coupling instability when compliance in the contact boundary allows elastic deformation. In order to throw light on some of the grey areas of the topic, the problem is framed in a rigorous physical and mathematical setup. Kinematic constraint instability is analysed from both rigid contact and compliant contact considerations with the objective of finding a point of convergence between the two formalisms. The minimal model of this kind of instability involves two structural modes. Even for this simple model, a complete picture of the stability characteristics is not available in literature. The present article tries to give a complete picture of the stability characteristics for two-mode interaction.

2. MATHEMATICAL MODEL AND STABILITY ANALYSIS

Friction induced self-excited instability is possible due to the mode-coupling effect. The minimal model explicating this effect involves two coupled structural modes of vibration.