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# On the Scaling Laws and Similarity Spectra for Jet Noise in Subsonic and Supersonic Flow

Max Kandula<sup>†</sup>

ASRC Aerospace, Kennedy Space Center, FL 32899, USA

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The scaling laws for the simulation of noise from subsonic and ideally expanded supersonic jets are reviewed with regard to their applicability to deduce full-scale conditions from small-scale model testing. Important parameters of scale model testing for the simulation of jet noise are identified, and the methods of estimating full-scale noise levels from simulated scale model data are addressed. The limitations of cold-jet data in estimating high-temperature supersonic jet noise levels are discussed. New results are presented showing the dependence of overall sound power level on the jet temperature ratio at various jet Mach numbers. A generalised similarity spectrum is also proposed, which accounts for convective Mach number and angle to the jet axis.

<sup>†</sup>Member of the International Institute of Acoustics and Vibration (IIAV)

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## Nomenclature

- $A_j$  – jet cross sectional area  
 $c$  – sound velocity  
 $d_j$  – jet exit diameter, characteristic length  
 $f$  – frequency  
 $F_t$  – thrust  
 $I$  – sound intensity ( $I = \overline{p^2}/\rho c$ )  
 $I'$  – normalised acoustic far field intensity  
 $L$  – characteristic length scale of eddies  
 $m$  – mass flow rate  
 $M$  – Mach number  
 $p$  – pressure  
 $P$  – sound power ( $P = 4\pi r^2 I$ )  
 $P'$  – sound power per unit volume  
 $r$  – distance from the sound source  
 $R$  – gas constant  
 $Re$  – Reynolds number ( $Re = \rho_j u_j d_j / \mu_j$ )  
 $s$  – entropy  
 $St$  – Strouhal number ( $St = f d_j / u_j$ )  
 $T$  – temperature  
 $u$  – velocity  
 $u_c$  – convective velocity  
 $x$  – axial distance from the nozzle exit plane  
 $y$  – radial distance from the jet axis  
 $v_i$  – turbulent velocity fluctuation  
 $W_m$  – mechanical power  
 $\delta_{ij}$  – Kronecker delta  
 $\mu$  – dynamic viscosity  
 $\rho$  – density  
 $\gamma$  – isentropic exponent  
 $\theta$  – angle from the jet axis (downstream)  
 $\eta$  – acoustic efficiency  
 $\omega_f$  – characteristic circular frequency of the eddies

## Subscripts

- $av$  – average  
 $c$  – chamber condition, convective  
 $j$  – jet  
 $p$  – peak

- ref – reference condition  
 $\infty$  – ambient fluid

## 1. INTRODUCTION

Noise from subsonic jets is mainly due to turbulent mixing, according to the early (original) theoretical model of Sir James Lighthill.<sup>1,2</sup> The turbulent mixing noise is primarily broadband. In perfectly expanded supersonic jets (nozzle exit plane pressure equals the ambient pressure), the large-scale mixing noise manifests itself primarily as Mach wave radiation<sup>3</sup> caused by the supersonic convection of turbulent eddies with respect to the ambient fluid. In imperfectly expanded supersonic jets, additional noise is generated on account of broadband shock noise emanating from shock-turbulence interaction<sup>4</sup> and screech tones<sup>5</sup>, with the tonal amplitude likely occasioned by shock-acoustic wave interaction.<sup>6</sup>

Scale models are often used in early design stages as a means of predicting the acoustical environment associated with flight vehicles. A detailed knowledge of the mechanisms of the noise generation and radiation by jets is essential in designing a scale model of the noise source.<sup>7</sup> In order to ensure complete similarity between the model and full scale, we need to satisfy similarity of flow, noise generation, and noise propagation. For a fuller discussion of the underlying physical mechanisms of jet noise, especially of sound generation, the following references may be consulted: Crighton<sup>8,9</sup>, Howe<sup>10</sup>, Dowling et al.<sup>11</sup>, and Ribner<sup>12</sup>. For recent work on the sources of jet noise, Bogey and Bailly<sup>13</sup>, and Tam et al.<sup>14</sup> may be consulted.

In practice, it is generally difficult to duplicate (simulate) all the characteristic parameters in the scale model. Model testing with very small rocket engines requires extensive safety precautions. Heated jet facilities also involve considerable complexity and cost. The use of inexpensive facilities or low gas temperatures, for example, would considerably simplify model testing.<sup>7</sup> The ability to conduct a scale model test with a substitute gas (air, nitrogen, helium, etc.) results in substantial savings (reduced costs of test facilities, test time) and other advantages. These substitute gas tests entail some compromise with the actual physics of the hot jet.