Use of a New Modified Acoustic Model to Investigate Mean Flow Effects on Underwater Sound Sources

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In this investigation, by introducing a relatively comprehensive acoustic equations system, the possibility of a more precise time and spatial pattern for sound wave propagation in fluid was revealed. Since the conservation equation is known as a fundamental equation for obtaining the wave equation, initially, by using scale analysis, the differential terms and weight coefficients are converted into the dimensionless form. Then, by assuming the amplitudes of the sound sources are small and by utilizing the perturbation technique, these dimensionless equations are converted into different orders based on the order of acoustical fluctuations. Consequently, it was shown that the obtained first order equations are representative of acoustical equations. Also, the results are indicative of the first order equations being coupled with the leading order ones. Comparison of the obtained acoustic the equations of the present study are capable of considering velocity, viscosity, and density changes of the background fluid flow. In the end, the effects of the flow velocity with a different Mach number on the acoustical distribution pattern that stemmed from different sound sources have been studied for several benchmarks.

1. INTRODUCTION

The linear equation of the wave was founded on the basis of the linear constitutive theory in fluid mechanics and the assumption of sound waves with a small amplitude. Linear constitutive theory of the fluid medium being used in the formulation of the linear equation of the wave includes the assumption of non-viscous and stationary background fluid flow with constant density. In light of these assumptions, linear wave equations in many fields of hydrodynamics and aerodynamics related to sound wave propagation are valid. Studies conducted on the noise generated by a hydrodynamic or aerodynamic occurrence in a time domain could be categorized into three groups. The first group is based on the suggested acoustic analogy by Lighthill. The linear acoustic wave which is placed on the left side of the Lighthill model is capable of evaluating wave propagation under the effect of sound sources placed on the right side of model. Ffowes Williams and Hawkings (FWH) extended this analogy by adding the effects of unsteady surface pressure. Seol and Salvator’s studies are amongst those related to the FWH model in the investigation of the underwater propeller’s noise. Although by using such models it is possible to identify patterns of wave propagation and directivity in the far field, this type of noise estimation methodology presents many assumptions such as those considered in linear acoustics, a low Mach number, and compressed sound sources.

Another group of noise estimation viewpoints includes research utilizing Direct Numerical Simulation (DNS) to model and simulate the hydrodynamics of fluid with its acoustic noise, directly and simultaneously. The advantage of using this method is in its limitless capability by which noise generated by all fluid flow such as a low Mach flow or a flow with a high Reynolds number could be obtained. At the same time, scaled use in aerodynamics and hydrodynamics simulations have much difference with scaled use in acoustic simulation. This inequality of scales has caused the utilization of DNS in aerodynamic and hydrodynamic fields to be very difficult. Moreover, using the DNS method is very time consuming. Seo, et.al chose the DNS method to find the noise of the cloud cavitation. They used the compressible Navier-Stokes equations and a homogeneous equilibrium model based on fluctuating density to simulate noise generation in a flow field.

The third point of view is, in fact, a hybrid method between the two fields of hydrodynamics and acoustics or between the two fields of aerodynamics and acoustics. Some of these hybrid methods are formed based on dividing the flow field into compressible hydrodynamic and compressible acoustic perturbation equations (or Perturbed Euler Equations). They also developed a set of revised Perturbed Euler Equations capable of calculating the effects of compressibility in the near field. They also developed a set of linearized, perturbed, compressible equations to overcome the occurred instability in numerical calculations caused by perturbed vorticity. Ewert and Schröder formulated several acoustic perturbation models based on the different sound sources derived from compressible flow simulation. They initially simulated the compressible flow and generated acoustic sources. Then on the basis of the type of formed sound sources, by using the suitable acous-
tic perturbation equation, they analysed the time and spatial wave propagation pattern. By using this hybrid method, Ewert also simulated the trailing edge noise of a propeller.\textsuperscript{16}

Alongside studies mentioned thus far, some investigated the nonlinear effects caused by a high sound pressure level. Walsh and Torres presented a weak formulation of nonlinear acoustic equations.\textsuperscript{1} They discretized and solved these governing equations by using the finite element method. Kuznetsov introduced classic wave equations containing the second order effects of nonlinearity.\textsuperscript{17} Söderholm has developed these equations by using the precise state equation.\textsuperscript{18} Also, Hoffelner, et al. demonstrated the effects of nonlinearity in high frequency such as ultrasonic increases, and as a result, considering these effects in high frequency is highly important.\textsuperscript{19}

