Wavelet Analysis of Vibration Signals Part 1: Wavelet Properties

David E. Newland
Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, England,
e-mail: den@eng.cam.ac.uk

Wavelet analysis allows the energy of a recorded signal to be decomposed into its contributions from different frequency bands and different locations. In contrast, Fourier analysis generates data which is the result of averaging over the duration of the record being studied; information about location can be obtained only by dividing a record into sections and Fourier analysing each section separately (which is what the short-time Fourier transform, STFT, does). For transient data analysis, the wavelet transform and the STFT produce similar results, but wavelet transforms have computational and other advantages which may be important in particular applications.

Part 1 of this paper reviews wavelet theory and describes wavelet properties and computational algorithms for vibration signal analysis. Part 2 compares wavelet analysis with time-frequency analysis using the STFT and discusses particular advantages of the harmonic wavelet transform in this respect.

1. INTRODUCTION

A wavelet is a short wave. Any waveform may be used provided that it is localised at a particular time (or position). For convenience, we shall regard time to be the independent variable, so that localisation means concentration at a particular time, but any physical variable can be used. If $w(t)$ is the function of time t which describes the wavelet, and $x(t)$ is the signal being studied, then the wavelet coefficient $a_w(t)$ is defined by the correlation equation

$$a_w(t) \int_{-\infty}^{\infty} x(\theta)w(t-\theta)d\theta. \qquad (1)$$

Knowledge of $a_w(t)$ provides information about the structure of $x(t)$ and its relationship to the shape of the analysing wavelet $w(t)$. When $x(t)$ correlates with $w(t)$, then $a_w(t)$ will be large; when they do not correlate, $a_w(t)$ will be small. More information is obtained if the process is repeated with a different wavelet function, $w_1(t)$.

Wavelets are essentially local functions so that $w(t) \to 0$ when $|t| \to 0$. Therefore, in practical terms, the integral in Eq. (1) needs to extend over only a limited range of θ, and $a_w(t)$ gives information about $x(\theta)$ in the vicinity of $x(\theta = t)$. This is how wavelet analysis differs from Fourier analysis. Fourier transforms are calculated by the same Eq. (1) when the wavelet function $w(t)$ is replaced by a harmonic function. In that sense, Fourier analysis may be thought of as an extreme case of wavelet analysis; the requirement that the wavelet should be a local function is ignored.

Much of the theory of wavelet analysis is about self-similar wavelets (which are wavelets of the same shape but different length scales) spaced as discrete intervals along the time axis. It turns out that, just as a periodic signal may be represented by the (infinite) sum of its harmonics in Fourier analysis, so a transient signal may be represented as the (infinite) sum of the (weighted) amplitudes of its component wavelets. The component wavelets all belong to a doubly infinite set of self-similar wavelets: each wavelet is discretely spaced along the infinite time axis and each wavelet is of a length scale that bears a discrete relationship to the length scales of all the other wavelets.

Because Fourier coefficients are obtained by averaging over the full length of a signal, Fourier analysis provides no information about how frequency composition may be changing with time. To obtain *local* frequency data, a record must be divided into sections and each segment analysed separately. The short-time Fourier transform (STFT) does this and is essential if the changing frequency composition of a transient signal is to be examined. Frequency coefficients computed by the STFT depend on the length and time (or position) of the short record that the calculation process assumes is one period of a periodic signal. Various authors have studied how results computed by the STFT depend on how a record is subdivided and making a good choice may be crucial (see, for example, Hodges et al. 1985). But the difficulty remains that infinitely-long harmonic functions are being used to decompose a transient signal. It is usually better to employ a set of short functions as the basis for decomposing transient records. Wavelets are such short functions.

The practical embodiment of wavelet transforms, as for the Fourier transform, requires a discrete calculation using sampled values of the continuous record $x(t)$. Just as the continuous Fourier transform is replaced by the discrete Fourier transform, so the continuous wavelet transform is replaced by the discrete wavelet transform. Let $X(j)$, j=1 to N, be sampled values of the record $x(t)$, and $A(k)$, k=1 to N, be sampled values of its wavelet transform (which are arranged in a logical order that takes account of the scale and position of the wavelet), then