Stability of a Nonlinear Quarter-Car System with Multiple Time-Delays

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This paper examines the dynamical behaviour of a nonlinear oscillator which models a two-degree-of-freedom quarter-car system forced by the road profile. The influence of two time delays in the system, which is generally due to the inherent dynamics of the actuator, is studied. The asymptotic and technical stability domain is obtained by using Bogusz's stability criterion for a two-degree-of-freedom system. The results obtained from Lyapunov's and Bogusz's stability criterions are compared. The numerical results obtained are found to be in good agreement with the analytical predictions.

1. INTRODUCTION

Li et al. investigated a possible chaotic motion in a nonlinear vehicle suspension system, which is subject to a multi-frequency excitation from a road surface.[5] Litak et al. investigated global homoclinic bifurcation and the transition to chaos in the case of a quarter-car model excited kinematically by a road surface profile.[6] Siewe Siewe investigated resonance, stability, and the chaotic motion of a quarter-car model excited by a road surface profile.[11] All controllers exhibit a certain time-delay during operation. Many researchers have studied the behaviour of delayed differential equations. Zhang et al. studied the stability of the delayed differential equations.[13] Wirkus and Rand investigated the dynamics of two van der Pol oscillators with delay velocity coupling with special attention to the bifurcation accompanying the change in number and the stability of the solutions.[9] Gohary studied the vibration suppression of a dynamical system to multi-parametric excitations via a time-delay absorber.[3] Naik & Singru studied the stability of a single-degree-of-freedom system.[8]

The dynamics of a quarter-car model with nonlinear suspension characteristics was studied by Li et al. and Shen et al.[4,10] Recent efforts by Borowiec et al. have been focused on the excitation of the automobile by a road surface profile with harmful noise components.[2] Hysteretic nonlinear suspension was studied by Yang et al.[12]

This paper deals with the dynamics of a quarter-car model with an active vehicle suspension system. The influence of delay in the system, which is generally due to the inherent dynamics of the actuator, is studied. The time delay systems are usually of infinite dimension; an attempt is made to reduce them to a finite dimension since the operating time delays in the system under consideration are small (Processing Time & Actuator Delays). The technical stability domain is obtained by using Boguszs stability criterion on the finite time domain. The asymptotic stability is derived for unequal time delays.

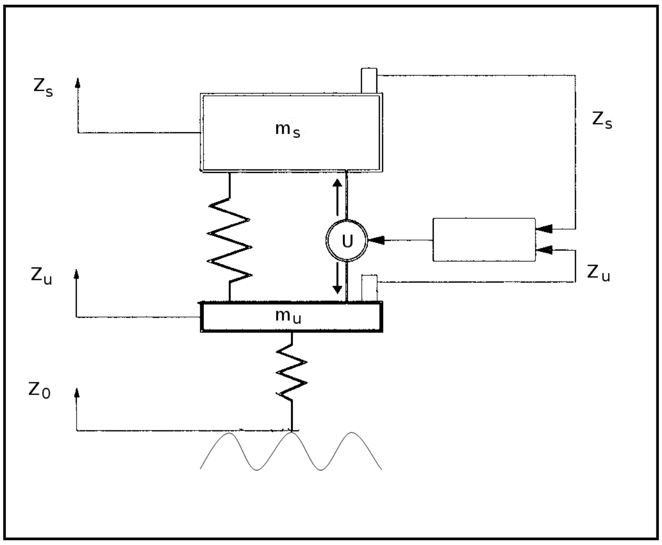

Figure 1. The quarter-car model active control.

The stability of the two models are investigated, and the results are compared. The results are validated by numerical simulations.

2. TWO-DEGREE-OF-FREEDOM QUARTER-CAR SYSTEM

Figure 1 shows the closed-loop active control for the vehicle system. This system represents an active vehicle suspension system with non-linearity in the dampers; the parameters considered are similar to that of the single-degree-of-freedom system considered in the previous section.

