Free Vibration and Frequency Calculation in Tall Buildings with Stepped Tube-in-Tube Systems

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(Received 23 November 2020; accepted 20 April 2021)

This study investigates the free vibration and frequency analysis of tall buildings with stepped cross-sections for stepped tube-in-tube structures. Dynamic equations were presented in accordance with the stepped structural system for free vibration and the differential equations were solved according to the application of boundary conditions and flexural and shear stiffness and stepped cross-section of different steps. The answer was converted to an 8×8 matrix and was obtained using the determinants of matrix and mathematical calculations of frequency. Nine mathematical computational models were designed for stepped tube-in-tube structures for a 50-story tube-in-tube building by increasing the height of the outer tube by 15 meters. Finally, the heights of the inner and outer tubes were equalized by increasing the height of the outer tube and then compared with the results of free vibration in the articles with the tube-in-tube structural system with equal height and characteristics. Using this method, the frequency of structures was calculated correctly and the obtained results from finite element analysis showed that this mathematical method accurately calculated the frequency. It was found that this method was accurate enough and the obtained computational error was very small. The results showed that the frequency decreases by adding to the outer tube and increasing its height.

1. INTRODUCTION

The tube-in-tube structural system is considered to be one of the systems of tall structures, which has led to many improvements in tall buildings. The use of stepped tube-in-tube architecture in tall structures has several advantages such as providing the possibility of lighting the adjacent buildings and not blocking the view in urban spaces. In free vibration, one of the important parameters in stepped narrowing constructions is the calculation of the natural vibrational frequency of the structure (ω). In this study, new relationships, formulas and mathematical methods for calculation are presented and compared with software calculations and other relevant studies.[1–3] The stepped tube-in-tube system is a type of framed tube consisting of an interior tube and an exterior tube. The exterior and interior tubes work together to withstand gravity and lateral loads in buildings. The interior tube differs from the exterior tube in height. The use of this system makes it possible to construct buildings with higher heights. It can be used in structures higher than 100 floors.[3,4]

The calculation of vibrational frequency and the analysis of free vibration play an important role in designing tall buildings, especially in the case of the first frequency. Free vibration of tube-in-tube structures has been considered by different researchers. Therefore, it is important to study the methods of calculating natural frequencies for stepped tube-in-tube systems in tall buildings. In free vibration analysis, tall structures can be considered as beams with a variable cross-section.[5,6] Wang studied the free flexural vibrations of a beam that had variable stiffness but with a uniform mass.[7] Lee et al., examined the free flexural vibrations of tall buildings with varying stiffness and mass. Many of these studies have led to simplifications in the calculation process, in a way that the differential equation of the structure can be solved. Lee et al., suggested an analytical method for the dynamic response of tubular structures based on D'Alembert's principle.[6,8] The Rayleigh-Ritz vibrational frequency method was presented by Maleknejad and Rahgozar.[9–13] They proposed an analytical method to determine the vibrational frequencies of Bernoulli and Timoshenko beams by converting the governing differential equation to attenuated integral equations.[14,15] An approximation method for free vibration analysis of tall tube-in-tube structures was developed using the series solution method.[16] In addition, free vibration analysis was solved using the DQM method, which is the governing differential equation for free vibration with shear walls by Bozdogan.[17,18] A new and simple solution was presented for determining the natural frequencies of wall-tube and tube-in-tube structures. Mohammad Nejad and Haji Kazemi in 2018 modeled a tall building by a beam with variable stiffness and mass along the height. Thus, the governing partial differential equation with variable coefficients was solved to calculate natural frequencies.[19] Frequency analysis of tall buildings was performed considering the effect of axial force.[20]

In this paper, an approximation method was developed for calculating the vibrational frequency of tall buildings with stepped tube-in-tube systems. The structure can be equivalent to a beam with a box cross section along the height. The differential equations for the vibration of the stepped tube system can be obtained using the vibrational principles of the continuous system. The structures with stepped tube-in-tube systems are replaced by an equivalent cantilever beam, which has a bending and shear stiffness and variable mass as a step along the height. The answer to the vibrational equation governing the stepped tube system was obtained using mathematical methods. Then, natural frequencies can be determined in the system by applying boundary conditions at the site of ground connection (abutment) where the stepped cross-section changes and at the end of the system. A numerical sample of free vibrational analysis and evaluation of natural frequencies were performed for a number of tall buildings with stepped tube-in-tube systems. Further, various comparisons with the results in published papers and a finite element analysis were performed to compare the validity of the method. In order to discuss the effects of this method, natural frequencies for uniform and non-uniform cross-sections of the stepped tube-in-tube system of tall buildings were calculated with flexural stiffness, shear stiffness and variable stepped mass along the height. The frequency diagram was plotted in steps according to height.

