Statistical Evaluation of Complex Sound Environment with Background Noise

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(Received 21 January 2013; revised 6 December 2013; accepted 7 May 2014)

In order to evaluate noise in a sound environment, it is necessary to estimate the sound levels at evaluation points based on the observations at a reference point. In this study, a method is derived based on the observations contaminated by a background noise to estimate system parameters reflecting several orders of correlation information between the evaluation and reference points in a complex sound environment. Furthermore, a statistical evaluation method for traffic noise under the existence of background noise is proposed. The effectiveness of the proposed method is experimentally confirmed by applying it to the traffic noise data measured in a complex sound environment.

1. INTRODUCTION

In the evaluation of a sound environment around a main line, it is necessary to estimate the sound levels at evaluation points based on the observations at a reference point from the viewpoint of establishing a monitor system for the actual complex sound environment. Furthermore, the internal physical mechanism of an actual sound environment is often difficult to recognize analytically, and it contains unknown structural characteristics. In a previous study,[1] it was found that complex sound environment systems are difficult to analyse by using usual structural methods based on the physical mechanism. Therefore, a nonlinear system model was derived in the expansion series form reflecting various types of correlation information from the lower order to the higher order between input and output variables.[1] The conditional probability density function contains the linear and nonlinear correlations in the expansion coefficients, and these correlations play an important role as the statistical information for the input and output relationships of sound environment systems.

On the other hand, the random noise in an actual sound environment usually exhibits multifarious and complex characteristics such as non-Gaussian distribution and non-linear and non-stationary properties relating to natural, social, and/or human factors. Furthermore, the observation data are usually contaminated by background noise with complex statistical properties. In this situation, in order to evaluate the sound environment, precise estimation of the system characteristics of the sound environment is required, considering the contaminated observed data.

In this study, a general type of complex sound environment is considered. An estimation method for the sound levels at evaluation points for the complex sound environment around a main line such as a highway and a railroad is proposed on the basis of the observations at a reference point under the existence of background noise. By adopting an expansion expression of the conditional probability distribution as the system characteristics, a method to estimate the system parameters reflecting several orders of correlation information between the evaluation and reference points is first derived. Furthermore, a prediction method for the probability distribution of traffic noise at the evaluation points is also considered.

The effectiveness of the proposed theory is experimentally confirmed by applying it to actual data of road-traffic noise measured around a national road in the city of Hiroshima and low-frequency noise observed in a complicated sound environment near the exit of a tunnel, which is generated by Shinkansen trains running through the tunnel.

2. EVALUATION OF TRAFFIC NOISE UNDER THE EXISTENCE OF BACKGROUND NOISE

2.1. Statistical Model for Sound Environment around a Main Line

In the evaluation of traffic noise in a sound environment around a main line such as a highway and a railroad, it is necessary to estimate the sound levels at multiple evaluation points based on the observation at a reference point because of the difficulties of monitoring the sound levels at all evaluation points and at every instantaneous time. Furthermore, in the measurement of the sound environment, the observation data are generally contaminated by background noise.

The intensity variables satisfying the additive property of the specific noise and the background noise are considered in this section. Let x and y be the sound intensities of a specific noise at an evaluation point and a reference point, respectively. The probability distribution of x has to be predicted on the basis of the observed data of y. Though the single evaluation point is considered, theoretically, for the simplification of the mathematical expression, the extension of the theory to the case of the multi-evaluation points is easy by considering multi-dimensional variable $\mathbf{x}$ instead of the single variable x.

All the information on linear and/or nonlinear correlations between x and y is included in the conditional probability density function $P(x|y)$.

In order to explicitly find the various correlation properties between x and y, let us expand the joint probability density function $P(x,y)$ into an orthogonal polynomial series, as follows:[2]

$$P(x,y) = P_0(x)P_0(y)\sum_{r=0}^{\infty}\sum_{s=0}^{\infty} A_{rs}\phi_r^{(1)}(x)\phi_s^{(2)}(y); \quad (1)$$

where $A_{rs} = \left\langle \phi_r^{(1)}(x)\phi_s^{(2)}(y)\right\rangle$ and $\langle\cdot\rangle$ denotes the averaging operation. $P_0(x)$ and $P_0(y)$ can be chosen arbitrarily as the probability density functions describing the dominant parts of the actual fluctuation pattern. Two functions $\phi_r^{(1)}(x)$ and $\phi_s^{(2)}(y)$ are orthogonal polynomials with the weighting functions $P_0(x)$ and $P_0(y)$. The information on the various types of linear and/or nonlinear correlations between x and y is reflected hierarchically in each expansion coefficient A_{rs}. In this section, the gamma distribution suitable for the random variables fluctuating within only the positive region such as the sound intensity is adopted.

