Natural Frequencies and Acoustic Radiation Mode Amplitudes of Laminated Composite Plates Based on the Layerwise FEM

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In this paper, the natural frequencies and acoustic radiation mode amplitudes of laminated composite plates are studied. The layerwise finite element model is imposed to determine the natural frequencies and velocity distributions of laminated composite plates. The amplitude of the laminated composite plates are then discussed based on the acoustic radiation mode, the effects of the panel orientation angle, the elastic modulus ratio, the width-depth ratio, and the damping ratio on the first acoustic radiation mode. A sixteen-layer laminated plate was used as an example, and the numerical simulations and experimental results show that the natural frequencies of the laminated composite plate can be analysed accurately using the proposed model. Furthermore, it is found that the effects of the panel orientation angle and width-depth ratio on the acoustic radiation mode amplitude of the laminated composite plates are significant.

1. INTRODUCTION

Laminated composite plates have been widely used in aerospace vehicles, maritime carriers, and wind turbine blades, where high strength, high stiffness, and low weight are important properties. When these composite structures are used in dynamic environments, vibration control and noise reduction become of great technical significance. The vibration of laminated composite structures is generally in low-frequency ranges. Thus, noise reduction of laminated composite structures in low-frequency ranges is very important. Although active vibration control could be used to decrease structural noise, low noise design is the most reliable means of reducing radiated noise. The most suitable function of a low noise design is in sound radiation power. A low noise design optimisation is considered with the goal of minimising the total sound power of the structure. The sound radiation power is related to the characteristics of the structure. Therefore, it is important to study the relation of the laminated composite structure parameters to the sound radiation power in low-frequency ranges.

For planar radiators, it is well known that the low-frequency sound radiation is directly related to the velocity distribution of the structure’s surface. Different velocity distributions have different contributions to the radiation sound power. Velocity distributions corresponding to certain mode shapes are more efficient radiators. It has been widely accepted that structural acoustic problems can be analysed based on so-called radiation modes. Radiation modes are sets of independent radiating velocity distributions. The dominant radiation modes are the first few order modes at low-frequency ranges. Controlling the first few order radiation modes’ amplitudes has been shown to reduce the total sound power efficiently in low-frequency ranges.

For laminated composite plates, the dynamic response of the structures must first be analysed in order to study the first few order radiation modes’ amplitudes of the plates in low-frequency ranges. Laminated plate theories are essential to providing an accurate analysis of laminated composite plates, and a variety of laminated plate theories have been developed and reported in the literature. A review of the various equivalent single layer and layerwise laminated plate theories can be found in Reddy’s work. Because of their complex behaviour in the analysis of laminated composite plates, some technical aspects must be taken into consideration. For example, the classical laminate plate theory (CLPT) is based on the Kirchhoff plate theory. It is the simplest theory, but the shear deformation effects are neglected. Furthermore, it results in an underestimate of the deflection and an overestimate of the natural frequencies. The first and higher order shear deformation theories are improvements over classical theories. For the first and higher order shear deformation theories, transverse shear deformation through the thickness of the structure is not ignored.

Another aspect in the analysis of composite structures is the existence of couplings among stretching, shearing, bending, and twisting. These couplings can significantly change the response of composite structures and need to be considered. The layerwise lamination theory assumes a displacement representation formula in each layer. The layerwise finite element theory can be seen as a three-dimensional theory. The assumed layerwise displacement field uses a linear Lagrange polynomial approximation for the thickness of each lamina and a constant transverse displacement for the entire thickness. The interlaminar stresses can be predicted accurately, and the layerwise finite element theory can better adapt to the combination of boundary conditions. In this paper, the dynamic responses of the laminated composite plates are examined, based on a layerwise finite element theory.