2. ESTIMATING THE BEHAVIOUR OF STEPPED TUBULAR STRUCTURES INCLUDING CANTILEVER BEAMS AND CALCULATING THE FREE VIBRATION EQUATION AND THE ANSWER OF THE DIFFERENTIAL EQUATION

Analyses of a stepped high-rise structure with a precise view of all the behavioral problems of members and building materials is practically impossible even if the properties of the materials and dimensions of the elements are all known. Additionally, the application of simplistic assumptions to reduce the volume of the problem is inevitable. In this regard, the most common types of hypotheses are introduced as follows: 1) the distance between columns and beams is uniform along the height of the building; 2) the cross-sectional dimensions of all beams and columns are the same at the height of the building; 3) the structural materials are linearly elastic, isotropic, homogeneous and obey Hooke's law; 4) the structural system in terms of plan and height in all these models is assumed to be symmetrical and rotational inertia is ignored; 5) rigid floors in the floorboards of tall buildings do not deform in their panels and do not move perpendicular to them; and, 6) small and ineffective deformations, axial and shear deformations, torsional deformations, bending deformations, shear and torsional deformations of roofs and axial deformations of building columns have been ignored.

In this part, structures with a tubular system (stepped tube-in-tube is shown in Fig. 1) are replaced with an equivalent cantilever beam as shown in Fig. 2, which has a variable bending stiffness, shear stiffness and mass along the height. The elastic parameters of equivalent cantilever beam, which include the modulus of elasticity and the shear modulus of the beam, are obtained according to Eqs. (1) and (2):[19]

$$E = E_m; \tag{1}$$

$$G = \frac{\frac{H}{st}}{\frac{\Delta_b}{Q} + \frac{\Delta_s}{Q}}; \tag{2}$$

where E_m represents the modulus of elasticity of consumed material, h is the height of column, H is the height of the whole structure, s shows the horizontal distance of the axes of the columns from each other, Q is considered as the load on the structure, t indicates the thickness of the walls of equivalent cantilever beam, Δ_s and Δ_b is shear and bending deformation of the structure due to the force Q, respectively. The Δ_b/Q and Δ_s/Q parameters are obtained from Eqs. (3) and (4):

$$\frac{\Delta_b}{Q} = \frac{(h - H_b)^3}{12E_m I_c} + \left(\frac{h}{d}\right)^2 \frac{(d - H_c)^2}{12E_m I_b}; \tag{3}$$

$$\frac{\Delta_s}{Q} = \frac{(h - H_b)}{G_m A_{sc}} + \left(\frac{h}{d}\right)^2 \frac{(d - H_c)}{G_m A_{sb}}. \tag{4}$$

The t parameter is obtained based on Eq. (5)

$$t = \frac{A_C}{S}. \tag{5}$$

In Eqs. (3) to (5), h is the height of the column, H is the height of the whole structure, H_o is the height of the steps of the structure, s is the horizontal distance from each other, d is the horizontal distance axes of columns from each other, H_b is the height of beam, H_c is the height of column, I_b is the moment of inertia of beam, I_c is the moment of inertia of column, A_{sc} is the cross-section of column, A_{sb} is the cross-section of beam, Q is the wide lateral load on the structure, t indicates the thickness of the walls of equivalent cantilever beam, G_m is the shear modulus of consumables, Δ_s and Δ_b are shear and bending deformations of the structure due to the force Q, respectively.

Equations governing the vibrational frequency of the system are given in Eq. (6):[12, 24, 25]

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right) - \frac{\partial}{\partial x}\left(GA(x)\frac{\partial w(x,t)}{\partial x}\right) + m(x)\frac{\partial^2 w(x,t)}{\partial t^2} = 0. \tag{6}$$

The x-axis is along the length of the building height. The boundary conditions are applied for the equations as follows. In the support (abutment), the displacement value is zero:

$$x = 0; \qquad w_1(x,t) = 0. \tag{7}$$

The amount of rotation on the support is zero:

$$x = 0; \qquad \frac{\partial}{\partial x} w_1(x,t) = \frac{\partial}{\partial x} w_2(x,t) = 0. \tag{8}$$

The amounts of displacement at the intersections are equal:

$$x = H - H_0; \qquad w_1(x,t) = w_2(x,t). \tag{9}$$

The amounts of rotations at the point of intersections are equal:

$$x = H - H_0; \quad \frac{\partial}{\partial x} w_1(x,t) = \frac{\partial}{\partial x} w_2(x,t) \to w_1'(x,t) = w_2'(x,t). \tag{10}$$

The amounts of shear forces at the intersections are equal:

$$x = H - H_0; \quad \frac{\partial}{\partial x}\left(EI_1(x)\frac{\partial^2 w_1(x,t)}{\partial x^2}\right) - GA_1(x)\frac{\partial}{\partial x}w_1(x,t) = \frac{\partial}{\partial x}\left(EI_2(x)\frac{\partial^2 w_2(x,t)}{\partial x^2}\right) - GA_2(x)\frac{\partial}{\partial x}w_2(x,t). \tag{11}$$

The amounts of bending anchors at the intersection area are equal:

$$x = H - H_0; \quad EI_1(x)\frac{\partial^2 w_1(x,t)}{\partial x^2} = EI_2(x)\frac{\partial^2 w_2(x,t)}{\partial x^2} \to EI_1(x)w_1''(x,t) = EI_2(x)w_2''(x,t). \tag{12}$$