$$P_0(x) = \frac{x^{m_x-1}}{\Gamma(m_x)s_x^{m_x}}e^{-\frac{x}{s_x}};$$
$$m_x = \frac{\mu_x^2}{\sigma_x^2}; s_x = \frac{\sigma_x^2}{\mu_x}; \mu_x = \langle x\rangle; \sigma_x^2 = \left\langle (x-\mu_x)^2\right\rangle; \quad (2)$$

$$P_0(y) = \frac{y^{m_y-1}}{\Gamma(m_y)s_y^{m_y}}e^{-\frac{y}{s_y}};$$
$$m_y = \frac{\mu_y^2}{\sigma_y^2}; s_y = \frac{\sigma_y^2}{\mu_y}; \mu_y = \langle y\rangle; \sigma_y^2 = \left\langle (y-\mu_y)^2\right\rangle; \quad (3)$$

where $\Gamma(\bullet)$ is a gamma function. Thus, orthogonal polynomials $\phi_r^{(1)}(x)$ and $\phi_s^{(2)}(y)$ are given by the Laguerre polynomial:[2]

$$\phi_r^{(1)}(x) = \sqrt{\frac{\Gamma(m_x)r!}{\Gamma(m_x+r)}}L_r^{(m_x-1)}(\frac{x}{s_x}); \quad (4)$$

$$\phi_s^{(2)}(y) = \sqrt{\frac{\Gamma(m_y)s!}{\Gamma(m_y+s)}}L_s^{(m_y-1)}(\frac{y}{s_y});$$
$$\left(L_n^{(\alpha)}(x) = \frac{e^x x^{-\alpha}}{n!}\frac{d^n}{dx^n}(e^{-x}x^{n+\alpha})\right). \quad (5)$$

By substituting Eq. (1) into the definition of the conditional probability, $P(x|y)$ can be expressed in an expansion series form as follows:

$$P(x|y) = \frac{P(x,y)}{P(y)} = \frac{P_0(x)\sum\limits_{r=0}^{\infty}\sum\limits_{s=0}^{\infty} A_{rs}\phi_r^{(1)}(x)\phi_s^{(2)}(y)}{\sum\limits_{s=0}^{\infty} A_{0s}\phi_s^{(2)}(y)}; \quad (6)$$

2.2. Countermeasure for Background Noise in the Sound Environment

In the measurement of the sound environment, effects of the background noise are inevitable. Then, based on the additive property of the intensity variable, the observed sound intensity z_k at a discrete time k is expressed as

$$z_k = x_k + v_k; \quad (7)$$

where x_k and v_k are sound intensities of the specific noise and background noise at an evaluation point. We assume that the statistics of the background noise are known. In this section, an estimation method for the expansion coefficients A_{rs} in Eq. (1), reflecting the correlation information between x and y, is derived on the basis of the observed data z_k. Considering the expansion coefficients A_{rs} as unknown parameter vector $\mathbf{a}$:

$$\mathbf{a} = (a_1, a_2, a_3, ...)(= (\mathbf{a}_{(1)}, \mathbf{a}_{(2)}, ...);$$
$$\mathbf{a}_{(r)} = (A_{r1}, A_{r2}, ...); (r = 1, 2, ...); \quad (8)$$

the simple dynamical model

$$\mathbf{a}_{k+1} = \mathbf{a}_k; (\mathbf{a}_k = (a_{1,k}, a_{2,k}, a_{3,k}, ...)$$
$$= (\mathbf{a}_{(1),k}, \mathbf{a}_{(2),k}, ...)); \quad (9)$$

is naturally introduced for the successive estimation of the parameter.

In order to derive the estimation algorithm of the parameter, attention is focused on Bayes' theorem for the conditional probability distribution:

$$P(\mathbf{a}_k|Z_k) = \frac{P(\mathbf{a}_k, z_k|Z_{k-1})}{P(z_k|Z_{k-1})}; \quad (10)$$

where $Z_k \equiv \{z_1, z_2, \cdots, z_k\}$ is a set of observation data up to time k. Based on Eq. (10), using a similar calculation process to the previously reported paper,[3] the estimate for an arbitrary polynomial function $f_{\mathbf{M}}(\mathbf{a}_k)$ of $\mathbf{a}_k$ with $\mathbf{M}$-th order can be derived as follows (cf. Appendix):

$$\hat{f}_{\mathbf{M}}(\mathbf{a}_k) = \langle f_{\mathbf{M}}(\mathbf{a}_k)|Z_k\rangle$$
$$= \frac{\sum\limits_{\mathbf{m}=\mathbf{0}}^{\mathbf{M}}\sum\limits_{n=0}^{\infty} B_{\mathbf{m}n}C_{\mathbf{Mm}}\theta_n^{(2)}(z_k)}{\sum\limits_{n=0}^{\infty} B_{\mathbf{0}n}\theta_n^{(2)}(z_k)}; \quad (11)$$

with

$$B_{\mathbf{m}n} = \left\langle \theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k)\theta_n^{(2)}(z_k)|Z_{k-1}\right\rangle;$$
$$(\mathbf{m} = (m_1, m_2, ...)). \quad (12)$$