The equation of motion is assumed to have the following form:

$$m_u \ddot{Z}_u - k(Z_s - Z_u) - k_t(Z_0 - Z_u) + U(t) = 0;$$
$$m_s \ddot{Z}_s + k(Z_s - Z_u) - U(t) = 0; \qquad (1)$$

where $U(t)$ is the active control force

$$U(t) = -C_2(\dot{Z}_s - \dot{Z}_u)^3 + K_{Au}Z_u(t-\tau_1) + K_{As}Z_s(t-\tau_2) + ... + C_{Au}\dot{Z}_u(t-\tau_1) + C_{As}\dot{Z}_s(t-\tau_2) - C_1(\dot{Z}_s - \dot{Z}_u); \quad (2)$$

$\omega_u^2 = \left(\frac{k+k_t}{m_u}\right)$; $\omega_s^2 = \left(\frac{k}{m_s}\right)$; $K_1 = \left(\frac{k}{m_u}\right)$; $C_1 = \left(\frac{c_1}{m_u}\right)$; $C_2 = \left(\frac{c_2}{m_u}\right)$; $K_{1u} = \left(\frac{K_{A1}}{m_u}\right)$; $K_{2u} = \left(\frac{K_{A2}}{m_u}\right)$; $C_{1u} = \left(\frac{C_{A1}}{m_u}\right)$; $C_{2u} = \left(\frac{C_{A2}}{m_u}\right)$; $C_3 = \left(\frac{c_1}{m_s}\right)$; $C_4 = \left(\frac{c_2}{m_s}\right)$; $K_{1s} = \left(\frac{K_{A1}}{m_s}\right)$; $K_{2s} = \left(\frac{K_{A2}}{m_s}\right)$; and $C_{1s} = \left(\frac{C_{A1}}{m_s}\right)$; $C_{2s} = \left(\frac{C_{A2}}{m_s}\right)$; where ω_u = unsprung mass natural frequency, ω_s = sprung mass natural frequency, K_{Au} are semi-active stiffness for unsprung mass, K_{As} are semi-active stiffness for sprung mass C_{Au} semi-active damping for unsprung mass, C_{As} semi-active damping for sprung mass, τ_u, τ_s are the time-delays associated with unsprung and sprung mass, and the parameters are as follows:[1] m_s = 200 kg; m_u = 40 kg; k = 9000 Nm^{-1} ; k_t = 160,000 Nm^{-1}; k_2 = -300,000 Nm^{-3}; c_1 = 10,000 Nsm^{-1}; c_2 = -25,000 Ns^{3}m^{-3}; C_{As} =1000 Nsm^{-1}; C_{Au} =-10,000 Nsm^{-1}; K_{As} = 250000 Nm^{-1}; K_{Au} = 50000 Nm^{-1}.

3. LYAPUNOV STABILITY FOR TWO-DEGREE-OF-FREEDOM-SYSTEM

The asymptotic stability as $t \to \infty$ is studied for the two-degree-of-freedom system, and the results are compared with the technical stability criterion. The time-delay systems are usually infinite dimensional. An attempt is made to reduce them to a finite dimension since the operating time delays in the system under consideration are small (Processing Time & Actuator Delays).

Expanding the function $U(t)$ into Taylor's series including only the first term of the function $U(t)$:

$$U(t) = -C_2(\dot{Z}_s - \dot{Z}_u)^3 + K_{Au}Z_u(t) + \tau_u K_{Au}\dot{Z}_u(t) + K_{As}Z_s(t) + \tau_s K_{As}\dot{Z}_s(t) + ... + C_{Au}\dot{Z}_u(t) + \tau_u C_{Au}\ddot{Z}_u(t) + C_{As}\dot{Z}_s(t) + \tau_s C_{As}\ddot{Z}_s(t) - C_1(\dot{Z}_s - \dot{Z}_u). \quad (3)$$