The amount of shear force at the end of the beam is zero

$$\begin{gathered} x = H; \\ \frac{\partial}{\partial x}\left(EI_2(x)\frac{\partial^2 w_2(x,t)}{\partial x^2}\right) - GA_2(x)\frac{\partial}{\partial x}w_2(x,t) = 0 \rightarrow \\ (EI_2(x)w_2'')' - GA_2(x)w_2' = 0. \end{gathered} \tag{13}$$

The amount of bending anchor at the end of the beam is zero

$$x = H; \quad EI_2(x)\frac{\partial^2 w_2(x,t)}{\partial x^2} = EI_2(x)w_2''(x,t) = 0. \tag{14}$$

Assuming the value of the following answer for differential Eq. (6):[12,24,25]

$$w(x,t) = W(x)e^{i\omega t}; \tag{15}$$

$$\begin{gathered} w_t' = \frac{\partial w}{\partial t} = i\omega W(x)e^{i\omega t}; \\ \frac{\partial^2 w}{\partial t^2} = i^2\omega^2 W(x)e^{i\omega t} = -\omega^2 W(x)e^{i\omega t}; \end{gathered} \tag{16}$$

$$\varepsilon = \frac{x}{H}; \quad 0 \le x \le H; \rightarrow 0 \le \frac{x}{H} \le 1; \rightarrow 0 \le \varepsilon \le 1. \tag{17}$$

The following parameters are substituted in Eq. (6)

$$\begin{gathered} \frac{d}{dx}w(x) = w'(x); \rightarrow \frac{dw(\varepsilon)}{d\varepsilon} \times \frac{d\varepsilon}{dx} = \frac{1}{H}w'(\varepsilon); \\ \frac{d^n}{dx^n}w(x) = \frac{1}{H^n}\frac{d^n}{dx^n}w(\varepsilon); \rightarrow \frac{d^n}{dx^n}w(\varepsilon) = H^n\frac{d^n}{dx^n}w(x). \end{gathered} \tag{18}$$

2.1. Vibrational Equation of the System

To analyze the free vibration of a building with a stepped tubular structural system and assuming the harmonic vibration of Eq. (15) and using Eqs. (15) to (18) and placing it in Eq. (6) and applying the boundary conditions in Eqs. (7) to (14), Eqs. (19) and (20) were obtained as follows, where $w(x)$ is the mode shape function, ω is the natural frequency, $EI(x)$ is the sum flexural rigidity of inner and outer tube steps, $m(x)$ is the mass per height unit, and $GA(x)$ is the sum of shear rigidities for inner and outer tube steps.

$$\begin{gathered} EI(x)w''''(x) - GA(x)w''(x) - m(x)\omega^2 w(x) = 0; \\ EI_o w''''(x) - GA_o w''(x) - m_o\omega^2 w(x) = 0; \\ w''''(x) - \frac{GA_o}{EI_o}w''(x) - \frac{m_o}{EI_o}\omega^2 w(x) = 0; \end{gathered} \tag{19}$$

$$\begin{gathered} a_1 = (H - H_0)\sqrt{\frac{GA_1}{EI_1}}; \qquad b_1 = (H - H_0)^2\sqrt{\frac{m_{o1}}{EI_1}}; \\ b_2 = H_0^2\sqrt{\frac{m_{o2}}{EI_2}}; \qquad a_2 = H_0\sqrt{\frac{GA_2}{EI_2}}; \\ w_1''''(\varepsilon) - a_1^2 w_1''(\varepsilon) - b_1^2\omega^2 w_1(\varepsilon) = 0; \\ w_2''''(\varepsilon) - a_2^2 w_2''(\varepsilon) - b_2^2\omega^2 w_2(\varepsilon) = 0. \end{gathered} \tag{20}$$

After solving differential Eq. (19) and (20), numbers are obtained by Eqs. (21) and (22) as follows:

$$\begin{gathered} w_1(\varepsilon) = c_1 e^{\frac{-\sqrt{2a_1^2 - 2\sqrt{a_1^4 + 4b_1^2\omega^2}}}{2}\varepsilon} + c_2 e^{\frac{\sqrt{2a_1^2 - 2\sqrt{a_1^4 + 4b_1^2\omega^2}}}{2}\varepsilon} + \\ c_3 e^{\frac{-\sqrt{2a_1^2 + 2\sqrt{a_1^4 + 4b_1^2\omega^2}}}{2}\varepsilon} + c_4 e^{\frac{\sqrt{2a_1^2 + 2\sqrt{a_1^4 + 4b_1^2\omega^2}}}{2}\varepsilon}; \\ 0 \le \varepsilon \le 1 - \frac{H_0}{H}; \qquad 0 \le \varepsilon \le 1 - k; \end{gathered} \tag{21}$$

$$\begin{gathered} w_2(\varepsilon) = c_5 e^{\frac{-\sqrt{2a_2^2 - 2\sqrt{a_2^4 + 4b_2^2\omega^2}}}{2}\varepsilon} + c_6 e^{\frac{\sqrt{2a_2^2 - 2\sqrt{a_2^4 + 4b_3^2\omega^2}}}{2}\varepsilon} + \\ c_7 e^{\frac{-\sqrt{2a_2^2 + 2\sqrt{a_2^4 + 4b_2^2\omega^2}}}{2}\varepsilon} + c_8 e^{\frac{\sqrt{2a_2^2 + 2\sqrt{a_2^4 + 4b_2^2\omega^2}}}{2}\varepsilon}; \\ 0 \le \varepsilon \le 1 - \frac{H_0}{H}; \qquad 0 \le \varepsilon \le 1 - k. \end{gathered} \tag{22}$$