Considering the points discussed, in light of the need for the presentation of precise mathematical models for the estimation of sound wave propagation, in this investigation, attempts have been made to present a complete and, at the same time, precise model to estimate the time and spatial pattern of sound wave propagation. This has been done by decomposing the conservation equations of mass, momentum, and energy including the state equation of a different order. This order separation is based on the fluctuation of acoustic sources and is done by using the perturbation method. Hence initially, by utilizing scale analysis, differential terms, and their coefficients, equations of conservation are converted into a dimensionless form. Then, by assuming the amplitude of sound source fluctuations as very small and utilizing the perturbation method, equations would decompose into different orders based on the order of acoustic fluctuations. Results show that first order equations are coupled with leading order equations. Comparing the obtained acoustic model with linear acoustic equations reveals that the presented equations in this study are capable of considering velocity, viscosity, and density variations of the fluid flow and the effects on wave propagation. Finally, the effect of flow velocity with different Mach numbers on the pattern of wave propagation for different sound sources has been analysed and studied during several benchmarks.

2. MATHEMATICAL FORMULATION OF ACOUSTIC MODEL

Since acoustics are an ingredient of fluid flow dynamics, it would be possible to describe and model the sound radiated from different events in a fluid medium in addition to the hydodynamics of them. This is done by using the governing conservation laws including conservation of mass, momentum, and energy for elementary particles of the fluid. Equations (1) through Eq. (3) represent general dimensional forms in the conservation of mass (continuity), momentum, and a combination of energy and state equations.\textsuperscript{20}

\[
\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u u) = -\nabla p + \mu \left( \nabla u + \left( \frac{1}{3} \frac{\nabla u}{\mu} \right) \nabla \left( \operatorname{div}(u) \right) \right) + \rho u Q; \quad (2)
\]

\[
\frac{\partial p}{\partial t} + (\rho u \cdot \nabla)(\rho u) = -\nabla p + c^2 \left( \frac{1}{\rho} \frac{\partial p}{\partial t} \right) + \beta \left( \frac{\partial \omega}{\partial t} \right) \left( \nabla \cdot u \right)^2 + 2\mu \left( \epsilon_{ij} \epsilon_{ji} - \frac{1}{3} \epsilon_{ii}^2 \right); \quad (3)
\]

In these equations, \( \rho(X, t) \) is the density, \( \vec{u}(X, t) \) is the velocity, \( Q(X, t) \) is the rate of fluid volume fluctuations, \( p(X, t) \) is the pressure of fluid, \( \mu \) is the viscosity, \( \mu_e \) is the bulk viscosity, \( \epsilon_{ij} \) is the tensor of strain rate, \( c \) is the speed of sound, \( c_p \) is the heat capacity at constant pressure, \( T \) is the temperature, and \( \beta = \frac{1}{\rho} \left( \frac{\partial p}{\partial \rho} \right) \) is the constant. Since in the present study the objective is to present an applied acoustic formulation for different problems in the fluid environment, it is necessary to convert all equations into a dimensionless form. In this way, depending on the problem, it is possible to disregard the less important terms by comparing the weight of each term.

2.1. Scale Analysis (Dimensional Analysis)

To make the governing equations dimensionless, the following relations are used:

\[
p = \hat{p} \rho^*; \quad \vec{u} = \hat{U} \vec{u}^*; \quad \rho = \hat{\rho} \rho^*; \quad t = \frac{t^*}{\omega}; \quad Q = \hat{Q} Q^*; \quad X, Y, Z = \hat{L} x, \hat{L} y, \hat{L} z \Rightarrow \nabla = \frac{\partial}{\partial x} = \frac{\partial}{\partial \hat{L} x}. \quad (4)
\]

In the above relations, terms containing the star index (*) are the order of one. Also, terms shown by the index of (') are scale parameters in need of being selected from available physical parameters in the studied problem. Finally, by applying the above relations in Eq. (1) through Eq. (3), differential terms are made dimensionless. Then by conducting several mathematical operations, coefficients of these differential terms become dimensionless. Equation (5) through Eq. (7) are indicative of the dimensionless form of conservation equations.