In recent years, there has been much research on the vibration and acoustical characteristics of laminated composite plates. For example, Li et al. deal with the structural vibration and acoustic radiation of a fluid-loaded laminated plate based on the first order shear deformation theory (FSDT) and the classical Kirchhoff-Love thin plate theory. Cao et al.
present the structural and acoustical characteristics of stiffened plates in the form of the transverse displacement spectra and sound pressure. The equations of motion for the composite laminated plate are derived on the basis of the first-order shear deformation plate theory. Acoustic radiation is analysed using the Fourier wave number transform and the stationary phase method. The numerical study of the vibration and acoustic response characteristics of a fibre-reinforced composite plate in a thermal environment is presented by Jeyaraj et al.,14 and the critical buckling temperature and vibration response are obtained using the finite element method based on the classical laminate plate theory (CLPT), while the sound radiation characteristics are obtained using a coupled FEM/BEM technique. Niu et al.2–4 propose optimizing the vibrating laminated composite plates for minimum sound radiation. However, little has been reported on the relation of the laminated composite structure parameters to the sound radiation in the low-frequency range. This study aims to disclose the vibrating features and acoustic radiation mode amplitudes of laminated composite plates so that effective composite structure parameter approaches can be implemented to address the low noise design problem of such structures in low-frequency ranges.

In this paper, the structure vibration and acoustic radiation mode amplitudes of laminated composite plates based on layerwise finite element models are presented. Furthermore, based on the acoustic radiation mode, the effects of the orientation angle, the elastic modulus ratio, the width-depth ratio, and the damping ratio on the amplitude of the first acoustic radiation mode for laminated composite plates are discussed.

2. RADIATION MODES THEORY

Consider a planar structure vibrating with an angular frequency $\omega$ and radiating sound into the upper half space $V$ exterior to the panel surface $S$ for $z > 0$. The density of the medium is $\rho$, and the sound speed is $c$. The surface area of the vibrating plate is $S$. The plate is divided into $J$ elements with equal areas. Assume that the elemental source is small compared to the acoustic wavelength. The vector of the normal velocities of each element is denoted as $U(\omega)$. The acoustic power can be expressed as

$$ W(\omega) = \frac{\rho c S}{2} |U(\omega)|^2 \cdot R \cdot U(\omega); $$

(1)

where the superscript $^H$ denotes the complex conjugate transpose. The matrix $R$ is a $J \times J$ matrix of the real part of the acoustic transfer impedance between each pair of elements. Because the matrix $R$ is real, symmetric and positive definite,1 the following eigenvalue decomposition is always possible: $R = QAQ^H$, where the superscript $^T$ denotes the transpose. The matrix $A$ is a diagonal matrix with eigenvalues $\lambda_i$, which are related to the radiation efficiency. $Q = [Q_1, Q_2, \ldots, Q_i, \ldots, Q_J]$ is a group of orthogonal surface vibration patterns, and $Q_i$ is a real vector representing the $i$th radiation mode shape. Because the matrix $R$ is related to the frequency, the radiation mode shapes $Q_i$ are also frequency dependent. It has been demonstrated that the radiation mode shapes can be chosen to be independent of the frequency at low-frequency ranges.15,16

Any vibration velocity distribution $U(\omega)$ has a representa-

Figure 1. Laminated plate geometry and coordinate system.

tion in terms of the radiation modes $Q_i$:

$$ U(\omega) = \sum_{i=1}^{J} v_i(\omega)Q_i; $$

(2)

where $v_i(\omega) = Q_i^T U(\omega)$

(3)

and $v_i(\omega)$ is called the $i$th radiation mode’s amplitude.1

Because $R = QAQ^H$, the total radiation acoustic power can be rewritten as

$$ W(\omega) = \frac{\rho c S}{2} \sum_{i=1}^{J} \lambda_i |v_i(\omega)|^2. $$

(4)

From Eq. (4), it can be demonstrated that each radiation mode contributes to the sound power independently. Equation (4) shows that the total radiation acoustic power $W(\omega)$ is the sum of the individual mode amplitudes squared, multiplied by their corresponding eigenvalues. The relative contribution of the first radiation mode to the radiated sound power is only dominant for low frequencies.1,17 At low-frequency ranges, the sound power of the first radiation mode is the main part of the total sound power. Moreover, after cancelling the sound power of the first radiation mode, the total acoustic power can be reduced significantly.1 Therefore, acquiring information on the amplitudes of the first radiation modes is important to control the acoustic radiation power of the vibrating structure. However, for higher frequencies, all radiation modes have similar radiation efficiencies, so more radiation modes have to be taken into account to reduce the radiated noise. Moreover, in order to obtain reduction at higher frequencies, i.e., $ka > 1$, where $k$ is the wave number and $a$ is the characteristic radius of the structure, more radiation modes are needed.