By substituting the parameters of the A, B, C, and D coefficients, the answers to the differential Eqs. (21) and (22) are obtained as follows:

$$\begin{gathered} w_1(\varepsilon) = c_1 e^{-A\varepsilon} + c_2 e^{A\varepsilon} + c_3 e^{-B\varepsilon} + c_4 e^{B\varepsilon}; \\ 0 \le \varepsilon \le 1 - \frac{H_0}{H}; \qquad 0 \le \varepsilon \le 1 - k; \end{gathered} \tag{23}$$

$$\begin{gathered} w_2(\varepsilon) = c_5 e^{-C\varepsilon} + c_6 e^{C\varepsilon} + c_7 e^{-D\varepsilon} + c_8 e^{D\varepsilon}; \\ 1 - \frac{H_0}{H} \le \varepsilon \le 1; \qquad 1 - k \le \varepsilon \le 1. \end{gathered} \tag{24}$$

In general, we have the following equation:

$$w(\varepsilon) = w_1(\varepsilon) + w_2(\varepsilon). \tag{25}$$

After applying the boundary conditions in Eqs. (21) to (25), a matrix of 8×8 is obtained, which represents a row which is mentioned in matrix Eq. (26), where the coefficients in matrix Eq. (26) are as follows:

$$\begin{aligned} a_{61} &= -A^3 e^{-A(1-K)} + a_1^2 A e^{-A(1-K)}; \\ a_{62} &= A^3 e^{A(1-K)} - a_1^2 A e^{A(1-K)}; \\ a_{63} &= -B^3 e^{-B(1-K)} + a_1^2 B e^{-B(1-K)}; \\ a_{64} &= B^3 e^{B(1-K)} + a_1^2 B e^{B(1-K)}; \\ a_{65} &= C^3 e^{-C(1-K)} - a_2^2 C e^{-C(1-K)}; \\ a_{66} &= -C^3 e^{C(1-K)} + a_2^2 C e^{C(1-K)}; \\ a_{67} &= D^3 e^{-D(1-K)} - a_2^2 D e^{-D(1-K)}; \\ a_{68} &= -D^3 e^{D(1-K)} + a_2^2 D e^{D(1-K)}. \end{aligned}$$

In Eqs. (6) to (26), A_o is the initial cross-section of the structure, m_o is the initial mass of the structure, I_o is the initial moment of inertia of the structure, I_1 is the moment of inertia of the first part of the structure, I_2 is the moment of inertia of the second part of the stairs, A_1 is the cross-section of the initial part of the structure, A_2 is the cross-section of the second part of the stairs of the structure, m_{o1} is the mass of the initial part of the structure, m_{o2} is the mass of the second part of the stairs of the structure, $w_1(x,t)$ is the displacement vector of the initial part of the structure, and $w_2(x,t)$ is the displacement vector of the second part of the stairs of the structure. Assuming harmonic oscillation $W(x,t) = W(x)e^{i\omega t}$ that $W(x)$ is a function of the shape of the mode, ω is the natural frequency of the structure, $\varepsilon = \frac{x}{H}$ is parameter without dimension, a_1, a_2, b_1, b_2 are the assumed parameters to solve the differential

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ -A & A & -B & B & 0 & 0 & 0 & 0 \\ e^{-A(1-K)} & e^{A(1-K)} & e^{-B(1-K)} & e^{B(1-K)} & -e^{-C(1-K)} & -e^{C(1-K)} & -e^{-D(1-K)} & -e^{D(1-K)} \\ -Ae^{-A(1-K)} & Ae^{A(1-K)} & -Be^{-B(1-K)} & Be^{B(1-K)} & Ce^{-C(1-K)} & -Ce^{C(1-K)} & De^{-D(1-K)} & -De^{D(1-K)} \\ A^2e^{-A(1-K)} & A^2e^{A(1-K)} & B^2e^{-B(1-K)} & B^2e^{B(1-K)} & -C^2e^{-C(1-K)} & -C^2e^{C(1-K)} & -D^2e^{-D(1-K)} & -D^2e^{D(1-K)} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} \\ 0 & 0 & 0 & 0 & C^2e^{-C(1-K)} & C^2e^{C(1-K)} & D^2e^{-D(1-K)} & D^2e^{D(1-K)} \\ 0 & 0 & 0 & 0 & -a_{65} & -a_{66} & -a_{67} & -a_{68} \end{bmatrix}; \tag{26}$$