\[
St \frac{\partial \rho^*}{\partial t^*} + \nabla \cdot (\rho^* \vec{u}^*) = \left( \frac{LQ}{U} \right) \rho^* Q^*; \quad (5)
\]

\[
St \left[ \frac{\partial (\rho^* \vec{u}^*)}{\partial t^*} \right] + \left[ \vec{u}^* \nabla \cdot (\rho^* \vec{u}^*) \right] = E_u [-\nabla p^*] + \frac{1}{Re} \left[ \nabla^2 \vec{u}^* + \frac{1}{3} \frac{\nabla u}{\mu} \left( \nabla \operatorname{div}(\vec{u}^*) \right) \right] + \left( \frac{LQ}{U} \right) \rho^* \vec{u}^* Q^*; \quad (6)
\]

\[
\left( St \right) \left( E_u \right) \left[ \frac{\partial \rho^*}{\partial t^*} \right] + \left( E_u \right) \left( \vec{u}^* \cdot \nabla \right) \left( \rho^* \right) = \left( \frac{c}{U} \right)^2 \left[ -\rho^* \operatorname{div}(\vec{u}^*) \right] + \left( \frac{c^2 \beta}{c_p} \right) \left[ \frac{1}{Re} \left( \nabla \cdot \vec{u}^* \right)^2 + \frac{1}{Re} 2 \left( \epsilon_{ij}^* \epsilon_{ji}^* - \frac{1}{3} \epsilon_{ii}^2 \right) \right]; \quad (7)
\]
where, \((Re), (St),\) and \((Eu)\) are Reynolds, Strouhal, and Euler dimensionless numbers. Now, in order to separate and decompose these equations that are dimensionless into two orders related to hydrodynamics and the acoustics of fluid flow, the Perturbation method will be used.

### 2.2. Decomposition of Governing Equations by Using the Perturbation Method

In light of the inequality between the space and temporal scales in the hydrodynamic solution of fluid flow with scales of its acoustic solution, using Direct Numerical Solution (DNS) in research for both aeroacoustics and hydroacoustics is very difficult and cumbersome. Moreover, using the DNS method is usually very time-consuming. On the other hand, considered assumptions in linear acoustics do not provide the possibility of considering effects like velocity, viscosity, and density changes of the background flow. Therefore, in the present study, it was attempted that by using the perturbation method, the governing equations of the hydrodynamics of flow became separated from equations governed in acoustics induced by that flow. Then, each one of these equations could be analysed and studied in the related computational domain.

In general, volume fluctuations in the fluid lead to the propagation of acoustic noise. On the other hand, the propagated sound waves from most phenomena in fluid with the exception of explosions or shock waves have fluctuations with small amplitude. Thus, it could be stated that the changes of volume sound sources in the fluid also have very small amplitude. The order of the governing equation in the acoustics of fluid flow is from the same order of the volume fluctuations. So, if wave propagation in the fluid flow is viewed as the perturbation approach, then the order of the equations used in the modelling of the flow is much larger than the acoustic equations of the flow. As a result, by using the perturbation method and based on the order of acoustic fluctuations, conservation equations could be converted into different orders. In the perturbation method, the perturbed term is shown with \(\varepsilon\). This term must be selected from the parameters of the considered problem. In light of the points mentioned and the objective of present research, which is separating equations on the basis of acoustic fluctuations, for the present study, \(\varepsilon = \frac{Q}{L}\) is considered. The following assumptions exist in equations:

\[
P^*(X,t,\varepsilon) = h_0(\varepsilon)p_0(X,t) + h_1(\varepsilon)p_1(X,t) + o(h_1(\varepsilon));
\]

\[
\bar{u}^*(X,t,\varepsilon) = f_0(\varepsilon)\bar{u}_0(X,t) + f_1(\varepsilon)\bar{u}_1(X,t) + o(f_1(\varepsilon));
\]

\[
P^*(X,t,\varepsilon) = g_0(\varepsilon)p_0(X,t) + g_1(\varepsilon)p_1(X,t) + o(g_1(\varepsilon));
\]

\[
Q^*(X,t,\varepsilon) = m_0(\varepsilon)Q_0(X,t) + m_1(\varepsilon)Q_1(X,t) + o(m_1(\varepsilon)).
\]

(8)