Two functions, $\theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k)$ and $\theta_n^{(2)}(z_k)$, are orthonormal polynomials with the weighting functions $P_0(\mathbf{a}_k|Z_{k-1})$ and $P_0(z_k|Z_{k-1})$. Furthermore, $C_{\mathbf{Mm}}$ is the coefficient when the function $f_{\mathbf{M}}(\mathbf{a}_k)$ is expanded as:

$$f_{\mathbf{M}}(\mathbf{a}_k) = \sum_{\mathbf{m}=\mathbf{0}}^{\mathbf{M}} C_{\mathbf{Mm}}\theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k) \quad (13)$$

As the concrete expression on the fundamental probability function for the parameter $\mathbf{a}_k$ fluctuating in both a positive and negative range, a standard Gaussian distribution is adopted. Furthermore, a gamma distribution is adopted as the probability function for the sound intensity $\mathbf{z}_k$:

$$P_0(\mathbf{a}_k|Z_{k-1}) = \prod_i \frac{1}{\sqrt{2\pi\Gamma_{i,k}}}e^{-\frac{(a_{i,k}-a_{i,k}^*)^2}{2\Gamma_{i,k}}}; \quad (14)$$

$$P_0(z_k|Z_{k-1}) = \frac{z_k^{m_k^*-1}}{\Gamma(m_k^*)s_k^*}e^{-\frac{z_k}{s_k^*}}; \quad (15)$$

with

$$a_{i,k}^* = \langle a_{i,k}|Z_{k-1}\rangle;$$
$$\Gamma_{i,k} = \left\langle (a_{i,k}-a_{i,k}^*)^2|Z_{k-1}\right\rangle;$$
$$m_k^* = z_k^{*2}/\Omega_k; s_k^* = \Omega_k/z_k^*;$$
$$z_k^* = \langle z_k|Z_{k-1}\rangle; \Omega_k = \left\langle (z_k-z_k^*)^2|Z_{k-1}\right\rangle. \quad (16)$$

Therefore, the orthogonal polynomials[2,4] with the weighting functions of Eqs. (14) and (15) are expressed as follows:

$$\theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k) = \prod_i \frac{1}{\sqrt{m_i!}}H_{m_i}\left(\frac{a_{i,k}-a_{i,k}^*}{\sqrt{\Gamma_{i,k}}}\right); \quad (17)$$

$$\theta_n^{(2)}(z_k) = \sqrt{\frac{\Gamma(m_k^*)n!}{\Gamma(m_k^*+n)}}L_n^{(m_k^*-1)}\left(\frac{z_k}{s_k^*}\right). \quad (18)$$

By considering Eq. (7) and independence of x_k and v_k, two parameters z_k^* and Ω_k in Eq. (16) can be given by

$$z_k^* = \langle x_k|Z_{k-1}\rangle + \langle v_k\rangle; \quad (19)$$

$$\Omega_k = \left\langle (x_k-x_k^*)^2|Z_{k-1}\right\rangle + \left\langle (v_k-\langle v_k\rangle)^2\right\rangle. \quad (20)$$

Considering Eq. (6) and the property of conditional expectation, the right sides of the above equations are expressed as follows:

$$\begin{aligned}\langle x_k|Z_{k-1}\rangle &= \langle\langle x_k|y_k, Z_{k-1}\rangle|Z_{k-1}\rangle \\ &= \left\langle \int x_k P(x_k|y_k)dx_k|Z_{k-1}\right\rangle \\ &= \frac{\sum_{r=0}^{1} e_{1r}\mathbf{A}_{(r),k}\Phi(y_k)}{\sum_{s=0}^{\infty} A_{0s}\phi_s^{(2)}(y_k)};\end{aligned} \quad (21)$$

$$\begin{aligned}&\left\langle (x_k-x_k^*)^2|Z_{k-1}\right\rangle \\ &= \left\langle \int (x_k-x_k^*)^2 P(x_k|y_k)dx_k|Z_{k-1}\right\rangle \\ &= \frac{\sum_{r=0}^{2} e_{2r}\mathbf{A}_{(r),k}\Phi(y_k)}{\sum_{s=0}^{\infty} A_{0s}\phi_s^{(2)}(y_k)};\end{aligned} \quad (22)$$

