Optimization of Nonlinear Passive Suspension System to Minimize Road Damage for Heavy Goods Vehicle

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Major contributors to the road damage are Heavy Goods Vehicles (HGV), resulting in high maintenance costs of roads. This high cost makes it necessary to look into the issue seriously for minimizing the road damage. An Automobile Engineer can reduce road damage through the efficient design of a suspension system. The design involves satisfying the two conflicting criteria of riding comfort and vehicle handling with the restriction on the suspension travel. This paper involves designing an automobile suspension system, to improve the performance of the vehicle without a significant change in the cost of the suspension system and minimize road damage. To achieve the aforesaid objective, the use of a nonlinear passive suspension is suitable as compared to a linear passive suspension system. For the analysis, a HGV model of vehicle suspension has been considered. The suspension system considered for investigation comprises of a cubical nonlinear spring and a linear damper. Road damage has been represented by the fourth power of the tire dynamic load. A genetic algorithm has been used to optimize the half truck model to minimize road damage. The solution has been obtained using MATLAB and SIMULINK.

1. INTRODUCTION

The main function of a vehicle suspension system is to support the vehicle body as well as to provide riding comfort to the passenger by rejecting vibrations induced due to irregular road surface. A vehicle suspension should also maintain the adequate vertical load required to provide vehicle stability when the vehicle turns or brakes.[1,2] Vehicle stability and riding comfort have mutually adverse effects, therefore passive suspensions which are widely used in vehicles, could not satisfy the riding comfort as well as driving stability simultaneously.[3] This problem can be solved by using an active suspension system, but the cost of an active suspension is high.[4] The performance of an active suspension is better than that of a passive one; however, it has the disadvantage of higher energy consumption, as fuel prices are increasing at a rapid rate.[5] Thus, due to the above condition, there is a need to make the passive suspension system more and more attractive.[6] This can be done by using a nonlinear passive suspension to meet the suspension trade-off in a better manner than linear passive suspension.[7]

The automobile chassis is mounted on the axles through the suspension system. The suspension system works as the essential interface between the vehicle and the road.[8,9] The design of an automobile vehicle suspension has been the subject of research for a long time. Generally speaking, a decent suspension must provide a comfortable ride and good vehicle handling within a reasonable range of suspension deflection.[10] The problem can be solved by an efficient design of automobile suspension to satisfy the above requirements. The optimum configuration of a suspension system handles differently for a variety of road, speed, and loading conditions. The following requirements are imposed on the suspension of a heavy-duty vehicle suspension:[11]

- To maintain a firm grip between road and wheel so that skid or slip is avoided, while the vehicle is traveling over an uneven road surface.
- To keep suspension travel within limits.
- To keep acceleration of the sprung mass within reasonable limits from the point of view of the driver and the transported load.
- To isolate human and goods from the vibration received due vehicle traveling on uneven road.
- To resist roll of chassis.

All these requirements result in a conflicting design situation. Some requirements may be satisfied by using a soft damper and spring suspension while others may involve the use of a hard damper and spring. Soft suspension spring and damper isolates the vehicle body from the vibrations resulting from uneven surface of the road or due to acceleration/braking.[12] A hard suspension controls both the vehicle body and wheel motion. While designing passive suspension systems, efforts are made to achieve a compromise between these requirements and an optimum ride control performance. Passenger and goods safety are achieved within the available limits of suspension travel.[13]

In the present study, the model utilized is an HGV hence emphasis has been given to minimizing the road damage. As

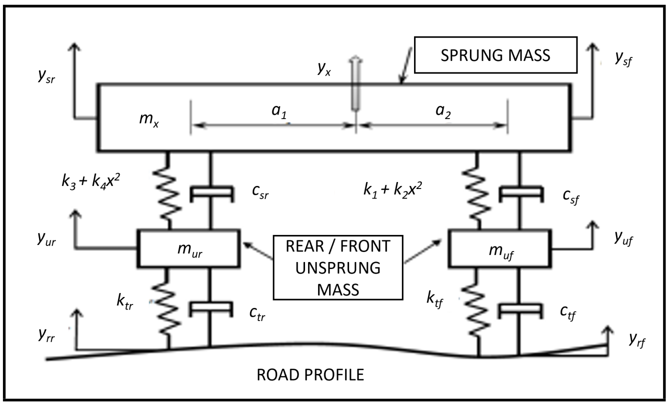

Figure 1. Half truck vehicle model.

discussed in the section of the performance, minimum road damage requires a soft spring, which is also required for ride comfort. Hence the above suspension minimizes road damage while improving ride comfort. Road holding is related to road damage; hence a road-friendly suspension will have good road holding.The hurdle in the design of the suspension system is the interlinking of performance metrics. Apart from the restriction on the travel of the suspension system, improving the ride comfort and vehicle handling is affected adversely.[14] This work hopes to achieve meeting the conflicting requirements of a suspension system improvement through an optimum solution to the problem by using a nonlinear passive suspension system instead of a linear passive suspension system, for a half-car model, resulting in minimum road damage.[15–19] The objective function for optimizing the vehicle suspension which is always noisy, and in such cases, the use of a Genetic Algorithm has a higher probability of reaching a global minimum. The Genetic Algorithm may not give the exact global minimum; however, it converges very close to the global minimum.