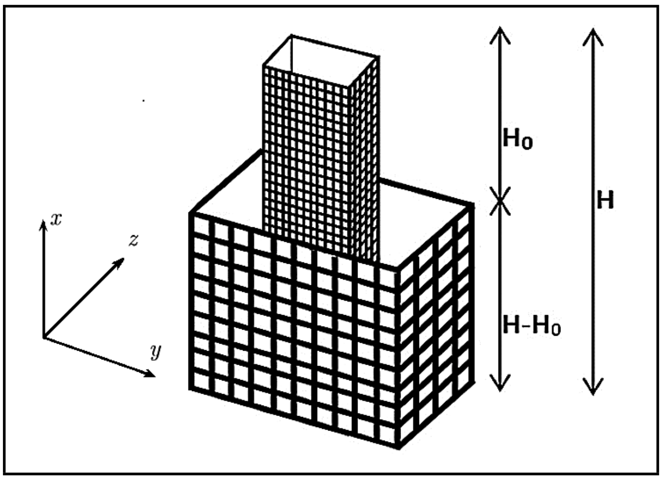

Figure 1. Stepped tube-in-tube structural system.

Table 1. Specifications of cross section, mass, length, flexural and shear stiffness of a 25-story building.

EI_o (KN/m^2)	EI_i (KN/m^2)	AG_t (KN/m^2)	m (ton/m)	L (m)
35.29×10^9	7.5538×10^9	3.99×10^7	325.83	75.9

equation, A, B, C, D are the assumed parameters for the answer of the equation, $k = \frac{H_0}{H}$ is the length of ratio parameter, $w(\varepsilon)$ is the answer of the whole equation, and c_1 to c_8 are the constant coefficients of the answer to the differential equation.

3. VALIDATION AND COMPARISON WITH OTHER ARTICLES WITH PROVIDED EXAMPLES

In this section, mathematical examples are provided to show application, efficiency, and computational accuracy. In the presented examples, the vibrational frequency of tall structures with a different tube-in-tube system is calculated by considering the value of the parameters related to the structure. Figures 1 and 2 show a typical tube structure with an interior tube. Each tube consists of four plates with an approximately uniform thickness of t equivalent. Therefore, the stepped tube-in-tube model can form a continuous system consisting of two tubes. Floor and roof plates are considered as hard diaphragms inside the structure. In the stepped tube-in-tube system, the addition of an external tube increases the stiffness. The changes in the height of the external tube on the frequency of the whole system are obtained by assuming that the height of the inner tube is constant.

Example 3.1. The vibrational frequency of the tube-in-tube system is given in Fig. 3 and Table 1 (specifications of cross section, mass, length, flexural and shear stiffness of a 25-story building) using the proposed method above.

Table 2. The obtained frequency using the mathematical model and comparison with other studies.

Method	Frequency
Mathematical model	3.7234
Effects axial forces[20]	3.214
Top displacement[21]	3.157
Mode superposition[21]	3.279
Finite element[22]	3.715
Variational method[23]	3.462
Sturm-Liouville equation[7]	3.463
Ordinary differential equation[24]	3.461
Power-series solution[16]	3.518
D'Alembert's principle[9]	3.705

Table 3. Dimensional specifications of a 50-story building with a framed tubular system.

Tube dimension	Spacing columns	Height of story	Tube dimension	Spacing columns
$2W_f$ (m)	S_f (m)	h (m)	$2W_w$ (m)	S_w (m)
50	2.5	3	30	2.5

The answer is calculated after placing the relevant A, B, a_1, a_2, C, D parameters in the determinants of matrix Eq. (26) and equating the determinant to zero. The obtained answer by the analytical mathematical method in Fig. 4 is accurate and correct compared to the previous studies and finite element. The frequency was obtained by using the mathematical model on Fig. 4: $\omega_1 = 3.7234$, $\omega_2 = 16.2099$, $\omega_3 = 41.0890$. The obtained frequency using the mathematical model and comparison with other studies are shown in Table 2.

Example 3.2. A mathematical calculation was provided to analyze the free vibration of symmetrical buildings with a single tube system to validate the calculation method.

In the second example, a building is analyzed with a reinforced concrete tube with a height of 50 floors. The size of the members of the beam and column is 0.8×0.8 meters. The height of each floor is 3 meters and the distance of column from the center to center is 2.5 meters. The thickness of the floor plates is 0.25 meters. The geometric parameters of the building are given in Table 3. The stiffness properties of the structural components are used from the model proposed by Maleknejad and Rahgozar, as shown in Tables 3 and 4.[12]

The obtained answer by analytical mathematical method for the equation is shown in Fig. 5. The obtained answer from the calculations for the tubular structural system is listed in Table 5.