with

$$\Phi(y_k) = (\phi_0^{(2)}(y_k), \phi_1^{(2)}(y_k), ...)^t;$$
$$\mathbf{A}_{(r),k} = (A_{r0}, \mathbf{a}_{(r),k}^*)(r=1,2,...);$$
$$\mathbf{A}_{(0),k} = \mathbf{A}_{(0)} = (A_{00}, A_{01}, A_{02}, ...);$$
$$\mathbf{a}_{(r),k}^* = \left\langle \mathbf{a}_{(r),k}|Z_{k-1}\right\rangle; \quad (23)$$

where t denotes the transpose of a matrix. The coefficients e_{1r} and e_{2r} in Eqs. (21) and (22) are determined in advance by expanding x_k and $(x_k-x_k^*)^2$ in the following orthogonal series forms:

$$x_k = \sum_{r=0}^{1} e_{1r}\phi_r^{(1)}(x_k); (x_k-x_k^*)^2 = \sum_{r=0}^{2} e_{2r}\phi_r^{(1)}(x_k) \quad (24)$$

Furthermore, using the definition of the Laguerre polynomial and Eqs. (6) and (7), the expansion coefficient $B_{\mathbf{m}n}$ can be calculated as in Eq. (25) (see the top of the next page), where e_{r_2r} is the expansion coefficient in the following expansion series:

$$x_k^{r_2} = \sum_{r=0}^{r_2} e_{r_2r}\phi_r^{(1)}(x_k). \quad (26)$$

From Eqs. (19)–(22) and (25), it can be found that the parameters z_k^*, Ω_k and the expansion coefficient $B_{\mathbf{m}n}$ are given by the predictions of the unknown parameter $\mathbf{a}_k$, the statistics of the background noise v_k, and the observations y_k at the reference point. By considering Eq. (9), the predictions to perform the recurrence estimation can be given for an arbitrary polynomial function $g_{\mathbf{N}}(\mathbf{a}_{k+1})$, with $\mathbf{N}$ th order of $\mathbf{a}_{k+1}$, can be expressed as:

$$\begin{aligned}g_{\mathrm{N}}^*(\mathbf{a}_{k+1}) &= \langle g_{\mathbf{N}}(\mathbf{a}_{k+1})|Z_k\rangle \\ &= \langle g_{\mathbf{N}}(\mathbf{a}_k)|Z_k\rangle = \hat{g}_{\mathbf{N}}(\mathbf{a}_k).\end{aligned} \quad (27)$$

2.3. Prediction of Specific Noise at the Evaluation Point

Because the conditional probability density function $P(x|y)$ can be considered as an invariant system characteristic, reflecting mainly the proper correlation relationship between the two sound intensities y and x at the reference and evaluation points, the probability distribution $P_s(x)$ of the sound intensity at the evaluation point corresponding to the random noise observed at the reference point can be predicted, as: $P_s(x) = \langle P(x|y)\rangle_y$. Thus, based on Eq. (6) and using the estimated parameter $\hat{\mathbf{a}}_k = \left(\hat{A}_{11}, \hat{A}_{12}, \cdots\right)$, the probability density function $P_s(x)$ at the evaluation point can be predicted from the observed data y at the reference point, as follows:

$$P_s(x) = P_0(x)\sum_{r=0}^{\infty}\left\langle \frac{\sum_{s=0}^{\infty}\hat{A}_{rs}\phi_s^{(2)}(y)}{\sum_{s=0}^{\infty}A_{0s}\phi_s^{(2)}(y)}\right\rangle_y \phi_r^{(1)}(x). \quad (28)$$

3. APPLICATION TO TRAFFIC NOISE IN AN ACTUAL SOUND ENVIRONMENT

The effectiveness of the proposed theory in Section 2 is confirmed experimentally by applying it to the actual data of road-traffic noise and low-frequency noise observed in a complicated sound environment near a national road and the entrance of a tunnel.

In order to evaluate the sound environment around the main line such as a highway and a railroad, the sound level at an evaluation point has to be estimated on the basis of the observation at a reference point. For the road-traffic noise, the reference point and the evaluation point were chosen at the positions being 1 m and 20 m apart from one side of the road as shown in Fig. 1. Since there are fences, a river, and buildings between the reference point and the evaluation point, the surrounding environment shows very complex characteristics, and it is difficult to identify the sound propagation characteristics based on the physical mechanism. By applying the proposed method, the probability density function of the sound level at the evaluation point was predicted on the basis of the observation at the reference point. Road-traffic noise was measured

$$B_{\mathbf{m}n} = \sqrt{\frac{\Gamma(m_k^*)n!}{\Gamma(m_k^*+n)}} \sum_{r_1=0}^{n} (-1)^{r_1} \begin{pmatrix} n \\ r_1 \end{pmatrix} \frac{1}{n!} \frac{\Gamma(m_k^*+n)}{\Gamma(m_k^*+r_1)} \frac{1}{s_k^*}$$

$$\cdot \sum_{r_2=0}^{r_1} \sum_{r=0}^{r_2} \frac{e_{r_2 r} \left\langle \theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k)(A_{r0}, \mathbf{a}_{(r),k}) | Z_{k-1} \right\rangle \Phi(y_k)}{\sum_{s=0}^{\infty} A_{0s} \phi_s^{(2)}(y_k)} \left\langle v_k^{r_1-r_2} \right\rangle ; \qquad (25)$$

Table 1. Statistics of the specific noise and the background noise.

