On the acoustic matching of straight, curved and twisted tubes

Luis M.B.C. Campos and Pedro G.T.A. Serrão

Centro de Ciências e Tecnologias Aeronáuticas e Espaciais (CCTAE)
Seccção de Mecânica Aeroespacial (SMA)
Instituto Superior Técnico (IST)
1049-001 Lisboa Codex, Portugal

(Received 4 April 2007; revised 22 February 2008; accepted 17 June 2008)

The acoustic matching of straight, curved and twisted rigid-walled tubes with the same constant rectangular cross-section is considered. The starting point is the acoustic pressure in an helicoidal tube of constant rectangular cross-section, which is specified exactly by Bessel functions of non-integer order. By expanding the Bessel functions in Fourier Series, the acoustic fields can be readily matched at the junction of the straight, curved or twisted tubes. The matching conditions are the continuity of acoustic pressure and particle displacement. In the case of matching of two straight tubes there is a reflection and a transmission coefficient, which depends only on the ratio of free wave impedances in the incidence and transmission media; in the case of matching involving curved (bent or twisted) tubes, e.g. straight to twisted, twisted to straight or twisted to twisted, there is one reflection coefficient and a series of partial transmission coefficients into each mode. This is illustrated by plotting the partial transmission coefficients for all modes with no less than 1% of the total energy, for the acoustic matching of a straight and a twisted tube, with various cross-sections, and radius of curvature of flexion and torsion.

1. INTRODUCTION

One of the major areas of acoustics,[1–3] is sound propagation in tubes,[4–6] e.g. horns or bars of variable cross-section[7–9] or variable-area nozzles.[10–12] Another topic of considerable interest is the case of curved ducts,[13] which has been considered both for electromagnetic waveguides[14,15] and sound waves.[16–19] The comparison with experiment[20,21] has confirmed acoustical theories of sound propagation in flat bends of rectangular cross-section based on cylindrical waves.[22,23] Numerical approaches have also been used,[24,25] including for a variation of the problem consisting of a flat bend with a partition.[26] Another approach to duct acoustics is the multimodal decomposition,[27–31] which has been applied to a flat bend of varying cross-section like an 'elephant trunk'.[32]

The present paper considers the acoustical matching of straight, curved and twisted tubes (Section 1). The starting point (Section2) is the use of helicoidal rectangular coordinates (Section 2.1) to write the rigid wall boundary conditions at the inner and outer (Section 2.2) and lower and upper (Section 2.3) sides of the duct. The wave equation (Section 3.1) is solved in the same system of coordinates, to specify the sound fields (Section 3) satisfying the two pairs of boundary conditions (Section 3.2 and 3.3). Both propagating and standing waves (Section 4) are considered (Section 4.1), the particular cases being (Section 4.2) the flat bend and straight tube. The matching conditions at the junction of the tubes, whether twisted, bent or straight, are the continuity (Section 4.3) of the acoustic pressure perturbation and of the particle displacement. This approach to acoustical matching of tubes (Section 5) is applied in three cases: (Section 5.1) propagation from a twisted to a straight tube; (Section 5.2) vice-versa; (Section 5.3) matching of twisted tubes with different curvatures of flexion and torsion.

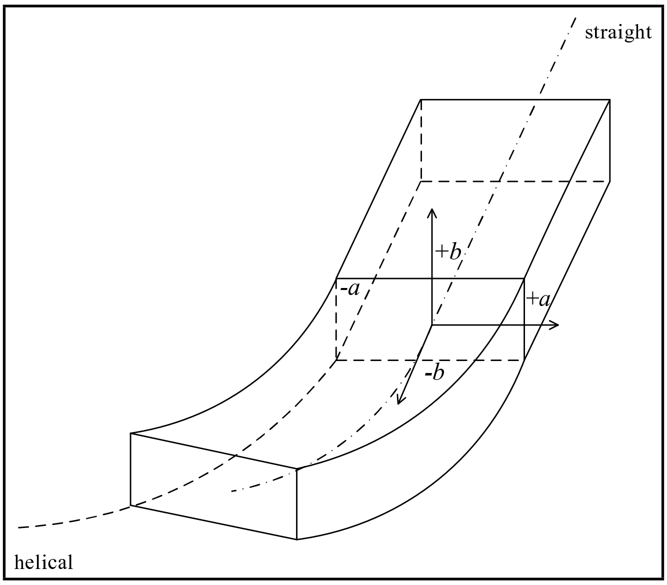

Figure 1. Acoustical matching of sound waves incident from an helicoidal tube and transmitted to a straight tube with the same rectangular cross-section.

Table 1. Matching of wavenumbers in three directions.

Direction	Coordinate	Wavenumber
Axial	θ	l
	x	Kr_0
Vertical	s	m
	z	m
Lateral	r	n
	y	n

The flat bend treated in the literature[14–25] is the particular case of a twisted bend[33] with zero torsion, and the latter, more general case, is chosen (Fig. 1 and Table 1) as an example (Section 6), plotting the reflection coefficient into the