Free Vibration Analysis of Rotating Functionally-Graded Cantilever Beams

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(Received 23 May 2012; accepted 20 August 2013)

The increasing needs of the industry involved in development of components for aerospace and power sector demand the engineering community to develop new concepts and strategies to improve the functional requirements of structures and to enhance the strength of materials. This is particularly essential in the cases of rotating beams that are subjected to severe vibration under large pressure loadings, high rotating accelerations, centrifugal forces, geometric stiffening, etc. A theoretical investigation of the free vibration characteristics of rotating cantilever beams, made of a functionally-graded material (FGM) consisting of metal and alumina, is presented in this study. It was assumed that the material properties of the FGM beam were symmetric, but varied continuously in the thickness direction from the core at the mid section to the outer surfaces, according to a power-law relation. Equations of motion were derived from a modelling method, which employed the hybrid deformation variable. The natural frequencies were determined using the Rayleigh-Ritz method. The effect of parameters such as the power law index, the hub radius, and the rotational speed on the natural frequencies of functionally-graded rotating cantilever beams were examined through numerical studies and then compared with the numerical results reported in earlier works.

NOMENCLATURE

$\vec{a}^P$	Acceleration vector of the generic point P
A	Cross-sectional area of the beam
b	Width of the beam
$E_{(z)}$	Youngs modulus
h	Total thickness of the beam
$\hat{\imath}, \hat{\jmath}, \hat{k}$	Orthogonal unit vectors fixed to the rigid hub
J_{11}^E	Axial rigidity of the beam
J_{11}^ρ	Mass density per unit length
$J_{22,yy}^E, J_{22,zz}^E$	Flexural rigidities of the functionally-graded beam
L	Length of the beam
n	Power law index
$\vec{P}$	Vector from point O to P_0
$P_{(z)}$	Effective material property
$P_{(m)}$	Metallic material property
$P_{(c)}$	Ceramic material property
q_{1i}, q_{2i}, q_{3i}	Generalized co-ordinates
r	Radius of the rigid frame
$\rho_{(z)}$	Mass density per unit volume
s	Arc length stretch of the neutral axis
T	Reference period
u, v, w	Cartesian variables in the directions of $\hat{\imath}$, $\hat{\jmath}$, and, $\hat{k}$
U	Strain energy of the functionally-graded beam
$\vec{v}^O$	Velocity of point O
$\vec{v}^P$	Velocity vector of the generic point P
x	Spatial variable
γ	Ratio of the angular speed of the beam to the reference angular speed
δ	Hub radius ratio
Θ	Constant column matrix characterizing the deflection shape for synchronous motion
μ_1, μ_2, μ_3	Number of assumed modes corresponding to q_{1i}, q_{2i}, and q_{3i}
τ	Dimensionless time
$\phi_{1j}, \phi_{2j}, \phi_{3j}$	Modal functions for s, v and w
$\vec{\omega}^A$	Angular velocity of the frame A
Ω	Angular speed of the rigid hub
$(')$	Partial derivative of the symbol with respect to the integral domain variable
$('')$	Second derivative of the symbol with respect to the integral domain variable

1. INTRODUCTION

Functionally-graded materials are special composites whose properties change spatially in one or more directions. Functionally-graded structures are being widely applied in extremely high temperature environments like those occurring