2. MATHEMATICAL VEHICLE MODEL

A half HGV model (four degrees of freedom) as shown in Fig. 1 is considered for analysis. The vehicle is assumed to be traveling over a sinusoidal road. The wheels (front/rear) are modeled using a linear spring and linear damper, while the main suspension is modeled using a cubic nonlinear spring and linear damper for both front/rear portion of the vehicle. Also, for optimization, it is assumed that the vehicle speed is constant and the interaction between the tire and road is a single point contact.

The vehicle body is considered rigid. The model has four degrees of freedom with the base excitation. The tire damping is very low as compared to the main suspension damping for both front/rear. The dynamic equations of motion for half HGV model shown in the Fig. 1 are given as follows:

$$m_s\ddot{y}_s + c_{sf}(\dot{y}_{sf} - \dot{y}_{uf}) + c_{sr}(\dot{y}_{sr} - \dot{y}_{ur}) + k_1(y_{sf} - y_{uf}) + k_2(y_{sf} - y_{uf})^3 + k_3(y_{sr} - y_{ur}) + k_4(y_{sr} - y_{ur})^3 = 0; \tag{1}$$

$$I_s\ddot{\theta}_s + a_1[c_{sf}(\dot{y}_{sf} - \dot{y}_{uf}) + k_1(y_{sf} - y_{uf}) + k_2(y_{sf} - y_{uf})^3] - a_2[c_{sr}(\dot{y}_{sr} - \dot{y}_{ur}) + k_3(y_{sr} - y_{ur}) + k_4(y_{sr} - y_{ur})^3] = 0; \tag{2}$$

Table 1. Vehicle Parameter.

Symbol	Parameters	Value
m_s	Sprung Mass	9000 kg
m_{uf}	Front unsprung Mass	450 kg
m_{ur}	Rear unsprung Mass	550 kg
I_s	Moment of inertia	20439 kgm^2
k_{tf}	Front tire stiffness	101115.0 N/m
k_{tr}	Rear tire stiffness	101115.0 N/m
c_{tf}	Front tire damping	700 Ns/m
c_{tr}	Rear tire damping	700 Ns/m
a_1	Front suspension distance from COG	2.340 m
a_2	Rear suspension distance from COG	2.885 m

$$m_{uf}\ddot{y}_{uf} + c_{sf}(\dot{y}_{sf} - \dot{y}_{uf}) - k_1(y_{sf} - y_{uf}) - k_2(y_{sf} - y_{uf})^3 + k_{tf}(y_{uf} - y_{rf}) + c_{tf}(\dot{y}_{uf} - \dot{y}_{rf}) = 0; \tag{3}$$

$$m_{ur}\ddot{y}_{ur} + c_{sr}(\dot{y}_{sr} - \dot{y}_{ur}) - k_3(y_{sr} - y_{ur}) - k_4(y_{sr} - y_{ur})^3 + k_{tr}(y_{ur} - y_{rr}) + c_{tr}(\dot{y}_{ur} - \dot{y}_{rr}) = 0; \tag{4}$$

$$y_s = \frac{(a_2 y_{sf} + a_1 y_{sr})}{l}; \tag{5}$$

$$\theta_s = \frac{(y_{sf} - y_{sr})}{l}; \tag{6}$$

$$l = a_1 + a_2; \tag{7}$$

where m_s is the sprung mass (kg), Is is the moment of inertia in the pitch plane (kgm^2), m_{uf} and m_{ur} are the front and rear unsprung mass respectively (kg), k_1 and k_3 are the front and rear suspension linear spring coefficient respectively (N/m), k_2 and k_4 are the front and rear suspension nonlinear spring coefficient respectively (N/m^3), k_{tf} and k_{tr} are the front and rear tire-stiffness respectively (N/m), c_{sf} and c_{sr} are the front and rear suspension damping coefficient (N-s/m), ctf and ctr are the front and rear tire damping coefficient respectively (N-s/m), y_s is the sprung mass vertical displacement (m), $\theta_s$is the rotary angle of the vehicle body at the centre of gravity sprung (rad), y_{uf} and y_{ur} are the front and rear unsprung mass displacement respectively (m), y_{rf} and y_{rr} are the front and rear road profile displacement respectively (m), a_1 and a_2 are the distance of the front and rear suspension location, with reference to the centre of gravity of the vehicle body (m), y_r is the amplitude of input excitation (m) and t_d is the time lag between front and rear input excitation (s). The half truck model with cubic nonlinear spring and linear damper is modeled in MATLAB/ SIMULINK using blocks available in SIMULINK library. The MATLAB/SIMULINK uses an ode 45 solver which is based on the fourth and fifth-order Runge–Kutta method. The simulation time considered is 10 seconds. The vehicle parameters considered are from[20] and are given in Table 1.