Since the non-homogeneous equation's stability is preserved when the stability of the homogeneous equations hold, the stability of the homogeneous equations are investigated, and we neglect the third power. Substituting Eq. (3) into Eq. (1) and considering only linear terms we obtain:

$$(1+\tau_u C_{1u})\ddot{Z}_u + (K_{1u}+\omega_u^2)Z_u(t) + (C_1 + C_{1u} + \tau_u K_{1u})\dot{Z}_u(t) + (K_{2u} - K_1)Z_s(t) + (\tau_s K_{2u} - C_1 + C_{2u})\dot{Z}_s(t) + \tau_s C_{2u}\ddot{Z}_s(t) = 0; \quad (4)$$

$$(1-\tau_s C_{2s})\ddot{Z}_s + (\omega_s^2 - K_{2s})Z_s + (C_3 - \tau_s K_{2s} - C_{2s})\dot{Z}_s(t) - (C_3 + \tau_s K_{1s} + C_{1s})\dot{Z}_u(t) + (K_{1s} - \omega_s^2)Z_u - \tau_u C_{1s}\ddot{Z}_u(t) = 0; \quad (5)$$

$$\ddot{Z}_u + \alpha_1 Z_u(t) + \alpha_2 \dot{Z}_u(t) + \alpha_3 Z_s(t) + \alpha_4 \dot{Z}_s(t) + \alpha_4 \ddot{Z}_s(t) = 0; \quad (6)$$

$$\ddot{Z}_s + \beta_1 Z_s + \beta_2\dot{Z}_s(t) + \beta_3 \dot{Z}_u(t) + \beta_4 Z_u + \beta_5 \ddot{Z}_u = 0; \quad (7)$$

where $\alpha_1 = \frac{(K_{1u}+\omega_u^2)}{(1+\tau_u C_{1u})}$, $\alpha_2 = \frac{(C_1+C_{1u}+\tau_u K_{1u})}{(1+\tau_u C_{1u})}$
$\alpha_3 = \frac{(K_{2u}-K_1)}{(1+\tau_u C_{1u})}$, $\alpha_4 = \frac{(\tau_s K_{2u}-C_1+C_{2u})}{(1+\tau_u C_{1u})}$ and $\alpha_5 = \frac{\tau_u C_{2u}}{(1+\tau_u C_{1u})}$
and $\beta_1 = \frac{(\omega_s^2-K_{2s})}{(1-\tau_s C_{2s})}$, $\beta_2 = \frac{(C_3-\tau_s K_{2s}-C_{2s})}{(1-\tau_s C_{2s})}$
$\beta_3 = \frac{(K_{1s}-\omega_s^2)}{(1-\tau_s C_{2s})}$, $\beta_4 = \frac{(C_3+\tau_s K_{1s}+C_{1s})}{(1-\tau_s C_{2s})}$ and $\beta_5 = \frac{\tau_s C_{1s}}{(1-\tau_s C_{2s})}$.
Substituting Eq. (6) into Eq. (7) we obtain:

$$\ddot{Z}_u = a_1 Z_u + a_2 Z_s + a_3 \dot{Z}_u + a_4 \dot{Z}_s. \quad (8)$$

Substituting Eq. (7) into Eq. (6) we obtain:

$$\ddot{Z}_s = b_1 Z_u + b_2 Z_s + b_3 \dot{Z}_u + \beta_4 \dot{Z}_s; \quad (9)$$