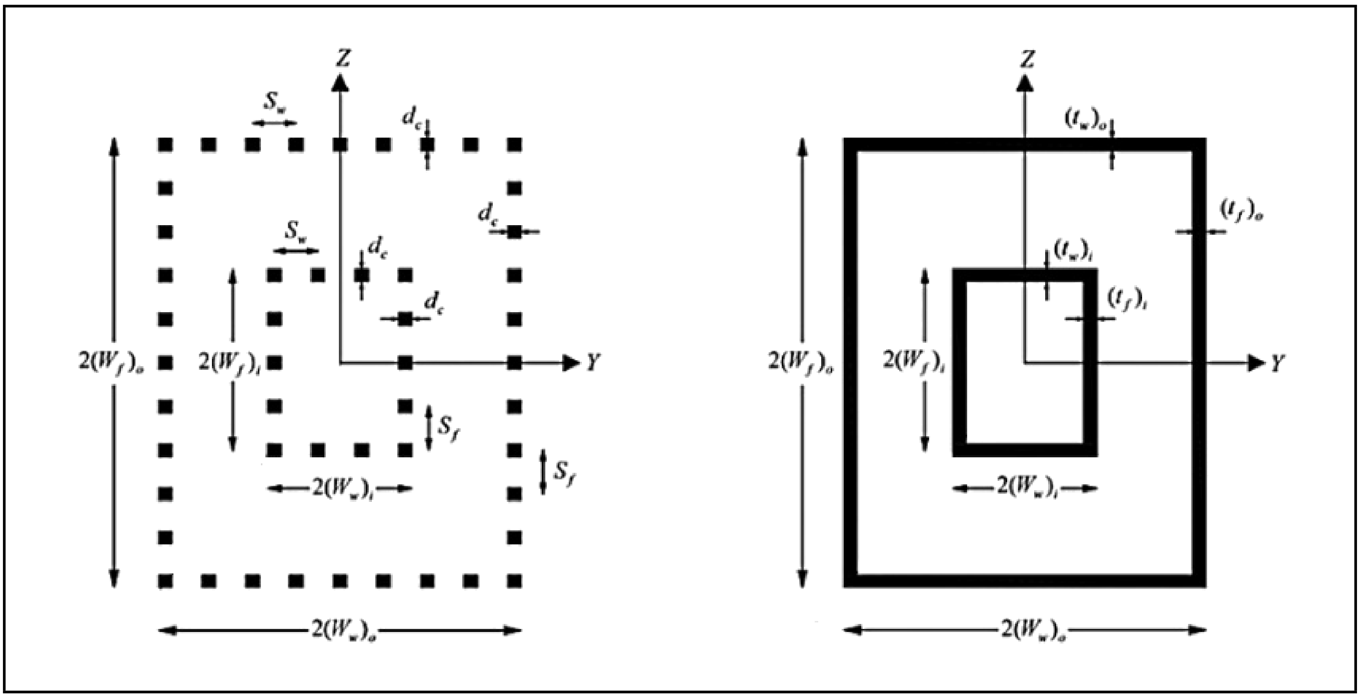

Figure 2. Plan of stepped tube-in-tube structural system with equivalent beam plan.

Table 4. Specifications of equivalent hardness for a 50-story building.

Actual structure		Equivalent membranes			Equivalent membranes		
Material		Webs			Flanges		
E (KN/m^2)	G (KN/m^2)	Ew (KN/m^2)	G_w (KN/m^2)	t_w (m)	E_f (KN/m^2)	G_f (KN/m^2)	t_f (m)
2×10^7	8×10^6	2×10^7	1.794×10^6	0.256	2×10^7	1.794×10^6	0.256

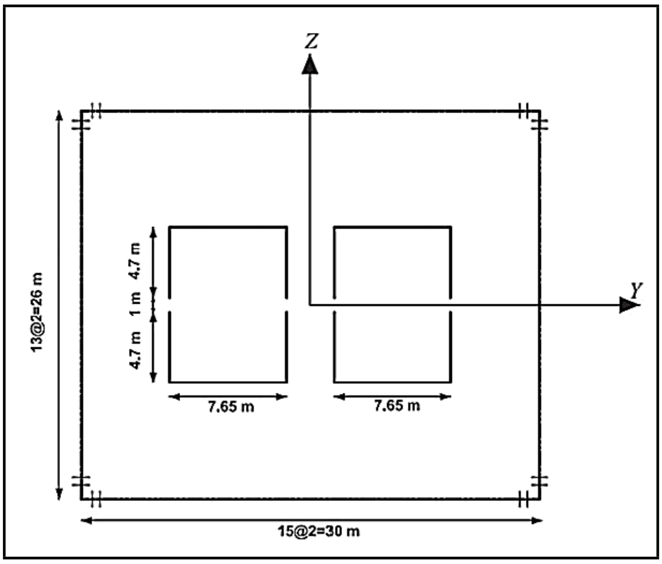

Figure 3. Plan of a 25-story tall building with a tube-in-tube system.

Table 5. The obtained answer from the calculations for the tubular structural system.

Structural system	Analytical method	Frequency (rad/sec)		
		ω_1	ω_2	ω_3
Tube	mathematical	1.2838	5.7536	14.5574
Tube	$R = 8$	1.105	5.434	
Tube	Malekinejad et al.[9]	1.239	5.707	
Tube	SAP 2000 FEM	1.148	5.758	

Table 6. Specifications of stepped external tube and specifications inner tube.

a_1	b_1	a_2	b_2
0	0	6.8	16.85
0.51416	0.179476	6.8	16.85
0.77124	0.403821	6.8	16.85
1.02832	0.717904	6.8	16.85
1.2854	1.121725	6.8	16.85
1.54248	1.615284	6.8	16.85
1.79956	2.198581	6.8	16.85
2.05664	2.871616	6.8	16.85
2.31372	3.634389	6.8	16.85
2.5708	4.4869	6.8	16.85

4. MODELING AND DISCUSSION RESULTS

After validating the proposed method, nine models with different heights were presented for the stepped tube-in-tube building, the dimensional specifications of which are given in Tables 6 to 8.