3. ROAD MODEL

Measurement of the road profile has shown that the road displacement can be considered as a stationary Gaussian random process with zero mean. Most of the researchers have used the following expression of the power spectral density (PSD) of road surface irregularities for analysis.[18–21]

$$\Phi(\omega) = (2\alpha\nu\sigma^2/\pi)/(\alpha^2\nu^2 + \omega^2); \tag{8}$$

where: σ_2 = variance of road irregularities (m^2); α = coefficient dependent on the shape of irregularity (m^{-1}); ω = temporal frequency (rad/s); ν = vehicle forward speed (m/s); The parameters α, σ describes the road irregularities. Their values depend on the type of road. The road surface excitations in the time domain are obtained using the first-order shape filter of the form;[17]

$$\dot{y}_r(t) = -\nu\alpha y_r + \zeta; \qquad (9)$$

where $\dot{y}_r$ and y_r, are road velocity and displacement respectively; ζ, is a zero-mean white noise process with a covariance function;

4. PERFORMANCE INDEX (PI)

The four main performance criteria (i.e., goals) of a suspension system for the vehicle are:

- Low levels of acceleration and jerk through the vehicle, especially for driver and cargo (i.e. better ride comfort);
- Limited variation in the type of deflection (i.e. better road holding);
- Limited variation in the suspension travel (i.e., less suspension travel); and,
- Limited variation in the dynamic type force (i.e. minimum road damage).

In the present study, only two performance criteria namely, a better ride comfort and minimum road damage for the suspension system of the vehicle is considered.

4.1. Ride Comfort

As per the ISO 2631-1, the basic vibration evaluation method involves the measurement of R.M.S. acceleration.

$$\text{R.M.S acceleration} = \left[1/T \int_0^T a^2(t)dt\right]^{1/2}; \qquad (10)$$

where $a(t)$ is acceleration (translational or rotational), as a function of time in (m/s^2) or (rad/s^2) respectively and T is the duration of measurement in seconds. An additional method is also obtained from ISO 2631-1, which takes into account the occasional shock and transient vibration. These are running R.M.S., maximum transient vibration value (MTVV) and vibration dose value (VDV).[18]

$$VDV = \left[\int_0^T \{a(t)^4\}dt^{1/4}\right]. \qquad (11)$$

As fourth power is used in the above expression, the VDV is more sensitive to peaks than r.m.s acceleration. The basic evaluation method, i.e., R.M.S. acceleration is applicable when the crest factor is less than or equal to nine. The crest factor is defined as the maximum instantaneous peak value of the acceleration signal to its R.M.S. value.

In the present study, the road is considered a concrete smooth highway road and the crest factor obtained is nearly four, hence R.M.S. sprung mass acceleration is used to measure ride comfort.[18]

$$\text{Ride comfort (PI)} = \left[E\{\ddot{Y}_s^2\}\right]^{1/2}. \qquad (12)$$

4.2. Road Damage

As discussed earlier, road damage is characterized by the fourth power of tire load force. The tire load force consists of both static and dynamic parts. But in the present study, when $t = 0$, i.e. initially all states including y_u are equal to zero and hence there is no static tire deflection. The vehicle suspension design affects dynamic tire load only, hence fourth of tire dynamic load is used to represent road damage for half HGV model.[18]

$$\text{Road damage (PI)} = \left[k_{tf}(y_{uf} - y_{rf})\right]^4 + \rho\left[k_{tr}(y_{ur} - y_{rr})\right]^4; \qquad (13)$$

where ρ is the relative weighting parameter for front and rear tire load damage. In the present study, ρ is taken as 1.

5. OPTIMIZATION PROBLEM

The purpose of suspension optimization is different for different applications. Depending upon the purpose, the objective function is selected. For an HGV, the road damage and for passenger cars, the ride comfort, are a concern. Vehicle suspension optimization is a stochastic optimization problem due to random road input. Road input can be considered as a stationary Gaussian random process. To reduce computational efforts, shape filters can be used, so that while computing the response, one has to deal with white noise excitations only. The vehicle model chosen should represent the dynamics of the actual vehicle as well as making it simple for analysis. The objective function for optimizing the vehicle suspension which is always noisy, in such cases, the Genetic Algorithm has a higher probability of reaching a global minimum. The Genetic Algorithm may not give the exact global minimum; however, it converges very close to the global minimum.

