
On Active Noise Reduction in a Cylindrical Duct with Flow

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An analytical approach to active-noise reduction is presented in the case of a line-source in a cylindrical enclosure, minimizing the noise in a sector, corresponding to (i) the passenger head area of an aircraft cabin for a cylindrical fuselage, in the absence of flow, and (ii) a sector or an annulus of noise reduction outside of a cylindrical duct carrying a uniform axial flow of an arbitrary Mach number. In both cases, the noise is assumed to consist of the superposition of modes, and the anti-noise is used to cancel the fundamental mode and/or specified harmonics. The total acoustic energy in the region of interest is calculated for the residual and original sound field, and their ratio specifies the noise-reduction function. The latter is minimized by adjusting the source position, and the noise reduction achieved is plotted versus the dimensionless radial wave number, taking into account mean flow effects. The case of the original noise field, consisting of the fundamental and the anti-noise source set to cancel this, is taken as the baseline for further comparison. Cases with several anti-noise line-sources set to cancel various combinations of the fundamental and harmonics are also considered; for example n anti-noise sources are used to cancel the fundamental and first $n-1$ harmonics. For a given noise field, the improvement in noise-reduction performance with the number of anti-noise sources is demonstrated, both for cylindrical and planar enclosures. The addition of anti-noise sources while reducing noise at low frequencies can cause an increase in noise at high frequencies; the latter may be countered by passive means. All results obtained follow from the calculation of the noise-reduction function in terms of Bessel functions, which can be evaluated with their zeros, using asymptotic methods, which are shown to be reasonably accurate.

1. INTRODUCTION

One of the major areas of classical acoustics¹⁻⁴ concerns sound propagation in ducts, of uniform or varying cross-section, i.e., horns,⁵⁻⁹ with or without mean flow, i.e., nozzles.¹⁰⁻¹⁹ The recent interest on active-noise reduction naturally focused on duct acoustics like propeller noise suppression in an aircraft fuselage;²⁰⁻²³ this is one aspect, viz. discrete tone suppression, of active-noise reduction in enclosures, which may also involve active suppression of structural vibration.²⁴⁻²⁷ Active-noise reduction has also been considered for sound in nozzles,^{28,29} although it is more often applied to structural acoustics.³⁰⁻³³ Thus, active-noise reduction in nozzles has been much less researched than other aspects, such as propagation in sheared^{10,11,34-37} or swirling³⁸⁻⁴⁰ mean flow, effect of uniform⁴¹⁻⁴³ or non-uniform⁴⁴⁻⁴⁷ wall impedance, and radiation from the nozzle exit across a vortex sheet^{48,49} or turbulent and irregular shear layer.^{50,51} The two main purposes of this paper are (i) to address active-noise reduction in a cylindrical duct with uniform axial flow, thus applying to nozzles, and as a particular non-flow case, to a cylindrical aircraft cabin; and (ii) to take analytical methods farther than usual, e.g., to calculate the noise reduction due to active-noise suppression in a closed form.

Active-noise reduction is considered (Section 2) in a cylinder for a given frequency and axial wave number and any superposition of radial and circumferential modes. The anti-noise source is taken to be (Section 2.1) a continuous, uniform, coherent line-source parallel to the axis of the cylinder (Fig. 1), whose position is arbitrary, i.e., it is specified by a distance from the axis and angle relative to the noise-reduction sec-

tor, which are subject to optimization (Fig. 2a). The criterion for optimization is (Section 2.2) the minimum total-acoustic energy in a circular sector (Fig. 2b) simulating the passenger head area or volume in an aircraft; in the case of a nozzle, the noise reduction may be considered over an angular sector if sound transmission is more sensitive in a particular direction. If sound transmission in all directions is of concern, the angular sector, which has arbitrary aperture, can be extended to a circular annulus. The calculations are explicitly made first for a noise field consisting only of a fundamental mode; the same method applied to any harmonic, and by linear superposition, to any combination of fundamentals and harmonics. A single anti-noise source may be tuned to cancel the fundamental or any harmonic. Using several anti-noise sources, the fundamental and several harmonics can be cancelled simultaneously. Even for a noise field consisting only of one mode, e.g., the fundamental and one anti-noise source tuned to cancel the fundamental, the anti-noise source also generates harmonics, and these will appear in the residual noise field.

In general, the residual noise field (Section 2.2) will be represented by a double series of radial plus circumferential harmonics, without some terms corresponding to cancelled modes; the associated acoustic-energy density is quadratic in the acoustic pressure, and thus, is specified by a four-fold series. The integration over the whole cross-section of the cylinder would considerably simplify the series, because the eigenfunctions are orthogonal; they are not orthogonal in the noise-reduction area of interest, and thus, the reduction from a four-fold to a single series is the main mathematical challenge. The noise-reduction function is defined as the ratio of