In these relations, \(p_0, \bar{u}_0, p_0, Q_0\) are dimensionless forms of pressure, velocity, density, and volume fluctuations in the leading order. Moreover, \(p_1, \bar{u}_1, p_1, Q_1\) are dimensionless forms of pressure, velocity, density, and volume fluctuations in the first order. Terms \(h_0, h_1, \ldots, m_0, m_1\) are indicative of the weight of dimensionless parameters. On the basis of the above definitions, in this study, only the separating of the equations in the two orders of the leading and the first order has taken place. After placing the relationships of Eq. (8) into Eq. (5) through Eq. (7), the resulting equations need to be decomposed into different orders. This calls for obtaining the relationships of each of the weighting terms \((h_0, h_1, \ldots, m_0, m_1)\) with the perturbed parameter \(\varepsilon\) from the stand point of order. Also, it needs to be considered that based on the applied definition in this study, it is presumed that the leading order equations must satisfy the conditions governing flow dynamics, and the first order equations must satisfy the conditions governing the acoustics of flow. In other words, relations between weighting terms of dimensionless parameters and perturbed terms have to be obtained such that after the decomposition of equations in different orders, the leading order equations can be capable of solving the flow field precisely. Moreover, the obtained first order equations should be indicative of linear wave equations after applying assumptions of linear acoustics (including nonviscous and static fluid). Alongside these points, the definition of \(x_{n+1}(\varepsilon) = o(x_n(\varepsilon))\) and the concept of minimum possible conditions has been used. Eventually, after simplification, the only condition agreeable with the physical parameters of velocity, density, pressure, and volume source fluctuations would be obtained as following:

\[
p^* = p_0 + \varepsilon p_1;
\]

\[
\bar{u}^* = \bar{u}_0 + \varepsilon \bar{u}_1;
\]

\[
\rho^* = \rho_0 + \varepsilon \rho_1;
\]

\[
Q^* = Q_0 + \varepsilon Q_1.
\]

(9)

Finally, these forms of parameters are applied in Eq. (5) through Eq. (7). Then, the obtained equations would be decomposed based on the order of the weight of the differential terms. Equations that are in the order of one \(\{O(1)\}\) indicate leading order equations. Also, equations of the order of \(\{O(\varepsilon)\}\) are first order or acoustic equations. The group of equations, Eq. (10) through Eq. (12), are indicative of leading order equations:

\[
(St) \frac{\partial p_0}{\partial t} + \nabla \cdot (\rho_0 \bar{u}_0) = 0;
\]

\[
(St) \left[ \frac{\partial (\rho_0 \bar{u}_0)}{\partial t} \right] + \left[ \bar{u}_0 \nabla \cdot (\rho_0 \bar{u}_0) \right] = (Eu) \left[ -\nabla p_0 + \frac{1}{Re} \nabla^2 \bar{u}_0 + \left( \frac{1}{3} + \frac{\mu_e}{\mu} \right) \left( \nabla \cdot \left( \rho_0 \bar{u}_0 \right) \right) \right];
\]

\[
(St)(Eu) \frac{\partial p_0}{\partial t} + (Eu)(\bar{u}_0 \cdot \nabla) p_0 = \left( \frac{c^2}{U} \right) \rho_0 (\nabla \cdot \bar{u}_0) + \left( \frac{c^2}{c_p} \right) \left[ \frac{1}{Re_v} (\nabla \cdot \bar{u}_0)^2 \right] + 2 \left[ \frac{1}{Re_v} \left( \frac{c^2}{c_p} \right) \left( \bar{u}_0 \cdot \nabla \bar{u}_0 \right) \right];
\]

(12)

As it can be observed, leading order equations are, in fact, the most general form of governing equations on the hydrodynamics of viscous and compressible fluid flow.21,22 Also,
Eqs. (13) through Eq. (15) are indicative of the obtained acoustical model.

\[
(St) \frac{\partial p_0}{\partial t} + \nabla \cdot (\rho_0 \vec{u}_1) + \nabla \cdot (\rho_1 \vec{u}_0) = \rho_0 Q_0; \tag{13}
\]

\[
(St) \left[ \frac{\partial (\vec{u}_0 \rho_1)}{\partial t} \right] + (St) \left[ \frac{\partial (\rho_0 \vec{u}_1)}{\partial t} \right] + \left[ (\vec{u}_0) \nabla (\rho_0 \vec{u}_1) \right] + \left[ (\vec{u}_0) \nabla (\rho_0 \vec{u}_0) \right] = (Eu) \left[ - \nabla (\rho_1) \right] + \frac{1}{Re} \left[ \nabla^2 (\vec{u}_1) + \frac{1}{3} \left( \frac{\mu_0}{\rho} \right) \left( \nabla \text{div} (\vec{u}_1) \right) \right] + \rho_0 \vec{u}_0 Q_0; \tag{14}
\]