As the vehicle has to satisfy different performance measures like ride comfort, road damage etc. generally the composite performance index with different weighting for each performance measure is used. The weightings depend on the intent of the design, for example in case of a HGV the highest weight is given to road damage measure, whereas for a luxury car highest weight is for ride comfort. It has been claimed in,[24] for the case of HGV only road damage performance can be used to design the suspension. In the present study, as the model utilized is an HGV hence emphasis has been given to minimum road damage. As discussed in the section of the performance, minimum road damage requires a soft spring, which is also required for ride comfort. Hence the above suspension minimizes road damage while improving ride comfort. Road holding is related to road damage; hence a road-friendly suspension will have a good road holding. Thus, an optimization problem can be presented as follows:

Minimize Road damage PI (R.D.))

$$= \left[k_{tf}(y_{uf} - y_{rf})\right]^4 + \rho\left[k_{tr}(y_{ur} - y_{rr})\right]^4;$$

Constraints

$$Max(y_{sf} - yuf)_{dynamic} \le 0.2\ m;$$
$$Max(y_{sf} - yuf)_{static} \le s;$$
$$Max(y_{sf} - yuf)_{dynamic} \le 0.2\ m;$$
$$Max(y_{sf} - yuf)_{static} \le s;$$

$$100\ kN/m \leq k_sf, k_sr \leq 1500kN/m;$$
$$5\ kNs/m \leq c_sf, c_sr \leq 60\ kNs/m;$$
$$0\ kN/m \leq k_1, k_3 \leq 500\ kN/m;$$
$$0\ kN/m \leq k_2, k_4 \leq 10^5\ kN/m. \qquad (14)$$

'S' is the allowable static deflection in meters. k_{sf}, and k_{sr} correspond to spring stiffness for linear suspension and c_{sf}, c_{sr}, k_1, k_2, k_3, k_4 correspond to spring stiffness for nonlinear suspension.

The vehicle parameter ranges are taken from.[22–24] The bounds on stiffness and damper are decided such that the minimum and maximum static deflection constraint is also satisfied. The minimum value of stiffness for nonlinear spring is taken as zero so that in any case if any one of them becomes zero during optimization, the static deflection constraint can still be satisfied.

6. SOLUTION PROCEDURE

For optimization, the Genetic Algorithm (GA) tool of MATLAB is executed using Optimtool 'GA' command. The steps involved in the optimization are as follows:

- Generation of design variable vector i.e. k_1, k_2, k_3, k_4, k_{sf}, k_{sr}, c_{sf} and c_{sr} are generated randomly using GA.
- For all design variables, the objective function value and constraint data are evaluated. Both the objective function and constraints are evaluated using response from the codes written in MATLAB using ode 45 solvers for the sinusoidal road input for 10 sec.
- GA operators produce the new generation.
- Step 2 and step 3 are repeated until the objective function converges to the minimum value (within the small-time limit), or a specified number of generations are finished.
- The above steps are repeated for another set of design variable values of first-generation.

For the optimization of linear and nonlinear suspension after exhaustive runs, the selected value of GA parameters is listed in Table 2.

Thus, the overall scheme of the problem is formulated for optimization.

Table 2. GA Parameter.

Parameters of GA	Linear Suspension	Nonlinear suspension
Crossover frequency	0.8	0.9
Population size	20	20
Elite count	2	2
Termination tolerance	10^{-6}	10^{-6}
Stall stop time	1000	1000
Generation	100	100

Table 3. Parameter used for linear half HGV model25.

Parameter	Value	Parameter	Value
k_1	66824.4 N/m	c_sf	1190 Ns/m
k_2	0 N/m^3	c_sr	1000 Ns/m
k_3	66824.4 N/m	c_tf	0 Ns/m
k_4	0 N/m^3	c_tf	0 Ns/m
k_{tf}	101115.0 N/m	y_r	0.0198 m
k_{tr}	101115.0 N/m	L	2.987 m
w	41.20 rad/s	V	50 km/h

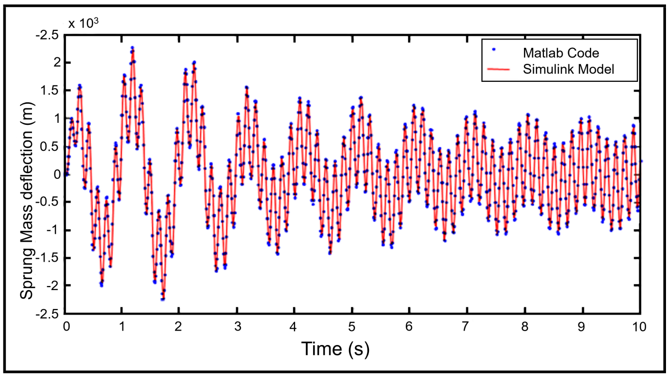

Figure 2. Validation of linear half HGV model.