In this paper, the vibration and sound radiations of a laminated composite plate at low-frequency ranges are presented. This will be discussed in the next section using the layerwise FEM.

3. THE NORMAL VELOCITY DISTRIBUTION

Based on the layerwise theory, each laminate in the direction of the thickness needs to be interpolated twice, and the laminated plate geometry and coordinate system are shown in Fig. (1). The displacement of the laminated plate can be written as:

$$ U(x, y, z, t) = \sum_{i=1}^{2n+1} \mu_i(x, y, t) \Psi_i(z); $$

(5)
\[ V(x, y, z, t) = \sum_{i=1}^{2n+1} v_i(x, y, t) \Psi_i(z); \]
\[ W(x, y, z, t) = \sum_{i=1}^{2n+1} w_i(x, y, t) \Psi_i(z). \]
Table 1. Material properties of the viscoelastic laminate plate.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (x direction)</td>
<td>0.9498 m</td>
</tr>
<tr>
<td>Length (y direction)</td>
<td>0.3480 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.00762 m</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>68.9 GPa</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.3, 0.49</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>0.67 GPa</td>
</tr>
<tr>
<td>Density</td>
<td>2740 kg/m³, 999 kg/m³</td>
</tr>
</tbody>
</table>

Table 2. Natural frequencies and the loss factor of the viscoelastic laminated plate.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency (Hz)</th>
<th>Loss factor</th>
<th>Ref. 16</th>
<th>Ref. 17</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80.3</td>
<td></td>
<td>0.190</td>
<td>0.1901</td>
<td>0.1758</td>
</tr>
<tr>
<td>2</td>
<td>115.4</td>
<td></td>
<td>0.203</td>
<td>0.2034</td>
<td>0.1924</td>
</tr>
<tr>
<td>3</td>
<td>130.6</td>
<td></td>
<td>0.203</td>
<td>0.2034</td>
<td>0.1924</td>
</tr>
<tr>
<td>4</td>
<td>178.7</td>
<td></td>
<td>0.181</td>
<td>0.1806</td>
<td>0.1700</td>
</tr>
<tr>
<td>5</td>
<td>195.7</td>
<td></td>
<td>0.174</td>
<td>0.1737</td>
<td>0.1562</td>
</tr>
</tbody>
</table>

Generally, the damping value of the composite laminate plate in the low-modal range is related to the degree of anisotropy and each mode of the composite laminate plate. The Rayleigh damping model is used in this paper, and the damping matrix C can be written as

\[
C = \alpha M + \beta K; \quad (35)
\]

where \( \alpha, \beta \) are the mass and stiffness factors, respectively. The element equivalent nodal force can be expressed as

\[
F = \int\int\int_{V} N^{T} p_{0} dx dy dz. \quad (36)
\]

From the unit stiffness matrix \( K \), mass matrix \( M \), damping matrix \( C \), and element nodal force \( F \), the total power equation can be obtained:

\[
M \ddot{X} + C \dot{X} + KX = F. \quad (37)
\]

From Eq. (37), the surface normal displacement vector \( X \) can be determined, and the normal velocity vector \( U_{X}(x, y, \omega) \) and natural frequency \( \omega \) can be obtained from \( |K - \omega^{2}M| = 0 \).

4. NUMERICAL EXAMPLES

In order to check the quality of the layerwise FEM method, the natural frequencies of a viscoelastic laminate plate and a T300 laminated plate are studied. The first acoustic radiation mode amplitude of a 16-layer laminate plate with a simply-supported boundary condition is presented.

4.1. Natural Frequencies of a Laminate Plate

The physical properties of a viscoelastic laminate plate under a simply-supported boundary condition are summarized in Table 1. The natural frequencies and loss factor of the viscoelastic laminate plate are determined: \( \omega = \sqrt{\text{Re}(\omega)^{2} \text{Im}(\omega)^{2}} \), \( \eta = \text{Im}(\omega)^{2} / \text{Re}(\omega)^{2} \). We used regular grids of 60x60 elements in the \( x \)- and \( y \)-directions.

