Parametric Instability of a Pretwisted Cantilever Beam with Localised Damage

S. C. Mohanty

Mechanical Engineering Department, National Institute of Technology, Rourkela, 769008 Orissa, India

(Received 28 March 2007; accepted 24 September 2007)

In the present work the effects of a localised damage on the dynamic stability of a pretwisted cantilever beam subjected to a time-dependent conservative end axial force is studied. The effects of parameters like the pretwist angle, the extent of damage, position of the damage and static load factor are studied. Three parameters are used to characterise the damaged region: location, size, and effective stiffness of the damaged region. Euler beam theory is used in the analysis. From the study it is revealed that angle of pretwist has significant effect on second and third instability zones. Increase in pretwist angle has a stabilising effect on the third instability zone and destabilising effect on the second instability zone. Variation in pretwist angle does not have significant effect on the first instability region. Extent of damage has always a destabilising effect for any pretwist angle. Localised damage has a greater destabilising effect when it is located near the fixed end than when it is towards the free end, irrespective of the pretwist angle. Increase in static load component has a destabilising effect irrespective of the value of pretwist angle.

Nomenclature

A – area of cross-section of the beam at any section x
c, d – X-coordinate of the end of the damaged region
EK_b – effective bending stiffness for the damaged region
f – X-coordinate of the centre of the damage portion
$[N_s]$ – shape function matrix for twisted beam
t – time coordinate
$\Theta = (\Omega/\omega_1)^2$ – non-dimensional excitation frequency
$\xi_b = EK_b/EI$ – extent of damage in bending sense
$\tau = (d-c)/L$ – size parameter of the damaged region
$\psi = f/L$ – non-dimensional position of damage

1. INTRODUCTION

Twisted beams have wide application in many industrial problems. Compressor blades, turbine blades, aircraft propeller blades, helicopter rotor blades, twist drill bits, etc. can be modelled as twisted cantilever beams. The dynamic stability analysis of these elements is of considerable importance. Though there are many alloys and composite materials having high strength to weight ratio have been developed, during the manufacturing of these materials, inclusion of flaws affects their structural strength. The effect of localised damages on the stability behaviour of structural elements is of great importance.

There are relatively a few studies on the dynamic behaviour and stability of pretwisted beams with localised damage. The earliest vibration analysis of pretwisted beams was reported by Troesch et al. and Anlinker and Troesch.[1,2] Gupta and Rao reported the natural frequencies of tapered Timoshenko beam using finite element method.[3] Carniege Thomas analysed the coupled bending-bending vibration of pretwisted cantilever Euler beams.[4] Subrahmanyam et al. studied the vibration of pretwisted cantilever beams using the Reissner method.[5] Banerjee studied the free vibration of twisted beams by dynamic stiffness method.[6] The stability of pretwisted columns under compressive axial loads was reported by Frisch-Fay.[7] Celep analysed the dynamic stability of a simply supported pretwisted column.[8] Gurgoze studied the dynamic stability of pretwisted beams with hinged-hinged, clamped-clamped and clamped-hinged boundary conditions.[9] Lee studied buckling and stability of spinning pretwisted beams under compressive axial loads.[10] Young and Gau investigated the dynamic stability of a pretwisted cantilever beam spinning along its longitudinal axis with a periodically varying speed and acted upon by an axial random force acting at its free end.[11] Sabuncu and Evan studied the dynamic stability of a rotating asymmetric cross-section Timoshenko beam subjected to an axial periodic force.[12]

The effect of localised damage on the static and dynamic stability of beams has been studied by many investigators. Parekh and Carlson employed the idea of effective stiffness to study the parametric excitation of a bar with a localised region of damage.[13] Datta and his colleagues extended this idealised model to study the dynamic stability behaviour of tapered bars with localised damage.[14,15] Chen and Ku studied the dynamic stability behaviour of a shaft disk system with flaws.[16] Mohanty and Kavi investigated the static stability of a tapered beam with localised damage subjected to an intermediate concentrated load.[17] Shear deformation was considered in their analysis. Reported literature show that the effect of localised damage on the dynamic stability of a pretwisted cantilever beam has not been investigated.

In the present work the effects of a localised damage on the dynamic stability of a pretwisted cantilever beam subjected to a time-dependent conservative end axial force is studied. The effects of parameters like the pretwist angle, the extent of damage, position of the damage and static load factor are studied. Three parameters are used to characterise the damaged region: location, size, and effective stiffness of the damaged region. Euler beam theory is used in the analysis. The equation of motion has been derived using finite element method. The principal instability regions are established by Floquet's theory.[18]