where $a_1 = -\left[\frac{\alpha_1-\alpha_5\beta_4}{1+\alpha_5\beta_5}\right]$, $a_2 = -\left[\frac{\alpha_2+\alpha_5\beta_3}{1+\alpha_5\beta_5}\right]$
$a_3 = -\left[\frac{\alpha_3-\alpha_5\beta_1}{1+\alpha_5\beta_5}\right]$, $a_4 = -\left[\frac{\alpha_4-\alpha_5\beta_2}{1+\alpha_5\beta_5}\right]$ and
$b_1 = -\left[\frac{\beta_4+\beta_5\alpha_1}{1+\alpha_5\beta_5}\right]$, $b_2 = -\left[\frac{\beta_1+\beta_5\alpha_3}{1+\alpha_5\beta_5}\right]$
$b_3 = -\left[\frac{\beta_5\alpha_2-\beta_3}{1+\alpha_5\beta_5}\right]$, $b_4 = -\left[\frac{\beta_2+\beta_5\alpha_4}{1+\alpha_5\beta_5}\right]$. In the state space form

$$\left.\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 \\ \dot{x}_4 &= b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 \end{aligned}\right\} \quad (10)$$

where $x_1 = Z_u$, $x_2 = \dot{Z}_u$, $x_3 = Z_s$ and $x_4 = \dot{Z}_s$, and the characteristic determinant associated with the Eq. (10) has the form

$$W = \begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ a_1 & a_2 & a_3-\lambda & a_4 \\ b_1 & b_2 & b_3 & b_4-\lambda \end{vmatrix}.$$

The determinant takes the form

$$W = \lambda^4 - (a_3+b_4)\lambda^3 - a_1\lambda^2 + (b_2 + a_1 b_4 - b_1 a_4)\lambda + a_2 b_3 - b_2 a_3 + a_1 b_2 - b_1 a_2. \quad (11)$$

The following characteristic equation is obtained:
$\lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0$,
where
$A_3 = -(a_3+b_4)$, $A_2 = -a_1$,
$A_1 = (b_2 + a_1 b_4 - b_1 a_4)$
and $A_0 = a_2 b_3 - b_2 a_3 + a_1 b_2 - b_1 a_2$. The necessary condition for the stability of the system is

$$A_3 > 0,\ A_2 > 0,\ A_1 > 0 \text{ and } A_0 > 0. \quad (12)$$

The sufficient conditions of the system require the positiveness of the determinants

$$D_1 > 0, D_2 > 0, D_3 > 0 \text{ and } D_4 > 0; \quad (13)$$

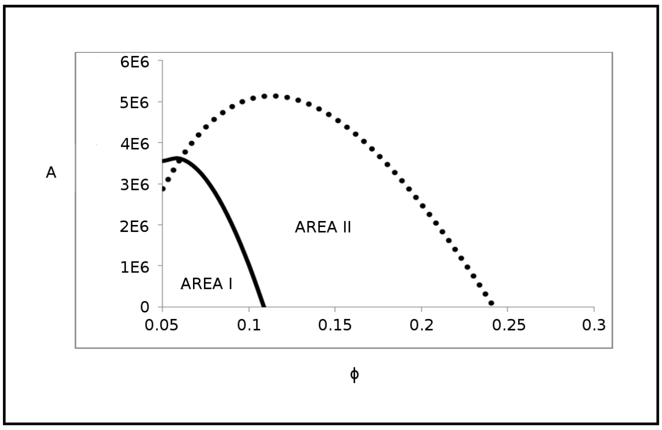

Figure 2. The stability region AREA I for $\tau_u > \tau_s$ and AREA II for $\tau_s > \tau_u$ (Lyapunovs Stability).

where $D_1 = A_3,\ D_2 = \begin{vmatrix} A_3 & 1 \\ A_1 & A_2 \end{vmatrix}$,

$$D_3 = \begin{vmatrix} A_3 & 1 & 0 \\ A_1 & A_2 & A_3 \\ 0 & A_0 & A_1 \end{vmatrix} \text{ and } D_4 = \begin{vmatrix} A_3 & 1 & 0 & 0 \\ A_1 & A_2 & A_3 & 1 \\ 0 & A_0 & A_1 & A_2 \\ 0 & 0 & 0 & A_0 \end{vmatrix}.$$

Now consider two cases

(i) The time delay associated with the sprung mass dynamics is greater than that of the unsprung mass dynamics i.e. $\tau_s > \tau_u$ and

(ii) The time delay associated with the unsprung mass dynamics is greater than that of the sprung mass dynamics i.e. $\tau_u > \tau_s$,

where the delay difference is $\phi = \tau_s - \tau_u > 0$ for $\tau_s > \tau_u$ and $\phi = \tau_s - \tau_u < 0$ for $\tau_u > \tau_s$.