Specific Noise		Background Noise	
Mean [watt/m^2]	Standard Deviation [watt/m^2]	Mean [watt/m^2]	Standard Deviation [watt/m^2]
1.7185×10^{-6}	2.0567×10^{-6}	1.7185×10^{-6}	6.734×10^{-8}

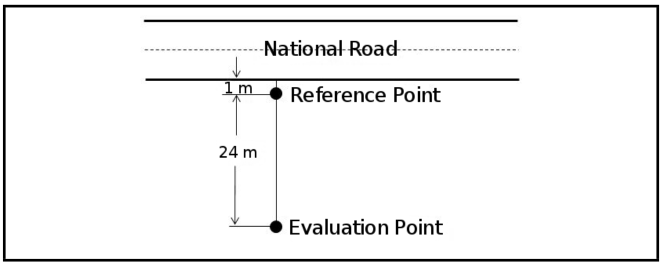

Figure 1. A schematic drawing of the experiment in a road traffic noise environment near a national road.

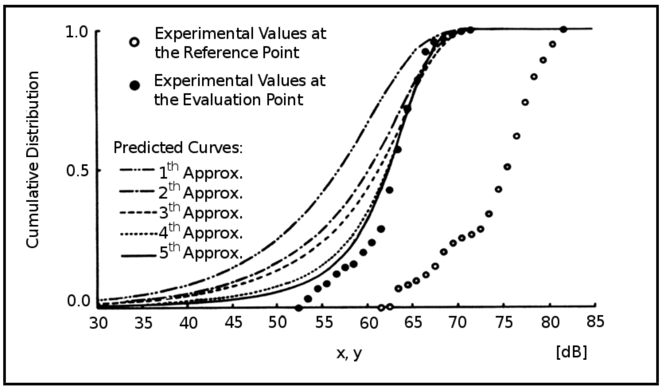

Figure 2. A comparison between predicted curves and experimental values of the probability distribution at the evaluation point for road-traffic noise.

at every 0.2 s using a sound-level meter (model NL-06 integral standard type, Rion Co.) under an A-characteristic and FAST response with a time constant of 0.125 s in an RMS circuit. Applying the proposed algorithm developed in Section 2 on the 400 data sample, the expansion coefficients in Eq. (1) were estimated. The statistics of the road-traffic noise and the background noise are shown in Table 1. In order to confirm the effectiveness of the proposed method, it is necessary to apply it to the data with the large amplitude of the background noise. Therefore, after separately recording the specific noise and background noise into a data recorder, by replaying the recorded two noises and mixing them in an anechoic chamber, the observation data were measured. Then, we adjusted the amplitude of background noise so as to have the same mean value as the specific noise.

Using Eq. (28) and the estimated expansion coefficients, the probability distribution at the evaluation point was predicted from the observation at the reference point. The 300 sampled data following the data used for the estimation of the expansion coefficients were adopted for predicting the probability distribution of the sound level at the evaluation point. For the purpose of confirming the prediction accuracy of the proposed method, it was only applied to the data during a short time interval as a trial. In the real assessment for the noise environment, the proposed method has to be applied to the data in appropriate time intervals, according to the purpose of real noise evaluation. Figure 2 shows the comparison between the theoretically predicted curves and the experimentally sampled points on the probability distribution. The cumulative distributions of sound level related to noise evaluation quantities L_x ($(100-x)$ percentile level) are shown. In the evaluation of the noise environment, the prediction of the cumulative distribution connected with the noise evaluation index L_x is important.[5] The "1$^{\mathrm{st}}$ Approx." considers only the first term in Eq. (28), and the predicted curves from the "2$^{\mathrm{nd}}$ Approx." to the "5$^{\mathrm{th}}$ Approx." consider the estimated expansion coefficients $\hat{A}_{11}$, $\hat{A}_{12}$, $\hat{A}_{21}$, and $\hat{A}_{22}$, additionally. It can be observed that the theoretically predicted curves approach the experimental values when expansion coefficients of several higher orders are considered. Several noise evaluation quantities L_x evaluated from Fig. 2 are shown in Table 2. It is obvious that the proposed method provides accurate predictions within ± 1 dB permissible errors in the measurement of environmental noises, and the effectiveness of the proposed method has been confirmed numerically.