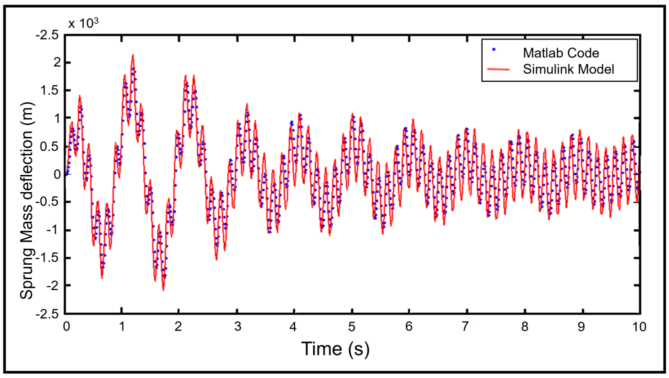

Figure 3. Validation of nonlinear half HGV model.

7. RESULT AND DISCUSSION

7.1. Validation of Half HGV Model

The vehicle model as shown in Fig. 1. is to be validated before proceeding to the results. In the dynamic equation which is obtained for the half truck model i.e. equations (1), (2), (3) and (4), if k_2 and k_4 is kept as zero, thus these equations reduce to the linear half truck model. The inputs are given in sinusoidal form, $y_{rf} = y_r \sin(wt)$ for the front and $y_{rr} = y_r \sin w(t - t_d)$ for rear wheel.

Where: y_r = Amplitude of sinusoidal road input. $t_d = l/v$ is the time lag between front rear input.

The response is obtained using codes written in MATLAB using ode 45 and SIMULINK model which are given in Appendix A. Response from both the models for the sprung mass deflection is compared for validation. The values selected for the stiffness of spring and damping coefficient for the damper are given in Tab. 3.

The sprung mass displacements are given in Fig. 2. There was a perfect match between MATLAB code using ode 45 and SIMULINK solution. When k_2 and k_4 are non-zero the model becomes a nonlinear half HGV model. For validation of the nonlinear half HGV model, the values selected for parameters are given in Table 4. The response of the nonlinear suspension vehicle model was prepared in SIMULINK and was compared with the MATLAB code using ode 45. The response obtained from both MATLAB and SIMULINK were compared in terms of sprung mass displacement as shown in Fig. 3. There was a perfect match between the response obtained from MATLAB codes and SIMULINK solution for the sprung mass deflection.

Table 4. Parameter used for nonlinear half HGV model25.

Parameter	Value	Parameter	Value
k_1	66824.4 N/m	$c_s f$	1190 Ns/m
k_2	3098210 N/m^3	$c_s r$	1000 Ns/m
k_3	66824.4 N/m	$c_t f$	700 Ns/m
k_4	2098120 N/m^3	$c_t f$	700 Ns/m
k_{tf}	101115.0 N/m	y_r	0.0198 m
k_{tr}	101115.0 N/m	L	2.987 m
w	41.20 rad/s	V	50 km/h

Table 5. Optimized nonlinear suspension parameters for road damage.

Static Deflections Constraint 's'(m)	Optimized value of nonlinear suspension						Road PI
	k_1	k_2 /10^4	k_3	k_4 /10^4	c_{sf}	c_{sr}	(N^4/10^{1}3)
0.05	361.08	5.25	198.56	2.70	14.19	5.02	2.17
0.07	196.59	1.42	400.67	9.39	10.88	24.49	2.72
0.09	323.47	8.37	172.13	6.42	17.64	11.03	2.85
0.11	243.71	2.76	376.13	8.40	5.65	42.90	2.29
0.13	131.79	9.85	481.05	2.75	6.94	32.00	2.57
0.15	304.49	5.85	301.49	8.99	5.00	42.35	2.34
0.17	222.58	7.52	146.88	3.31	6.16	37.90	2.52
0.19	390.98	5.04	193.52	2.87	7.02	14.09	2.35
0.21	291.28	2.19	245.59	9.77	7.83	15.14	2.29
0.23	359.59	6.54	451.38	6.41	8.66	7.03	1.76

Table 6. Optimized linear suspension parameters for road damage.

Static Deflections Constraint 's'(m)	Optimized value of nonlinear suspension				Road damage PI
	k_{sf} (kN/m)	k_{sr} (kN/m)	c_{sf} (kNs/m)	c_{sr} (kNs/m)	(N^4/10^{1}3)
0.05	2.02	270.53	5.00	5.00	4.91
0.07	2.02	284.49	5.00	31.09	6.68
0.09	2.00	365.28	5.00	57.91	6.07
0.11	2.00	432.48	5.00	5.00	4.97
0.13	100.00	450.98	5.00	5.00	4.23
0.15	2.01	412.81	5.00	5.00	4.94
0.17	2.02	216.92	5.00	5.00	4.92
0.19	2.00	336.19	5.51	5.00	5.02
0.21	58.33	417.243	5.00	5.00	4.00
0.23	2.00	83.421	5.00	41.09	6.09

7.2. Optimization of Road Damage For Sinusoidal Road Input

As previously discussed, optimization is done using the road damage performance index (PI) alone as an objective function or fitness function for the genetic algorithm. The road damage PI is given in equation (14). Optimization of a half truck model suspension system for minimizing road damage was carried out for various static deflection constraints (s) as mentioned in the solution procedure. The maximum dynamic deflection for the front and rear wheel was fixed at 0.2 m. Table 5 and Table 6 illustrate Optimized nonlinear and linear suspension parameters respectively for road damage of the vehicle model while optimizing the stiffness of spring and damping force of damper for both front and rear parts of the vehicle when the road profile is sinusoidal.