The conditions in Eqs. (12) and (13) are plotted by using MATLAB 11 by considering the parameters given in Section 3 above. Figure 2 indicates that the stability regions are the areas below the two curves where A is the amplitude of vibrations. Comparing both of the curves, one can observe that in the case of $\tau_s > \tau_u$, the area below the curve (AREA II) is greater than the area for $\tau_u > \tau_s$ (AREA I). The domain located below the curve, where the control is efficient is greater for $\tau_s > \tau_u$. Therefore vibration control is efficient for $\tau_s > \tau_u$. It can be easily found from Fig. 2 that for the delay difference $\phi = \tau_s - \tau_u <$ 0.24, the control is efficient for $\tau_s > \tau_u$, and for $\tau_u > \tau_s$ the control is efficient for $\phi = \tau_s - \tau_u < 0.11$.

The nonlinear differential Eq. (1) is numerically simulated to obtain the Poincaré section (plot of velocity v/s displacement) by using the dde23 module of MATLAB with a numerical accuracy of 10^{-8} . Figure 3 indicates the regular motion of the sprung mass for $\tau_u > \tau_s$ and ϕ = 0.02 . Figure 4 and Figure 5 indicate the periodic motion of the sprung mass and the unsprung mass for $\tau_s > \tau_u$ and $\phi = 0.2$. Figure 6 indicates the chaotic motion of the sprung mass for $\tau_s > \tau_u$ and $\phi = 0.24$. Figure 7 indicates the chaotic motion of the sprung mass for $\tau_u > \tau_s$ and $\phi = 0.15$, which is much lower compared to $\tau_s > \tau_u$. Figure 8 indicates the chaotic motion of the unsprung mass for $\tau_u > \tau_s$ and $\phi = 0.15$.

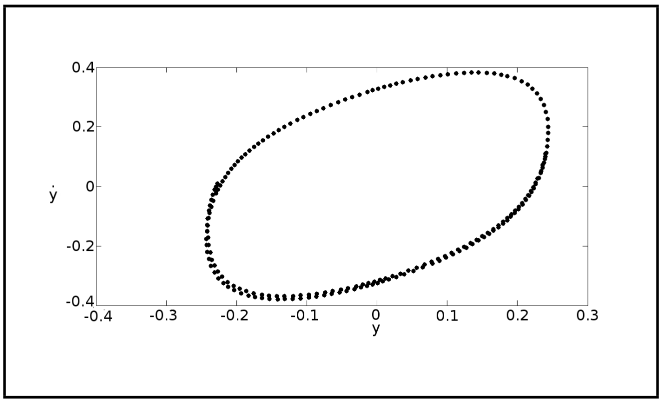

Figure 3. Poincaré section of sprung mass for $\tau_u > \tau_s$, $\phi = 0.02$ (Period one).

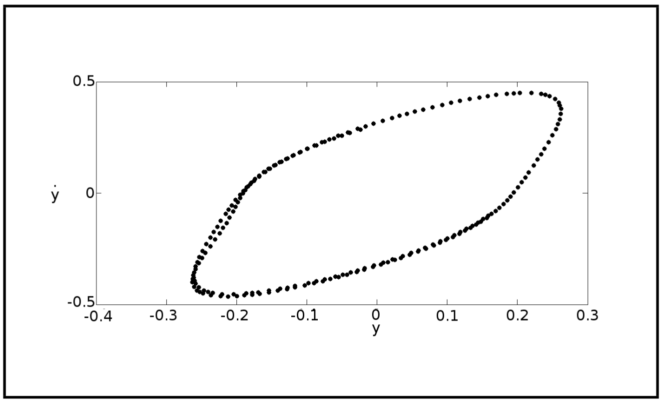

Figure 4. Poincaré section of sprung mass for $\tau_s > \tau_u$ and $\phi = 0.2$ (Period one).