The result is obtained from Eq. (37) and is compared with the result in references 19 and 20. A comparison of the results is shown in Table 2. The results show good agreement for both the frequencies and loss factors in this paper. Reference 19 takes a first-order shear deformation theory to describe the deformation of the faces. The first-order shear deformation theory

![Figure 2](image)
Young's modulus is...mer signal are simultaneously measured by the modal analysis software. The transfer functions of the sixteen points are analyzed in the modal analysis software. Curve fitting the sixteen transfer functions give the natural frequencies of the structure. A typical measured transfer function is shown in Fig. (3). All experiment apparatus are demarcated, and all measurement signals are transformed by the engineering unit and limited to a frequency range below 800 Hz. The numerical simulations and experimental results of the natural frequencies are listed in Table 4. Table 4 shows that there are errors between the experimental and numerical simulation results. The first reason is because the boundary condition is slightly different in the analytical model and in the experimental set-up. The second reason is that the density and Poisson ratio of the laminated composite plate are not constant.

4.2. Factors of the Acoustic Radiation Mode Amplitude

The physical properties of the 16-layer laminate simply-supported plate are as follows: $L_x = 0.40$ m, $L_y = 0.25$ m, the single thickness is 0.125 mm, the total thickness is $h = 2$ mm, Young’s modulus is $E_1 = 181$ GPa, and $v_{12} = v_{13} = v_{23} = 0.28$. One point force (force amplitude of 10 N), located at $x = 0.5 \cdot L_x$, $y = 0.5 \cdot L_y$, is applied to excite the plate. Assume that the bonding layer materials damping ratio is ignored.

4.2.1. The Effect of the Laying Angle

Assume that the laminated plate damping ratio is 0.2, the shear modulus is $G_{23} = G_{21} = G_{13} = 7.17$ GPa, and $E_2 = E_3 = 10.3$ GPa. The four orientation angles are $\alpha = (-15/15)_{\circ}$, $(-30/30)_{\circ}$, $(-45/45)_{\circ}$, and $(-90/90)_{\circ}$. The natural frequencies and the first acoustic radiation mode amplitude of the laminated plates with different orientation angles are shown in Table 5 and Fig. (4). It is found that the first acoustic radiation mode amplitude of the laminated simply-supported plate is only related to the (odd, odd) order structure mode. Frequencies corresponding to the maximum amplitude are equal to frequencies of the (odd, odd) order structure mode. These characteristics are the same as those of the acoustic radiation of the general single-layer structure. In addition, Table 5 and Fig. (4) show that the acoustic radiation mode is only related to the geometrical shape of the radiating structure and is independent of the material properties. Furthermore, besides $(-90/90)_{\circ}$, the smaller the orientation angle of a laminated plate is, the smaller is the natural frequency of the corresponding structure mode values. This means that the more natural the frequency number is, the greater the first-order acoustic radiation mode amplitude is and the greater the corresponding radiated power value is at the same external excitation frequency. Therefore, if a 45 degree orientation angle for the laminated plate is used, the laminated plate noise can be reduced significantly.

4.2.2. The Effect of the Young’s Modulus Ratio

Assuming that the orientation angle of the laminated plate is $\alpha = (-45/45)_{\circ}$ and the damping is 0.2, the shear modulus is $G_{23} = G_{21} = G_{13} = 0.6 E_2$ and Young’s modulus is $E_2 = E_3$. In addition, the elastic modulus ratio $E_1/E_2$ is, respectively, 5, 10, 15, and 20 for the four orientation angles. The natural frequency and the first order acoustic radiation mode amplitude of the laminated plates with different Young’s modulus ratios are shown in Table 6 and Fig. (5).