\[
(St) \left[ (Eu) \frac{\partial p_1}{\partial t} \right] + (Eu) (\vec{u}_0 \cdot \nabla) p_1 + (Eu) (\vec{u}_1 \cdot \nabla) p_0 = - \frac{c^2}{U} \rho_0 (\nabla \cdot \vec{u}_1) - \frac{c^2}{U^2} \rho_1 (\nabla \cdot \vec{u}_0) + \frac{c^2 \beta}{2} \frac{1}{Re} \cdot \left[ \nabla \cdot (\nabla \cdot \vec{u}_0) \right] \nabla \vec{u}_1 + \frac{1}{Re} \left[ \frac{1}{4} \frac{\partial u_{0i}}{\partial x_j} \frac{\partial u_{1j}}{\partial x_i} + \frac{1}{4} \frac{\partial u_{0i}}{\partial x_j} \frac{\partial u_{1i}}{\partial x_j} + \frac{1}{4} \frac{\partial u_{0i}}{\partial x_j} \frac{\partial u_{0j}}{\partial x_i} + \frac{1}{3} \left( \frac{2}{3} \cdot \left( \nabla \cdot \vec{u}_0 \right) \right) \left( \nabla \vec{u}_1 \right) \right]. \tag{15}
\]

Considering the present terms in Eq. (13) through Eq. (15), it could be concluded that the obtained acoustic equations system is coupled with the leading order ones. The observed coupling, in fact, is a kind of static coupling and is unilateral. In other words, the solving of the acoustic equations is only in need of determining the terms of the leading order like \( \vec{u}_0, \rho_0, p_0, Q_0 \). Also, there is no need to solve the leading and first order equation simultaneously. Generally, it would be possible to find these terms from an individual solution of the leading order equations. By applying the assumptions of linear acoustics in Eqs. (13) through Eq. (15), it could be shown that these equations will also end up as a linear wave formulation. This point can be considered as validity of the obtained acoustic model. Despite everything discussed, in proceeding, first, the validation of the obtained acoustic model will take place for a benchmark problem. Then, during the two other benchmarks, this acoustic model will apply for different sound sources, including dipole and quadrupole. Finally, the effect of the background fluid velocity on the sound wave propagation pattern in several benchmark problems will be reviewed and studied.

### 3. NUMERICAL RESULTS OBTAINED FROM ACOUSTIC MODEL

Benchmark problems considered in the present study include the presence of the spherical sound sources, which are located along the fluid flow with different velocities. In light of the existing symmetry in the problem, to present the numerical solution, it was attempted to use cylindrical axisymmetric coordination. Also, in order to solve the acoustical system of equations, the finite element method has been used. So, first, the momentum equation must be decomposed into two equations along \( r \) and \( z \) based on the velocity axis in the form of \( \vec{u}_1 = u_{1r} \vec{e}_r + u_{1z} \vec{e}_z \). Then, by disregarding terms along \( \theta \), acoustic equations in cylindrical axisymmetric coordinates will be in the following forms:

\[
\begin{align*}
\frac{\partial p_1}{\partial t} &+ \left( u_{1r} \frac{\partial p_1}{\partial r} + u_{1z} \frac{\partial p_1}{\partial z} + \rho_0 \left( \frac{\partial u_{1r}}{\partial r} + \frac{u_{1r}}{r} + \frac{\partial u_{1z}}{\partial z} \right) \right) + \\
\left( u_{0r} \frac{\partial p_1}{\partial r} + u_{0z} \frac{\partial p_1}{\partial z} \right) &+ \rho_1 \left( \frac{\partial u_{1r}}{\partial r} + \frac{u_{1r}}{r} + \frac{\partial u_{1z}}{\partial z} \right) = \rho_0 Q_0; \\
\end{align*}
\]

\[
\begin{align*}
\left( \frac{\partial p_0}{\partial t} \right) &+ \left( u_{1r} \frac{\partial p_0}{\partial r} + u_{1z} \frac{\partial p_0}{\partial z} + u_{0r} \frac{\partial p_1}{\partial r} + u_{0z} \frac{\partial p_1}{\partial z} \right) + \\
\left( u_{0r} \frac{\partial p_0}{\partial r} + u_{0z} \frac{\partial p_1}{\partial z} \right) &+ \rho_0 \left( \frac{\partial u_{1r}}{\partial r} + \frac{u_{1r}}{r} \right) + \rho_1 \left( \frac{\partial u_{1z}}{\partial z} \right) = 0. \tag{19}
\end{align*}
\]