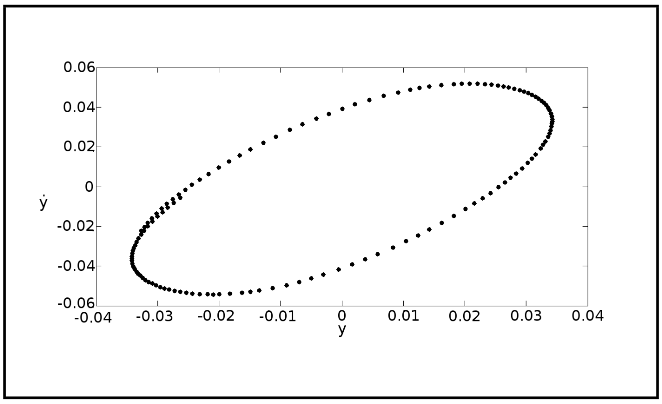

Figure 5. Poincar section of unsprung mass for $\tau_s > \tau_u$ and $\phi = 0.2$ (Period one).

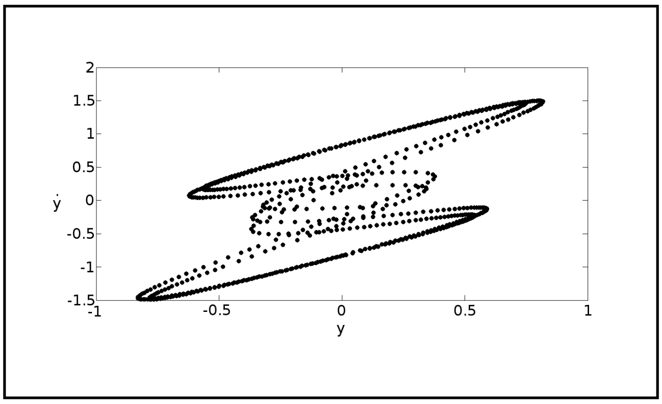

Figure 6. Poincaré section of sprung mass for $\tau_s > \tau_u$ and $\phi = 0.24$.

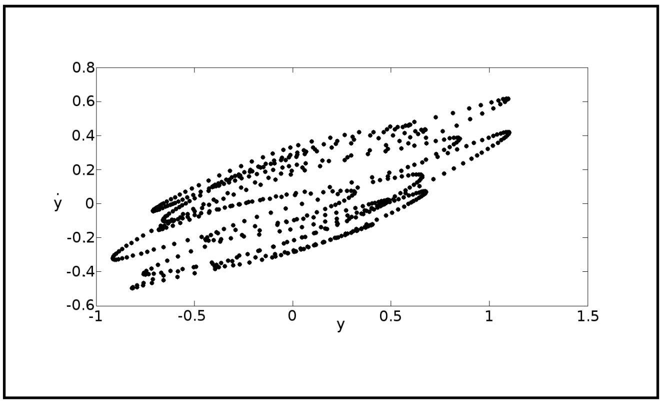

Figure 7. Poincar section of sprung mass for $\tau_u > \tau_s$, $\phi = 0.15$.

