# Cluster Control of Distributed-Parameter Structures

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When suppressing the vibration of a distributed parameter structure, control designers face the problem of its infinite number of vibration modes. It is, however, possible to group all the structural modes into a finite number of clusters, wherein all the structural modes belonging to a particular cluster have the same common attributes. If the structural modes within a given cluster are orthogonal to those in other clusters, the clusters may be controlled independently, thereby enabling cluster control with a simple control strategy without causing spillover problems. Grouping all the structural modes into a finite number of clusters is called cluster filtering, while independent control of each cluster is termed cluster actuation. Utilizing both cluster filtering and cluster actuation, cluster control may be performed. Cluster control offers the benefits of stability and control law simplicity analogous to low authority control (LAC), while providing the high control performance and some flexibility of control gain assignment of high authority control (HAC) — hence middle authority control (MAC). Some examples are demonstrated for the purpose of clarifying cluster control. By expanding on the concept of cluster control, this paper further presents a control strategy that enables the creation of a stable, vibration-free state in the designated region of a targeted structure. To this end, a cluster vector that serves as the common link between cluster filtering and cluster actuation is introduced. It is shown that the suppression of a performance index, expressed in terms of the cluster vector, leads to the generation of a vibration-free state, whereas the suppression of conventional orthogonal contributors, such as radiation modes (sometimes termed power modes), does not.

## 1. INTRODUCTION

A difficulty faced in actively controlling vibration is the infinite number of structural modes that exist in the real system. At some point, the control system designer must truncate the set of modes used in the model of the system, typically extending consideration to just a handful of low order, low frequency system modes. When the control system is then implemented, spillover effects appear. In particularly bad cases, spillover can lead to destabilization of the control system.[1] One active vibration control approach that avoids the spillover problem is direct velocity feedback (DVFB) with collocated sensors and actuators.[2] DVFB augments the damping of structural modes over a very wide (theoretically infinite) frequency range without spillover. However, DVFB implementations treat all structural modes equally, and they are not able to preferentially damp modes that are more bothersome while ignoring modes that are inconsequential to the final result. In effect, the overall result is spread thinly over all modes, leading to the label "low authority control" (LAC).[3] One area where the limitations of LAC are particularly troublesome is in the active control of structural acoustic radiation using vibration inputs. It is well known[4,5] that minimization of structural kinetic energy does not necessarily lead to the minimization of total acoustic power on a given structure; therefore, any control implementation should preferentially target attenuation of modes with the greatest radiation efficiency. Spreading the control effort evenly over all modes will not give the optimum result.

Control approaches that can specially target the most vexatious modes are often labeled "high authority control" (HAC).[6] HAC approaches, which include observer-based optimal control[7] and robust control,[8] generally aim to suppress the targeted modes in a very narrow frequency band without collocation. However, the risk of significant control spillover effects is high, and, particularly in the case of robust control; system complexity can limit practicality. It would be highly desirable to build a control system that is simple, effective, and robust — one that can avoid spillover without relinquishing the ability to preferentially target the most bothersome modes. The key to moving towards this result lies in the sensing system.

As mentioned, the most common way to handle the problem of an infinite number of structural modes in the control design stage is to truncate on the basis of frequency. Low-pass filters are then installed on the inputs in an attempt to limit observation spillover. However, a more efficient way to approach the problem, particularly when structural acoustic radiation is of interest, is to "cluster" structural modes with similar properties and truncate the number of clusters under consideration. Clustering can be approached using criteria as rudimentary as shape. For example, like-index modes on a rectangular plate (odd-odd modes, odd-even modes, etc.) could be grouped. However, a more rigorous approach to the exercise is to formulate a quadratic measure of the unwanted phenomenon, such as radiated acoustic power, and use the eigenvectors of the defining weighting matrix to generate the clusters. This is the approach taken in generating "power modes"[9–12] or "radiation modes",[13–15] which are groups of structural modes that radiate independently (from other groups).

This paper discusses a novel active control approach referred to as "cluster control",[16–22] where emphasis is placed on controlling designated groupings of structural modes. The approach falls into a category of MAC (Middle Authority Control) between conventionally-used LAC and HAC. The ap-