### 3.1. Validation of the Acoustic Model

As mentioned earlier, by placing the assumptions of linear acoustics in the presented acoustic models, the linear wave equation could be found. Nevertheless, in order to validate the equations and the algorithm of the solution, initially, a simple benchmark problem is used. This problem contains single frequency oscillations of a spherical sound source in a static fluid flow. In the proposed benchmark problems in this study, the scale parameters used are based on water as the background fluid with the following properties in the form of \( L = 130 \text{ mm}, \quad \bar{\omega} = L/\bar{U}, \quad \bar{U} = 1500 \text{ m/s}, \quad \bar{\rho} = 1000 \text{ kg/m}^3, \quad \bar{P} = \rho \bar{U}^2 \) with the given values. The oscillating sphere and the computational domain shape in the cylindrical axisymmetric coordinate is as


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a half-circle. Figure 1 depicts a view of the computational domain beside the oscillating sphere in the cylindrical axisymmetric coordinate.

To prevent the effects of wave reflection and return, the computational domain extends to the point where, in light of sound wave velocity, during the time of the numerical analysis of the problem, the sound wave won’t reach the end of the computational domain. Also, in order to prevent numerical instability, the Laplace of acoustic pressure and density will be considered zero on the end boundary of the computational domain. The boundary condition on the oscillating sphere with a dimensionless radius of $r_0 = 0.035 (m)/L$ is $\vec{u} \cdot \vec{n} = \frac{0.005}{U_0} \sin \left(2\pi \times \left( \frac{515}{\tau} \right) t^* \right)$. This problem is a model of a monopole sound source in the static fluid oscillating in the sinusoidal form with a frequency of $f = 515$ Hz. Moreover, in order to review independence from the mesh grid, the above problem was repeated for four different mesh grids. In the selected mesh grid, the number of applied elements in the finite element solution is equal to 104,880. The dimensionless acoustic pressure obtained from this mesh grid compared to those obtained from a mesh grid containing 152,600 elements presented 0.3% relative error. Therefore, to reduce the calculation time, the mesh grid with the lower number of elements was chosen as a final mesh grid. Figure 2(a) is indicative of the time history of dimensionless acoustic pressure for the two sensors located within the distances $r_1 = 7 (m)/L$ and $r_2 = 21 (m)/L$. Figure 2(b) depicts the frequency response of dimensionless acoustic pressure for the same sensors.

According to Fig. 2(a), considering the sinusoidal pattern of the sound source fluctuations, the time response also possesses a sinusoidal form. On the other hand, in light of the frequency response obtained from an analytical relation. According to the Distance Law, the following relation exists for a point source:

$$
\text{Amplitude} = \frac{p_2}{r_2} \frac{r_1}{r_2} = \frac{20 \cdot \log (p_2)_{\text{db}}}{20 \cdot \log (p_1)_{\text{db}}} + 20 \cdot \log \left( \frac{r_1}{r_2} \right)_{\text{db}}
$$

The above equations are indicative of the relationship of effective acoustic pressure $p_1$, $p_2$ between 2 points at distances $r_1$ and $r_2$ from the sound source, in terms of pressure unit and decibel. Now, if in light of simulation results, effective pressure $p_1$ at distance $r_1 = 7 (m)/L$ is presumed as known, then, by using the distance law, effective pressure at distance $r_2 = 21 (m)/L$ could be estimated. So, for comparing a numeri-
Table 1. Comparison of results of obtained effective pressure from numerical and analytical solutions at distance $r_2 = 21$ (m)/L.

| Numerical results of obtained effective pressure level at distance of $r_2 = 21$ (m)/L and in dB unit | -336.707 |
| Analytical results of obtained effective pressure level at distance of $r_2 = 21$ (m)/L and in dB unit | -349.099 |
| Relative error | 3.550% |