Table 2. Comparison between the experimental values and theoretically predicted values for several noise evaluation quantities in dB evaluated from Fig. 2.

Noise Evaluation Quantities	L_5	L_{10}	L_{50}	L_{90}	L_{95}
Experimental Values	67.3	66.1	62.9	55.6	53.6
Predicted Curve (1st Approx.)	66.2	64.7	56.7	41.7	35.5
Predicted Curve (2nd Approx.)	68.2	66.7	60.0	45.5	40.0
Predicted Curve (3rd Approx.)	68.6	67.3	61.1	47.3	41.1
Predicted Curve (4th Approx.)	67.9	66.7	62.1	52.0	46.4
Predicted Curve (5th Approx.)	67.6	66.7	62.3	53.9	49.8

For the low-frequency noise generated by the Shinkansen trains running through the tunnel, a reference point and an evaluation point were chosen at the positions that were 10 m and 25 m apart from the entrance of the tunnel as shown in Fig. 3. By applying the proposed method in Section 2, the probability density function of the low-frequency noise at the evaluation point was predicted. Four kinds of low-frequency noise data was generated from the Shinkansen trains: 1) Up "Hikari", 2) Down "Hikari", 3) Up "Nozomi", and 4) Down "Nozomi" were measured by using a ceramic microphone with a low-frequency sound pressure level meter (model NA18A, Rion Co.) under a FLAT-characteristic (i.e., a flat frequency

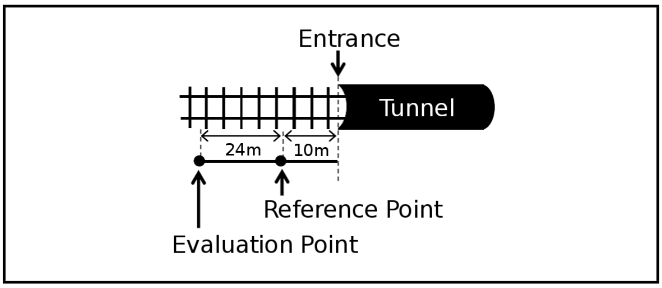

Figure 3. A schematic drawing of the experiment in low-frequency noise near an entrance of a tunnel.

Table 3. Statistics of the specific noise.

Train	Mean [watt/m^2]	Standard Deviation [watt/m^2]
Up "Hikari"	2.5295×10^{-5}	3.3607×10^{-5}
Down "Hikari"	3.0542×10^{-5}	4.6687×10^{-5}
Up "Nozomi"	3.3074×10^{-5}	6.8644×10^{-5}
Down "Hikari"	4.5607×10^{-5}	9.7969×10^{-5}

characteristic within the range of 1 Hz to 100 Hz) at every 0.1 s. In the measurement of low-frequency noise, the effects of a wind noise are inevitable.[6–8] Therefore, by applying the proposed estimation algorithm in Eq. (11) to the observation data for the Up "Hikari" train affected by a wind noise, the expansion coefficients in Eq. (1) were first estimated. The statistics of the low-frequency noise and the background noise are shown in Tables 3 and 4, respectively. Based on the estimates of the expansion coefficients, the probability distributions for the sound level of the low-frequency noises generated from the i) Down "Hikari", ii) Up "Nozomi" and iii) Down "Nozomi" trains were predicted by measuring the sound-level data at the reference point and using Eq. (28). Figures 4, 5 and 6 show comparisons between the theoretically predicted curves and experimental values for the probability distributions at the evaluation point for the Down "Hikari" train, Up "Nozomi" train, and Down "Nozomi" train, respectively. The cumulative distributions of the sound level are shown in these figures. When cboxa sufficient number of expansion coefficients of higher order are taken into consideration, the theoretically predicted curves approach the experimentally sampled values for the probability distribution of the low-frequency noise.

Furthermore, in order to discuss the precision of the proposed prediction method of the probability distribution, comparisons between the theoretically predicted values and the experimental values for noise evaluation quantities L_x (x =5, 10, 50, 90, 95) obtained from Figs. 4, 5 and 6 are shown in Tables 5, 6 and 7. In an evaluation for low-frequency noise with the fluctuation, L_5, and L_{10} correspond approximately to the peak or maximum values of the fluctuation, and L_{50} corresponds to the median. Furthermore, L_{90} and L_{95} correspond to the background noise levels. From these results, the effectiveness of the proposed prediction method for the probability distribution at evaluation points has been confirmed numerically.