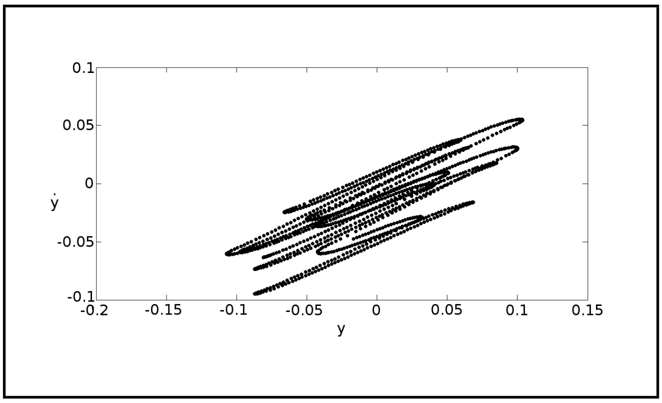

Figure 8. Poincaré section of unsprung mass for $\tau_u > \tau_s$, $\phi = 0.15$.

4. TECHNICAL STABILITY FOR TWO-DEGREE-OF-FREEDOM-SYSTEM

The observation time is often limited and relatively short due to the type of excitation. In such cases, we are interested in the first few seconds of the system's motion.[1] From a practical point of view, the possibility of defining the proximity of the solutions would be advantageous. Let the initial conditions field Γ for $t = t_0 = 0$ be:

$$\Gamma \equiv \left\{c_1\, x_1^2 + c_2 x_2^2 + c_3 x_3^2 + c_4 x_4^2 < \; r_0^2\right\}. \tag{14}$$

The field of acceptable solutions λ for $t \le T$, is assumed as:

$$\lambda \equiv \left\{c_1\, x_1^2 + c_2 x_2^2 + c_3 x_3^2 + c_4 x_4^2 < \; R_0^2\right\}; \tag{15}$$

while values $r_0 < R_0$ so that condition $\Gamma \in \lambda$ is fulfilled. Bogusz's function will take the form of:

$$V(x_1, x_2, x_3, x_4) = \frac{1}{2}\,\left(A\; x_1^2 + B\, x_2^2 + C\; x_3^2 + D\, x_4^2\right); \tag{16}$$

where $A > 0$, $B > 0$, $C > 0$, and $D > 0$. If there exists a number C_0 such that

$$\begin{aligned} C_0 &= \inf_{x_i \in \Gamma}\, [V(x_1, x_2, x_3, x_4)] \\ &= \frac{1}{2}\,\left(A\,.\; r_1^2 + B\,.\, r_2^2 + C\,.\; r_3^2 + D\,.\, r_4^2\right); \end{aligned} \tag{17}$$

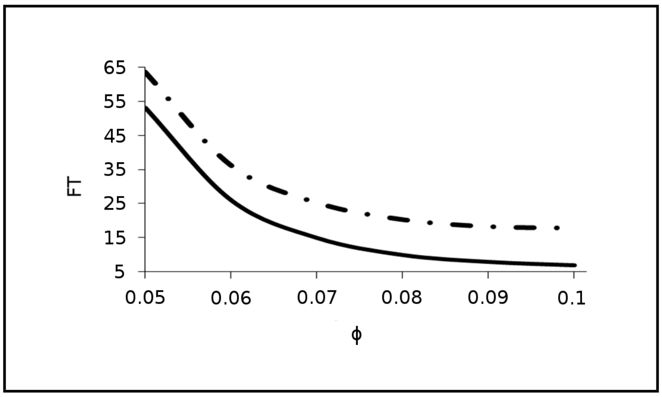

Figure 9. The stability region for $\tau_u > \tau_s$ is shown by the solid curve, and for $\tau_s > \tau_u$ it is shown by the dotted curve.

