A Simplified Formula for Calculating the Sound Power Radiated by Planar Structures

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Based on the wave number transform, the sound power radiated by a planar structure embedded in a baffle can be derived theoretically as a double integral within the range of supersonic wave numbers. Due to the specific form of the integrand, a singularity problem is encountered at the integration boundary that leads to difficulties in evaluating the sound power directly. This technical note discusses a mathematical method for simplifying the original formula to avoid this singularity problem by using a technique of substituting variables. Compared to its original counterpart, the simplified formula not only eliminates the singularity points but also converges much more rapidly when evaluating the double integrals numerically.

1. INTRODUCTION

In noise control engineering, the sound power generated by a structure often needs to be evaluated analytically for design and/or troubleshooting purposes. A classical model that is typically used in practice to describe the sound radiation behaviour of planar structures consists of a plate embedded in an infinite baffle. In the frequency domain, the sound power radiated by this finite plate is normally expressed as a two-dimensional integral within a finite area as shown below:[1]

$$W(\omega) = \frac{\omega\rho}{8\pi^2} \iint\limits_{k>\sqrt{k_x^2+k_y^2}} \frac{|V(k_x,k_y)|^2}{\sqrt{k^2-(k_x^2+k_y^2)}} dk_x dk_y; \quad (1)$$

where W is the sound power, ω is the radian frequency, ρ is the ambient medium density, k is the acoustic wave number of the ambient medium that is given as $k = \omega/c$; c is the speed of sound in the medium, k_x and k_y are x and y components of the flexural wave number of the plate, and $V(k_x, k_y)$ is the two-dimensional velocity wave number transform of the plate, which can be expressed as

$$V(k_x,k_y) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} v(x,y) e^{ik_x x} e^{ik_y y} dx dy; \quad (2)$$

in which $v(x, y)$ is the velocity distribution on the plate and $i = \sqrt{-1}$ is the unit imaginary number.

Based on the acoustical theory, only the supersonic wave number components contribute to the sound power radiation.[2] Thus, the original formula in Eq. (1) requires that the integral is calculated within the range in which the flexural wave number of the structure ($\sqrt{k_x^2 + k_y^2}$) is less than the acoustical wave number (k) in the ambient medium. This requirement induces a singularity problem in the integral when the denominator of the integrand is zero at the boundary of $k = \sqrt{k_x^2 + k_y^2}$. Therefore, to obtain reliable results, we must handle this singularity problem carefully when evaluating the sound power directly in accordance to Eq. (1). One possible way to avoid the singularity is letting the integration range infinitesimally approach but never reach the boundary. That is, the original integration can be approximated as

$$W(\omega) = \frac{\omega\rho}{8\pi^2} \int_{k_y=-\sigma k}^{k_y=\sigma k} \int_{k_x=-\sigma k_y}^{k_x=\sigma k_y} \frac{|V(k_x,k_y)|^2}{\sqrt{k^2-(k_x^2+k_y^2)}} dk_x dk_y; \quad (3)$$

where σ is a weighting coefficient whose value is between 0 and 1. Although Eq. (3) does not suffer from the singularity problem, it inevitably introduces a truncation error in the double integral since the weighting coefficient σ can never be 1. Obviously, the closer the value of σ to 1, the more accurate the result will be.

Although other approaches such as the elemental-radiator method[3,4] may also be considered to estimate the sound power, the current study focuses on a mathematical method for simplifying the original formula so that the singularity problem is removed without introducing the above-mentioned truncation error. The advantages of the simplified formula are also demonstrated by comparing the convergence rates of the two methods using a numerical example.