The effective pressure $\tilde{p}$ is considered for the dipole and quadrupole sources. In this paper and the analytical model referred to as Distance Law, effective pressure $p_2$ at distance $r_2 = 21$ (m)/L would be predicted. For this goal, initially, the effective pressure at the distance of $r_1 = 7$ (m)/L must be specified. In this investigation, it is chosen from numerical results and from time series pressure data. The Root Mean Square (RMS) of these dimensionless time pressures, which is indicative of effective sound pressure data. The Root Mean Square (RMS) of these dimensionless results. Comparing the numerical and analytical results of the obtained effective pressure at a distance $r_1 = 7$ (m)/L, is obtained. For analytical results, this value is considered as $p_1$ in Distance Law and is used for calculating $p_2$. On the other hand, by calculating RMS of the sound pressure time series data at distance $r_2 = 21$ (m)/L, numerically and directly, $p_2$ is estimated. Results of the obtained effective pressure directly from the present study are compared with results of the analytical solution at a distance of $r_2 = 21$ (m)/L and are presented in Table 1.

It is noteworthy that the obtained values are based on dimensionless results. Comparing the numerical and analytical results of the effective pressure level of sound in the problem shows that the acoustic model used in this study has been capable of modelling divergence losses of sound energy, effectively.

Figure 3 and Fig. 4 are indicative of a sound wave propagation pattern at the dimensionless time of $t_2^* = 400$ and $t_2^* = 600$.

Figure 3(a) and Fig. 4(a) are indicative of the spatial pattern of sound wave propagation at axisymmetric coordinates. After solving the problem at axisymmetric coordinates and displaying the wave pattern in axisymmetric coordinates, the complete view of the wave propagation must be obtained. This has been done through an 180$^\circ$ rotation of the computational domain along the $r$-axis. By this rotation, the half sphere will be generated. Presented patterns in Fig. 3(b) and Fig. 4(b) are 2D views of this half sphere from the front view. It is obvious that the results indicate the spherical (can be circular in 2D figure) propagation of sound waves produced by the oscillation of a monopole sound source. It is noteworthy that during these dimensionless problems, the wave velocity is equal to one. So, according to the figures, at a time of $t_2^* = 400$, sound waves have reached the dimensionless distance $r_1^* \approx 400$, and at the time of $t_2^* = 600$, sound waves have reached the dimensionless distance $r_2^* \approx 600$.

### 3.2. Modelling of Dipole and Quadrupole Sound Sources

After the validation of the acoustic model, the above problem is considered for the dipole and quadrupole sources. In order to do that, two and four semi-circle sources with the radii $r_1 = 0.035$ (m)/L are used in axisymmetric cylindrical coordinates. It is important to note that in dipole modelling, two spheres (two semi-circles in axisymmetric coordinates) with equal frequency ($f = 515$ Hz) and 180$^\circ$ phase differences are oscillating, simultaneously. Also, in the linear quadrupole sound source model, all four spheres (four semi-circles in axisymmetric coordinates) that are stranded in a line are oscillating together with the same frequency of ($f = 515$ Hz). The oscillation of the two inside spheres is considered to have the same phase, and they have 180$^\circ$ phase differences with the two side spheres. The spatial wave propagation pattern produced by sinusoidal oscillation of dipole and quadrupole sound sources is shown in Fig. 5 and Fig. 6, respectively. The amplitude of the oscillations is also considered depending on the oscillation phase of each source in the form of $\tilde{u} \cdot \tilde{n} = \pm \left[ \frac{0.002}{L} \times \sin \left( 2\pi \times \left( \frac{2}{L} \right) t^* \right) \right]$.

Figure 5(a) and Fig. 6(a) are indicative of the spatial propagation pattern of the sound wave pressure in axisymmetric coordinates. After solving the problem in axisymmetric coordinates and 180$^\circ$ rotating of the computational domain around the $r$-axis, the presented pattern in Fig. 5(b) and Fig. 6(b) can be obtained. Also, for better comparison between the dipole and quadrupole sound sources model, Fig. 7...
and Fig. 8 display the directivity pattern due to the dipole and quadrupole sound source.

In these figures, the oscillating spheres were located at the centre of these polar plots, and the main axes are horizontal (between 0 and 180°). These directivity patterns are those expected from the dipole and linear quadrupole sound sources at the far field. So, by the consideration of the spatial patterns of Fig. 5 and Fig. 6 besides the far field directivity patterns of Fig. 7 and Fig. 8, it is obvious that the results indicate the acoustic model introduced in this investigation is capable of modelling and solving different sound sources. At the same time, this model is also capable of considering parameters like velocity and viscosity of the background fluid flow. The effects of the background fluid flow will be analysed later on.