4. CONCLUSION

In this paper, an evaluation method of traffic noise in a complex sound environment under the existence of a background noise has been proposed. More specifically, a prediction method of the sound level at evaluation points based on the observations at a reference point has been theoretically derived. By paying attention to the intensity variables satisfying the additive property of the specific noise and the background noise, a method predicting the probability distribution of the sound intensity at evaluation points has been proposed. The proposed prediction method has been realized by introducing a sound environment model of the conditional probability type. The proposed method has then been applied to the estimation and prediction of actual road-traffic noise and low-frequency noise, and it has been experimentally verified that good results have been achieved with this method.

Table 4. Statistics of the wind noise.

Mean [watt/m^2]	Standard Deviation [watt/m^2]
1.2629×10^{-5}	4.5943×10^{-5}

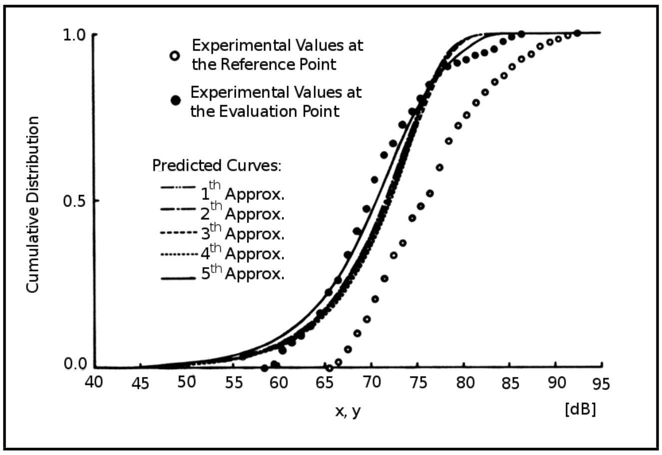

Figure 4. A comparison between predicted curves and experimental values of the probability distribution at the evaluation point for a low-frequency noise by Down "Hikari".

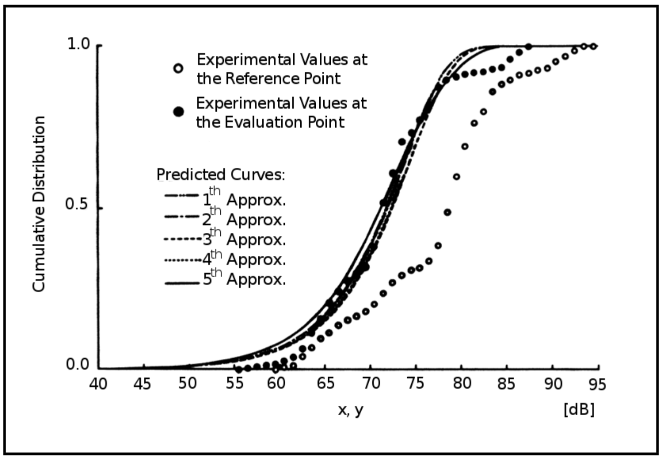

Figure 5. A comparison between predicted curves and experimental values of the probability distribution at the evaluation point for a low-frequency noise by Up "Nozomi".

Table 5. Comparison between the experimental values and theoretically predicted values for several noise evaluation quantities in dB evaluated from Fig. 4.

Noise Evaluation Quantities	L_5	L_{10}	L_{50}	L_{90}	L_{95}
Experimental Values	82.9	78.5	70.0	62.6	60.5
Predicted Curve (1st Approx.)	78.9	78.0	72.3	62.3	58.6
Predicted Curve (2nd Approx.)	79.1	77.7	72.1	62.1	58.4
Predicted Curve (3rd Approx.)	78.9	77.6	71.8	61.8	58.2
Predicted Curve (4th Approx.)	78.6	77.4	71.7	61.7	57.7
Predicted Curve (5th Approx.)	80.2	78.2	70.7	60.6	57.0

$$\hat{f}_{\mathbf{M}}(\mathbf{a}_k) = \int\int ... \int f_{\mathbf{M}}(\mathbf{a}_k) P(\mathbf{a}_k|Z_k) d\mathbf{a}_k = \frac{\int\int ... \int \sum_{\mathbf{m}'=\mathbf{0}}^{\mathbf{M}} C_{\mathbf{M}\mathbf{m}'} \theta_{\mathbf{m}'}^{(1)}(\mathbf{a}_k) P_0(\mathbf{a}_k|Z_{k-1}) \sum_{\mathbf{m}=\mathbf{0}}^{\infty} \sum_{n=0}^{\infty} B_{\mathbf{m}n} \theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k) \theta_n^{(2)}(z_k) d\mathbf{a}_k}{\sum_{n=0}^{\infty} B_{\mathbf{0}n} \theta_n^{(2)}(z_k)}. \tag{A3}$$