and a number C_1 such that

$$\begin{aligned} C_1 &= \sup_{x_i \in \Gamma}\, [V(x_1, x_2, x_3, x_4)] \\ &= \frac{1}{2}\,\left(A\,.\, R_1^2 + B\,.\; R_2^2 + C\,.\, R_3^2 + D\,.\, R_4^2\right); \end{aligned} \tag{18}$$

where
$\frac{\Gamma}{\lambda} \equiv \left\{r_1^2 < x_1^2 \le R_1^2, r_2^2 < x_2^2 \le R_2^2, r_3^2 < x_3^2 \le R_3^2,\right.$
$\left. r_4^2 < x_4^2 \le R_4^2\right\}$.
The derivative of Bogusz's function along the solutions of the studied system

$$\frac{dV}{dt} = \frac{\partial V}{\partial x_1}\,\dot{x}_1 + \frac{\partial V}{\partial x_2}\,\dot{x}_2 + \frac{\partial V}{\partial x_3}\,\dot{x}_3 + \frac{\partial V}{\partial x_4}\,\dot{x}_4. \tag{19}$$

Calculating the derivative of Eq. (19) by using Eq. (10) we obtain:

$$\begin{aligned} \frac{dV}{dt} = {} & C\,.\,a_3\,.\,x_3^2 + \; D.\,b_4\,.\,x_4^2 + \; D.\,b_1\,.\,x_1.\,x_4 \\ & +(A + C\,.\,a_1)\,.\,x_1.\,x_3 \\ & + C.\,a_2\,.\,x_2.\,x_3 + (B + D\,.\,b_2)\,.\,x_2.\,x_4 \\ & +(C\,.\,a_4 + D\,.\,b_3)\,.\,x_3.\,x_4. \end{aligned} \tag{20}$$

The condition of the system's stability at the stationary state fulfils the following inequality:

$$\sup_{x_i \in \Gamma/\lambda,\; t_1 \le t \le t_1 + T} \left\{\frac{dV}{dt}\right\} < (C_1 - C_0)/T. \tag{21}$$

The technical stability is defined by the factor *FT*

$$FT = \frac{(C_1 - C_0)}{T} - \frac{dV}{dt}. \tag{22}$$

If *FT* is maximum (positive) then the system is stable.

Figure 9 shows two curves; $\tau_u > \tau_s$ is shown by the solid curve, and $\tau_s > \tau_u$ is shown by the dotted curve. The curve for $\tau_s > \tau_u$ occupies a higher position in the plot, indicating better stability conditions. Figure 3 indicates that FT is lower for $\tau_u > \tau_s$. The control is efficient and is greater for $\tau_s > \tau_u$.

5. RESULTS AND DISCUSSION

Technical stability within definite time in Bogusz's sense was studied for finite time. Then the asymptotic stability was studied by using the Lyapunov method. We have considered two cases (i) $\tau_s > \tau_u$ and (ii) $\tau_u > \tau_s$, where the delay difference is $\phi = \tau_s - \tau_u < 0$ for $\tau_s > \tau_u$.
It was found that the vibration control is efficient for $\tau_s > \tau_u$ by using the Lyapunov stability criterion. It can be easily found that for the delay difference $\phi = \tau_s - \tau_u < 0.24$, the control is efficient for $\tau_s > \tau_u$, and for $\tau_u > \tau_s$ the control is efficient for $\phi = \tau_s - \tau_u < 0.11$. Therefore the control domain is better for $\tau_s > \tau_u$.

The chaotic and unstable motion of sprung and unsprung mass was found for $\tau_u > \tau_s$ and $\phi = \tau_s - \tau_u > 0.11$. The periodic motion of sprung and unsprung mass was obtained for $\tau_s > \tau_u$. The stability region is higher for higher a value of FT in the case of Bogusz's stability criterion. FT is higher in the case of $\tau_s > \tau_u$ and lower for $\tau_u > \tau_s$. The control is efficient and is greater for $\tau_s > \tau_u$. The above results are verified by using numerical simulations.

6. CONCLUSION

We are interested in the stability of the vehicle for finite time. Vibration control is efficient for $\tau_s > \tau_u$. Both Lyapunov's and Bogusz's stability criterion are useful for defining the stability of the given system. The present quarter-car model is able to capture the major dynamics that occur in the vehicle system.

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