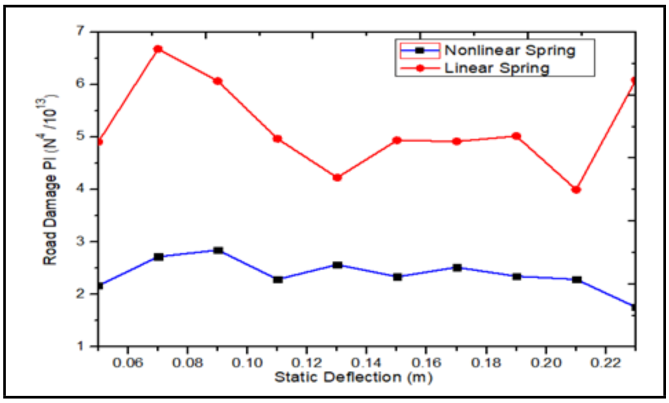

Figure 4. Comparison of road damage for linear and nonlinear suspension.

Table 7. Comparison with initial set up value.

Optimized passive suspension system	Comparison with initial set up value	
	Road damage PI	Ride comfort
Optimized value for nonlinear suspension(s=0.23) k_{1opt} = 359.59 kN/m k_{2opt} = 6.54×104 kN/m^3 k_{3opt} = 451.38 kN/m k_{4opt} = 6.41×104 kN/m^3 c_{sfopt} = 8.66 kNs/m c_{sropt} = 7.03 kNs/m	82.98% Less	1.96% Better
Optimized value for linear suspension(s=21) k_{sfopt} = 58.33 kN/m k_{sropt} = 417.24 kN/m^3 c_{sfopt} = 5.00 kNs/m c_{sropt} = 5.00 kNs/m	77.32% Less	15.67% Better

The road damage performances of both suspensions, linear and nonlinear, are compared in Fig. 4. It can be seen from Fig. 4 that a nonlinear suspension results in lesser road damage as compared to linear suspension for all value of static deflection. From Fig. 4 it is clear that minimum road damage for linear suspension was obtained at the static deflection of 0.21 m and minimum road damage for nonlinear suspension was obtained at the static deflection of 0.23 m. The optimized values for both linear and nonlinear suspensions are given in Table 7.

The initial set up value of road damage and ride comfort are compared in Table 7. The optimized value of spring stiffness and damping coefficients are compared in Table 8. Ride comfort was calculated for the optimized value of spring stiffness and damping coefficients, and was obtained by minimizing road damage function alone for both linear and nonlinear suspension and compared with the initial set up value. From Table 7 it is visible that the road damage PI was reduced by 82.98% by using the optimized value of nonlinear suspension when compared with the initial set up value. At the same time, it was found that ride comfort is 1.96% better. For the linear suspension road damage, PI was reduced by 77.32%, and ride comfort was a 15.67% improvement. As seen in Table 8., it was concluded that the road damage PI was reduced by 69.87%

Table 8. Comparison of linear with nonlinear suspension.

Optimized value of passive suspension		Comparison with optimized value of linear and nonlinear spring	
Optimized value for nonlinear suspension (s=0.23)	Optimized value for linear suspension (s=21)	Road damage	Ride comfort
$k1opt$ = 359.59 kN/m $k2opt$ = 6.54×104 kN/m^3 $k3opt$ = 451.38kN/m $k4opt$ = 6.41×104 kN/m^3 $csfopt$ = 8.66 kNs/m $csropt$ = 7.03 kNs/m	$ksfopt$ = 58.33 kN/m $ksropt$ = 417.24 kN/m $csfopt$ = 5.00 kNs/m $csropt$ = 5.00 kNs/m	69.87% Less	3.59% Less

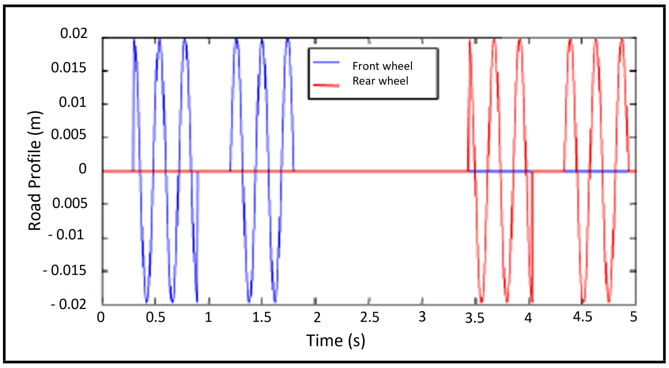

Figure 5. New Road profile.

when the optimized value of nonlinear suspension parameters was used as compared to when the optimized value of linear suspension parameters was used. At the same time, it was found that ride comfort was a 3.59% improvement.