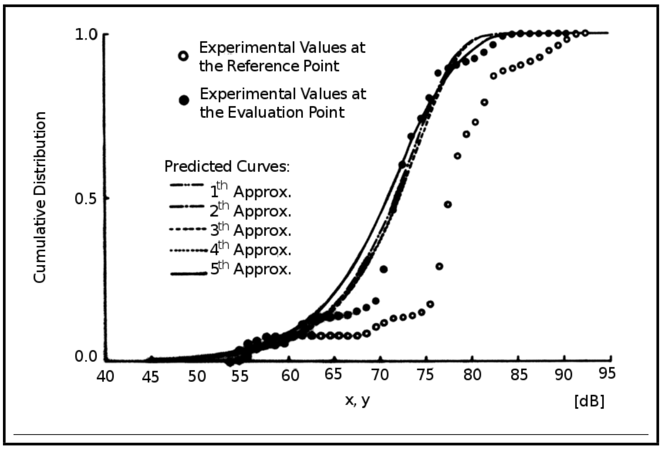

Figure 6. A comparison between predicted curves and experimental values of the probability distribution at the evaluation point for a low-frequency noise by Down "Nozomi".

Table 6. Comparison between the experimental values and theoretically predicted values for several noise evaluation quantities in dB evaluated from Fig. 5.

Noise Evaluation Quantities	L_5	L_{10}	L_{50}	L_{90}	L_{95}
Experimental Values	85.3	88.9	71.4	63.2	62.0
Predicted Curve (1st Approx.)	79.7	78.3	72.4	62.9	58.7
Predicted Curve (2nd Approx.)	79.7	78.3	72.3	62.6	58.7
Predicted Curve (3rd Approx.)	78.7	77.7	71.8	62.1	58.6
Predicted Curve (4th Approx.)	78.9	77.7	71.7	62.1	58.0
Predicted Curve (5th Approx.)	80.6	88.8	71.2	61.2	57.1

ACKNOWLEDGMENTS

The authors would like to thank Emeritus Professor M. Ohta for his valuable discussion and assistance during this study.

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Table 7. Comparison between the experimental values and theoretically predicted values for several noise evaluation quantities in dB evaluated from Fig. 6.

Noise Evaluation Quantities	L_5	L_{10}	L_{50}	L_{90}	L_{95}
Experimental Values	81.7	78.2	71.7	60.9	58.5
Predicted Curve (1st Approx.)	79.1	77.9	72.1	62.4	58.5
Predicted Curve (2nd Approx.)	79.1	77.9	72.1	62.4	58.5
Predicted Curve (3rd Approx.)	78.8	77.6	71.7	61.8	58.5
Predicted Curve (4th Approx.)	78.8	77.6	71.7	61.8	58.5
Predicted Curve (5th Approx.)	80.2	78.2	70.8	60.9	57.0

APPENDIX. DERIVATION OF THE ESTIMATE

The conditional joint probability density function of the parameter $\mathbf{a}_k$ and the observation z_k can be generally expanded in a statistical orthogonal expansion series:

$$P(\mathbf{a}_k, z_k|Z_{k-1}) = P_0(\mathbf{a}_k|Z_{k-1}) P_0(z_k|Z_{k-1}) \sum_{\mathbf{m}=\mathbf{0}}^{\infty} \sum_{n=0}^{\infty} B_{\mathbf{m}n} \theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k) \theta_n^{(2)}(z_k); \tag{A1}$$

Substituting Eq. (A1) into Eq. (10), the following expression can be obtained.

$$P(\mathbf{a}_k|Z_k) = \frac{P_0(\mathbf{a}_k|Z_{k-1}) \sum_{\mathbf{m}=\mathbf{0}}^{\infty} \sum_{n=0}^{\infty} B_{\mathbf{m}n} \theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k) \theta_n^{(2)}(z_k)}{\sum_{n=0}^{\infty} B_{\mathbf{0}n} \theta_n^{(2)}(z_k)}. \tag{A2}$$

By using the above equation and the relationship in Eq. (13), and taking the conditional expectation of the function $f_{\mathbf{M}}(\mathbf{a}_k)$, the estimate of $f_{\mathbf{M}}(\mathbf{a}_k)$ can be expressed as Eq. (A3). Furthermore, by using the orthonormal condition for the function $\theta_m^{(1)}(\mathbf{a}_k)$:

$$\int\int ... \int \theta_{\mathbf{m}}^{(1)}(\mathbf{a}_k) \theta_{\mathbf{m}'}^{(1)}(\mathbf{a}_k) P_0(\mathbf{a}_k|Z_{k-1}) d\mathbf{a}_k = \delta_{\mathbf{m}\mathbf{m}'}; \tag{A4}$$

Eq. (11) can be derived.