Geometrically Nonlinear Free Axisymmetric Vibrations Analysis of Thin Circular Functionally Graded Plates Using Iterative and Explicit Analytical Solution

Rachid El Kaak and Khalid El Bikri

Mohammed V University in Rabat, Ecole Normale Supérieure de l'Enseignement Technique de Rabat, Département de génie mécanique, LaMIPI, B.P. 6207, Rabat Instituts, 10100 Rabat, Morocco

Rhali Benamar

Mohammed V University in Rabat, Ecole Mohammadia d'Ingénieurs, LERSIM, Av. Ibn Sina, Rabat, Morocco.

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This paper deals with nonlinear free axisymmetric vibrations of functionally graded (FG) thin circular plates whose properties vary in thickness. The inhomogeneity of the plate is characterized by a power law variation of the Young's modulus and mass density of the material along the thickness direction, whereas Poisson's ratio is assumed to be constant. The theoretical model is based on Hamilton's principle and spectral analysis using a basis of admissible Bessel's functions to yield the frequencies of the circular plates under clamped boundary conditions on the basis of the classical plate theory. The large vibration amplitudes problem, reduced to a set of nonlinear algebraic equations, is solved numerically. The nonlinear to linear frequency ratios are presented for various values of the volume fraction index n showing a hardening type nonlinearity. The distribution of the radial bending stresses associated to the nonlinear mode shape is also given for various vibration amplitudes and compared with those predicted by the linear theory. Then, explicit analytical solutions are presented, based on the semi-analytical model previously developed by El Kadiri et al. for beams and rectangular plates. This model allows direct and easy calculation for the first nonlinear axisymmetric mode shape with its associated nonlinear frequencies and nonlinear bending stresses of FG circular plates, which are expected to be very useful in engineering applications and in further analytical developments. An excellent agreement is found with the results obtained by the iterative method.

1. INTRODUCTION

The concept of functionally graded materials (FGMs) was first introduced in 1984 as ultrahigh-temperature resistant materials for aircraft, space vehicles, and other engineering applications.[1] FGMs are nonconventional composite materials that are microscopically inhomogeneous, and their mechanical properties vary continuously in one or more directions. This is achieved by gradually varying the volume fraction of the constituent materials. The continuity of the material properties reduces the influence of the presence of abrupt interfaces and avoids high interfacial stresses. Furthermore, FGMs can be tailored to achieve particular desired properties, and the gradation in properties of materials can optimize the stress distribution.

Many studies have been devoted to FG plate vibrations in the literature. Allahverdizadeh et al.[2] investigated the nonlinear free and forced vibration of thin circular FG plates. Praveen and Reddy[3] conducted the nonlinear transient thermoelastic analysis of FG ceramic-metal plates using the finite element method. Yang and Shen[4] examined the dynamic response of initially stressed FGM rectangular thin plates subjected to impulsive loads. The effects of the volume fraction index, the foundation stiffness, the plate aspect ratio, the shape and duration of the applied impulsive load on the dynamic response of FGM plates have been studied in this work. Also, the vibration characteristics and the transient response of shear-deformable FGM plates made of temperature-dependent materials in thermal environments have been examined by Yang and Shen.[5] The differential quadrature technique, the Galerkin approach, and the modal superposition method have been used to determine the transient response of the plate subjected to lateral dynamic loads. Huang and Shen[6] discussed the nonlinear vibration and dynamic response of FG plates in a thermal environment by using an improved perturbation technique. The results reveal that the temperature field and the volume fraction distribution have significant effects on the nonlinear vibration and the dynamic response of simply supported rectangular plates with no in-plane displacements. Reddy and Cheng[7] studied the harmonic vibration problem of FG plates by means of a three-dimensional asymptotic theory formulated in terms of the transfer matrix. Prakash and Ganapathi[8] analyzed the asymmetric flexural vibration and thermoelastic stability of FG circular plates. They used a finite element method to solve the problem. Efraim and Eisenberger[9] studied the exact vibration analysis of thick annular isotropic and FGM plates of variable thicknesses. The motion equations that they obtained by the first order shear deformation theory have been solved by the exact element method. Dong[10] presented an analysis of three-dimensional free vibration of FG annular plates via Chebyshev-Ritz method. Malekzadeh et al.[11] discussed the in-plane free vibration of FG circular arches with