The obtained frequency for the stepped tube-in-tube system at different heights by mathematical method are listed in Table 9.

The obtained answer by analytical mathematical method for the equation are shown in Fig. 6.

As observed, the obtained answer in row 1 (Table 9) for the 50-story stepped tube-in-tube building is consistent with the study of Maleknejad and Rahgozar.[9] Therefore, the correctness of the method and the answer for the models in this study are confirmed.

The obtained frequency (Figs. 7 and 8) decreases while increasing the height. A 50-story structure with a stepped tubular system with different heights was analyzed (Tables 7, 8, and 9). The result of the first natural frequency was evaluated and compared with the results obtained by Maleknejad et al., as well as finite element analysis.[9] Tables 2 and 5 indicate the results for tube-in-tube structures. The specifications of a tubular system with a stepped tube at the height of the structure are given in Table 9 for nine models. In addition, the

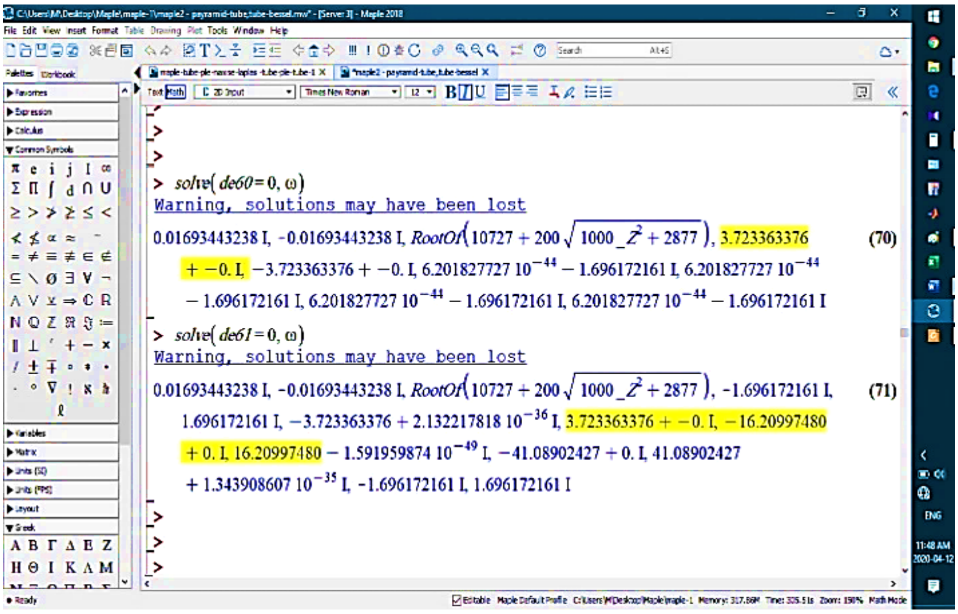

Figure 4. The obtained answer by using an analytical mathematical method.

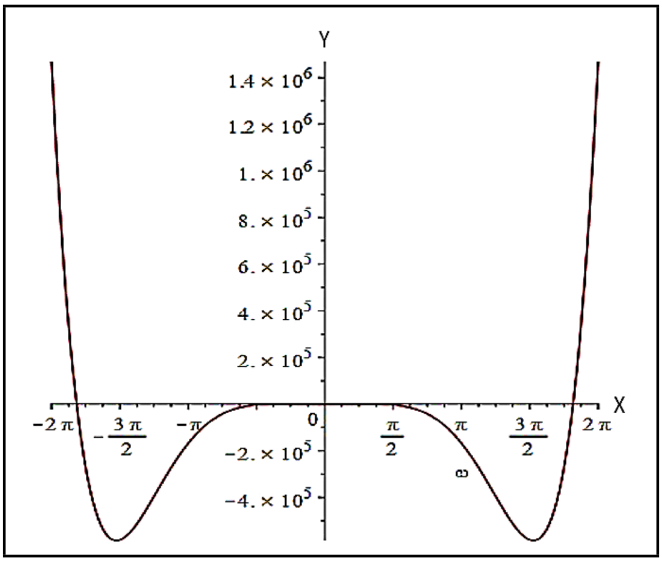

Figure 5. The obtained answer by analytical mathematical method for the equation.

stepped tube-in-tube structure was analyzed and the result of the first natural frequency, obtained by the proposed approximation method, is compared with an obtained answer by the finite element method from SAP 2000, as shown in Table 9 and Figs. 7 and 8.

Table 7. Specifications of stepped external tube and specifications inner tube in terms of height and k coefficient.

k	H (m)	$a_1^2/2$	b_1^2	$a_1^4/4$
0.2	30	0.13218	0.032212	0.017472
0.3	45	0.297406	0.163071	0.08845
0.4	60	0.528721	0.515386	0.279546
0.5	75	0.826127	1.258267	0.682485
0.6	90	1.189622	2.609142	1.415201
0.7	105	1.619208	4.833758	2.621835
0.8	120	2.114884	8.246178	4.472735
0.9	135	2.67665	13.20878	7.164456
1	150	3.304506	20.13227	10.91976

Table 8. Specifications of inner tube.

k	H (m)	a_2	b_2	$a_2^2/2$	b_2^2	$a_2^4/4$
0	150	6.8	16.85	23.12	283.9225	534.534

Table 9. The obtained frequency for stepped tube-in-tube system at different heights by mathematical method.