3.3. Study of Flow Velocity Effects on the Spatial Pattern of Sound Wave

In the previous problem, the background flow velocity was considered to be equal to zero. Here, attempts are made to investigate the sound wave pattern produced by monopole sound sources and in a flow with a different Mach number. It is important to point out that placement of a fixed sound source in a dynamic fluid flow, from the point of view of a sound pattern, is equivalent to assuming that the flow is stationary and the oscillating source in the fluid flow is moving. The patterns shown in Fig. 9 and Fig. 10 are indicative of the presence of a monopole sound source in the fluid flow with different velocities. In Fig. 9, the frequency of oscillation is equal to 5510 Hz. Also, the boundary condition on the wall of the oscillating source is in the form of \[ \mathbf{u} \cdot \mathbf{n} = 0.005 \tilde{U} \sin \left( 2\pi \times \frac{5510}{\tilde{c}} t^* \right) \]. It is necessary to point out that the results have been obtained based on equations and parameters being non-dimensional.

Similarly, the results are obtained on the basis of another monopole source oscillated with a frequency of 75 Hz. The boundary condition imposed on the wall of the oscillating source is in the form of \[ \mathbf{u} \cdot \mathbf{n} = 0.005 \tilde{U} \sin \left( 2\pi \times \frac{75}{\tilde{c}} t^* \right) \]. These results are shown in Fig. 10.

As it is evident from Fig. 9 and Fig. 10, the results were compared for three background flow velocities. The relation of the Mach number is \[ Ma = \frac{u}{c} \]. In light of all of the relations and the analysis in this study being dimensionless and the independent of the nature of the fluid flow, the velocity of the background flow in this investigation is considered such that the Mach numbers of \[ Ma = 0.067, 0.133, 0.267 \] are produced. Based on the obtained results, by any increase in the Mach number that was caused by the background flow velocity increase, the spherical pattern of the sound wave propagation would be dissipated. In order to precisely observe the outcome, some concentric circles centred on the monopole sound source are drawn, virtually. By comparing the pattern of propagation with these circles, it becomes evident that through increasing the Mach number, the sound propagation pattern shows more tendencies to exit from the spherical form. Also, it is obvious that acoustic pressure contours are more compressed at the downstream of the sound source, whereas, at the upstream, the flow is more rarefaction. This situation, in particular at the
Figure 8. Far field directivity pattern of linear quadrupole sound source with a frequency of \( f = 515 \) Hz at a dimensionless time of \( t^* = 400 \). Mach number of \( Ma = 0.267 \), is more vividly noticeable. The reason for this is that at the downstream of the source, the flow direction is opposite to the direction of the sound wave propagation. Consequently, this causes the pressure contours to be more concentrated. This concentration, in return, leads to an increase in the intensity of the acoustic pressure in this region.

4. DISCUSSION AND CONCLUSIONS

Conservation equations of mass, momentum, energy, and the equation of state are considered the fundamental equations in the formulation of a linear acoustic equation or a wave equation. Recognition and analysis of acoustic noise stemmed from the occurrence of a hydrodynamic phenomenon in a fluid flow depends on these equations being studied completely in the entire computational domain. Inequalities of time and space hydrodynamical and acoustical scale and the resulting time cost of the computational processes make this analysis very cumbersome or even impossible. On the other hand, many simplifying assumptions involved in the conservation equations used to obtain the wave equation are sometimes not suitable. Therefore, in the present study, by utilizing scale or dimensional analysis and the perturbation method, governing equations on the fluid flow dynamics are separated from governing equations of acoustics. Then, each one of these equations should be analysed and studied in a suitable domain. This means each equation is governed in a domain where the other equation is not to be able to affect it. So, separation of the equation at different orders and using each group of equations in the related computational domain leads to an enormous reduction in the computational processing for the hydrodynamic and hydroacoustic simulation. Based on the obtained results, the acoustic model of the present study is capable of modelling different kinds of acoustic sources effectively. Furthermore, the ability of this acoustic model in the recognition and the prediction of a pattern of wave propagation that stemmed from the presence of oscillating sound sources with desired frequency in the fluid flow with a different speed were proven and demonstrated.

REFERENCES


Figure 10. Spatial wave propagation pattern of acoustic pressure caused by the oscillation of a monopole sound source with a frequency of $f = 75$ Hz and the presence of a background flow with different velocities.