7.3. Optimization For Different Road Profile

It was interesting to study the performance of optimized suspension on different road profiles. Because of the above statement, consider the road profile as given in Fig. 5. The vehicle is considered to be moving with a constant speed of 50 km/hr. The road displacement with time for front and rear suspensions are shown in Fig. 5. The red profile is for the rear wheel while blue is for the front wheel.

7.4. Validation of the Vehicle Model

The vehicle model presented in Fig. 5 was again validated for this road profile. The parameters were the same as taken for the validation of the sinusoidal road profile. The response was obtained by codes written in MATLAB using ode 45 and SIMULINK model and both were compared for the sprung mass deflection for both linear half HGV model and nonlinear half HGV model. Simulation time was 5 sec for both. The sprung mass deflection for a linear half truck model is given in Fig. 6 and the sprung mass deflection for a nonlinear half truck model is given in Fig. 7. As seen from the above results, during the analysis of both linear and nonlinear suspension, it was evident that there is was almost a perfect match between the

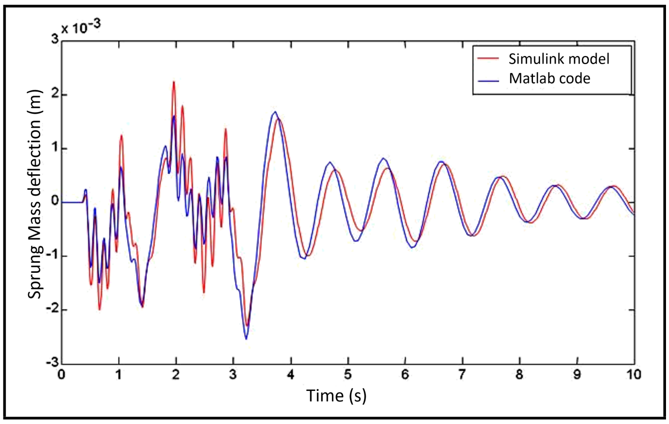

Figure 6. Validation of linear half HGV model.

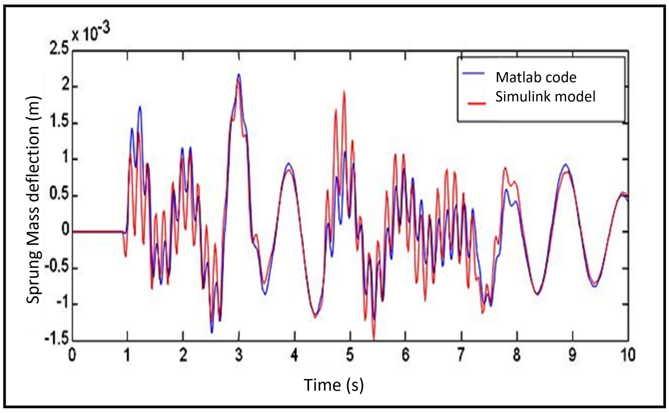

Figure 7. Validation of nonlinear half HGV model.

Table 9. Optimized nonlinear suspension parameters for road damage.

Static Deflections Constraint 's'(m)	k_1	k_2 /10^4	k_3	k_4 /10^4	c_{sf}	c_{sr}	Road PI (N^4/10^{1}3)
0.05	45.57	1.24	70.65	2.63	21.99	56.62	1.97
0.07	203.00	2.52	216.43	2.55	13.47	46.12	1.97
0.09	164.25	2.07	178.99	3.96	10.77	53.64	1.93
0.11	467.72	4.29	213.89	2.12	29.77	56.59	1.94
0.13	28.00	4.38	454.40	1.89	8.72	52.42	1.93
0.15	395.53	4.68	374.43	2.44	16.97	49.12	1.96
0.17	498.52	3.70	461.43	2.21	7.15	55.97	1.88
0.19	484.94	2.56	477.13	1.77	6.41	54.54	1.90
0.21	302.14	3.39	302.23	1.56	11.09	59.08	1.89
0.23	326.19	3.54	309.16	3.20	8.88	54.55	1.91

results derived from MATLAB using ode 45 and SIMULINK solution.

For the optimization of this road model, again Genetic Algorithm has been used and the Genetic Algorithm parameter taken is the same as that taken for sinusoidal road input previously. The constraints are also the same as taken earlier for sinusoidal road input. The optimized value of road damage and optimized value of suspension for nonlinear and linear suspension systems are given in Table 9 and Table 10. respectively.

The road damage performances of both suspensions linear and nonlinear are compared in Fig. 8. It can be seen from the Fig. 8. that the nonlinear suspension results has lesser road

Table 10. Optimized linear suspension parameters for road damage.