Table 5. Natural frequencies of the laminated plate with different orientation angles.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\alpha = 15^\circ$</th>
<th>$\alpha = 30^\circ$</th>
<th>$\alpha = 45^\circ$</th>
<th>$\alpha = (0/90)^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>139</td>
<td>157</td>
<td>152</td>
<td>151</td>
</tr>
<tr>
<td>2</td>
<td>298</td>
<td>291</td>
<td>288</td>
<td>287</td>
</tr>
<tr>
<td>3</td>
<td>476</td>
<td>442</td>
<td>429</td>
<td>442</td>
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<tr>
<td>4</td>
<td>510</td>
<td>489</td>
<td>481</td>
<td>477</td>
</tr>
<tr>
<td>5</td>
<td>635</td>
<td>614</td>
<td>605</td>
<td>601</td>
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<tr>
<td>6</td>
<td>803</td>
<td>755</td>
<td>737</td>
<td>728</td>
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<tr>
<td>7</td>
<td>871</td>
<td>853</td>
<td>845</td>
<td>824</td>
</tr>
<tr>
<td>8</td>
<td>1001</td>
<td>912</td>
<td>880</td>
<td>841</td>
</tr>
<tr>
<td>9</td>
<td>1152</td>
<td>945</td>
<td>887</td>
<td>862</td>
</tr>
</tbody>
</table>

Table 6. Natural frequencies of the laminated plate with different $E_1/E_2$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$E_1/E_2 = 5$</th>
<th>$E_1/E_2 = 10$</th>
<th>$E_1/E_2 = 15$</th>
<th>$E_1/E_2 = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>139</td>
<td>157</td>
<td>152</td>
<td>151</td>
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<td>2</td>
<td>298</td>
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<td>476</td>
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<td>9</td>
<td>1152</td>
<td>945</td>
<td>887</td>
<td>862</td>
</tr>
</tbody>
</table>
4.2.3. The Effect of the Thickness Ratio

Assuming that the orientation angle of the laminated plate is $\alpha = (-45/45)^8$ and the Young’s modulus is $E_2 = E_3 = 10.3$ GPa, the shear modulus is $G_{23} = G_{21} = G_{13} = 0.6E_2$. In addition, $b/h$ is, respectively, 150, 125, 100 and 75 for the four orientation angles, where $b$ is the defined side length of the plate. The natural frequency and the first order acoustic radiation mode amplitude of the laminated plates with different $b/h$ ratios are shown in Table 7 and Fig. (6).

Table 7 and Fig. (6) show that the $b/h$ ratio has a great effect on the acoustic radiation mode amplitude. The corresponding natural frequencies of the same order structure mode vary with the $b/h$ ratio, and the larger the $b/h$ ratio is, the smaller the natural frequencies of the same order structure mode are. The acoustic radiation mode amplitude decreases as the peak corresponding to the frequency increases, and the total radiated sound power decreases accordingly when the thickness of the laminated plate increases. Therefore, a smaller $b/h$ ratio is better in order to reduce the laminated plate noise. However, increasing the thickness is limited by the material and process.

4.2.4. The Effect of Damping

Assuming that the Young’s modulus of the laminated plate is $E_2 = E_3 = 10.3$ GPa, the shear modulus is $G_{23} = G_{21} = 0.6E_2$ and the orientation angle of the laminated plate is $\alpha = (-45/45)^8$. The first-order acoustic radiation mode amplitude and the acoustic power of the laminated plates with different dampings are shown in Fig. (7).

It is well known that the damping does not affect the structure of the natural frequency, which is also reflected in Fig. (7). In addition, the figure shows that the damping ratio is larger and that the first order acoustic radiation mode amplitude and the acoustic power of the laminated plate are smaller at the same external excitation frequency.

5. CONCLUSIONS

In this paper, the natural frequency and acoustic radiation mode amplitude of laminated composite plates have been investigated. Based on the layerwise finite element theory, the natural frequency and dynamic response of laminated composite plates are obtained. The effects of the laminated composite plate’s parameters on the first-order acoustic radiation mode amplitude of the laminated composite plates are discussed when the excitation point is located at the centre of the plate. The numerical calculation and experimental results show that the natural frequencies of the laminated composite plate can be analysed accurately using the layerwise finite element model. In addition, the layerwise finite element model has been successfully used to analyse the acoustic radiation of clamped laminated composite plates. The proposed method can provide the structural parameters of the optimal design of low-noise laminated plates.

There is still a need to further investigate the influence of the different excitation points on the results. Furthermore, the case
in which the multi-excitation points simultaneously impact the laminated composite plate may be considered in future work.

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Bibliography