Static Deflections Constraint 's'(m)	Optimized value of nonlinear suspension k_{sf} (kN/m)	k_{sr} (kN/m)	c_{sf} (kNs/m)	c_{sr} (kNs/m)	Road damage PI $(N^4/10^13)$
0.05	458.76	265.20	5.25	39.21	2.18
0.07	29.20	200.34	6.74	49.38	2.12
0.09	23.28	449.64	8.87	51.80	2.10
0.11	124.87	20.24	5.00	45.07	2.05
0.13	108.63	363.44	5.01	52.32	2.08
0.15	56.07	351.04	6.24	57.02	2.02
0.17	173.46	392.78	10.77	41.47	2.23
0.19	246.99	275.33	9.94	56.32	2.13
0.21	168.95	54.45	11.89	55.37	2.19
0.23	88.04	483.67	11.28	51.95	2.16

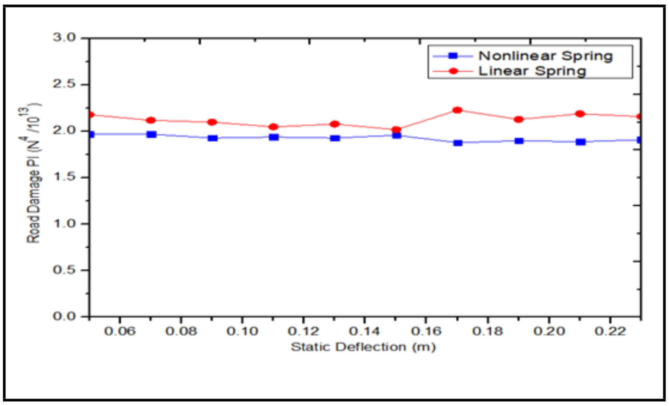

Figure 8. Comparison of road damage for linear and nonlinear suspension.

damage as compared to linear suspension for all values of static deflection.

Ride comfort is calculated for the optimized value of the parameters and is obtained by minimizing the road damage function for both linear and nonlinear suspension and compared with the initial set up value. From Table 11, it is clear that by using the optimized values of non-linear suspension, the road damage PI has been reduced by 18.96% on comparing with the initial set up values. At the same time, it was found that ride comfort is 28.07% better. For linear suspension road damage, PI is reduced by 13.67% and ride comfort is 16.11% better.

8. CONCLUSION

The following are the major conclusions from the present study:

- It was found that while using the optimized value of parameters that are obtained by optimizing road damage, the road damage can be minimized.
- The results were obtained by solving linear half HGV and nonlinear HGV by SIMULINK and MATLAB code that resulted in nearly a perfect match.
- During optimization using genetic algorithm, the constraint function used in MATLAB gives better results than the constraint function used in SIMULINK.

Table 11. Comparison with initial set up value.

Optimized passive suspension system	Comparison with initial set up value: Road damage PI	Ride comfort
Optimized value for nonlinear suspension(s=0.) k_{1opt} = 498.52 kN/m k_{2opt} = 3.70×1e08 kN/m^3 k_{3opt} = 461.43 kN/m k_{4opt} = 2.21×1e08 kN/m^3 c_{sfopt} = 7.15 kNs/m c_{sropt} = 55.97 kNs/m	18.96% Less	28.07% Better
Optimized value for linear suspension(s=0.15) k_{sfopt} = 56.07 kN/m k_{sropt} = 351.04 kN/m^3 c_{sfopt} = 6.04 kNs/m c_{sropt} = 57.02 kNs/m	13.67% Less	16.11% Better

- For the road hump profile, it was found that the use of the linear half HGV resulted in lesser road damage as compared to continuous sinusoidal road input. For nonlinear half HGV road damage, it was almost the same for both road profiles.

Hence it can be concluded from the study that as compared to linear suspension, cubic nonlinear suspension gives better ride comfort and less road damage.

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APPENDIX A

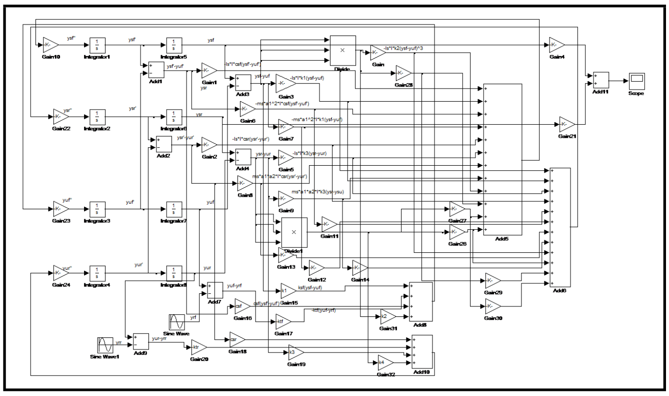

Figure A1. Comparison of road damage for linear and nonlinear suspension.