No.	Heights inner tube (m)	Heights outer stepped tube (m)	Frequency ω (rad/s)
1	150	150	1.39735
2	150	135	1.56800
3	150	120	1.80813
4	150	105	2.16023
5	150	90	2.70534
6	150	75	3.61369
7	150	60	5.29494
8	150	45	8.94535
9	150	30	19.41863

5. CONCLUSIONS

The goal of the present study was to calculate the vibrational frequency of tall buildings with a stepped tube-in-tube system for mass and stiffness with stepped decreasing. Finite element analytical methods are based on discrete modeling and a large number of linear equations are needed for calculating the results of the structural analysis due to a large number of elements. The proposed methods and equations provided a free vibration analysis for tall stepped tubular structures. In this mathematical study, the structure was modeled in a continuous environment. This was closer to the reality of a stepped structure which needed the solution of a differential equation which was easily programmed in computational software. The results can be used to reduce modeling errors. Differential equations were used to calculate the natural frequencies of a stepped tubular structure with variable stepped mass and stiffness which was solved by a strong form. The results were calculated correctly and they indicated that the changes in bending and shear stiffness had the greatest effect on the first fre-

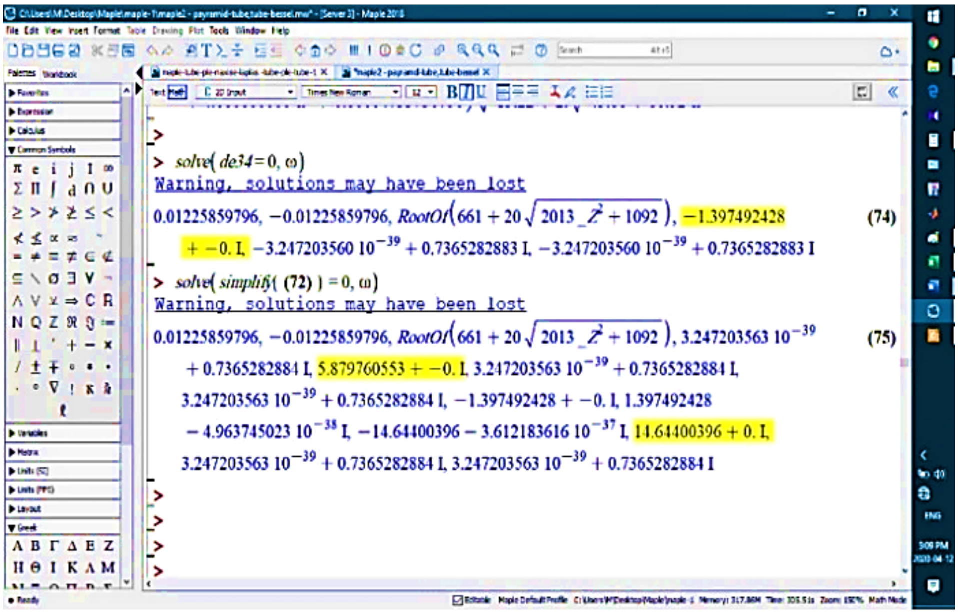

Figure 6. The obtained answer by analytical mathematical method for equation.

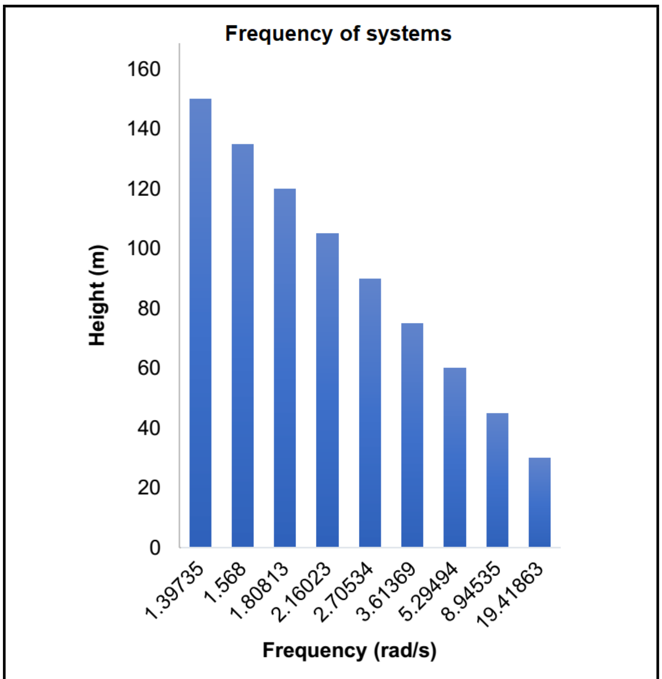

Figure 7. Bar diagram of vibration frequency changes for stepped tube system models.

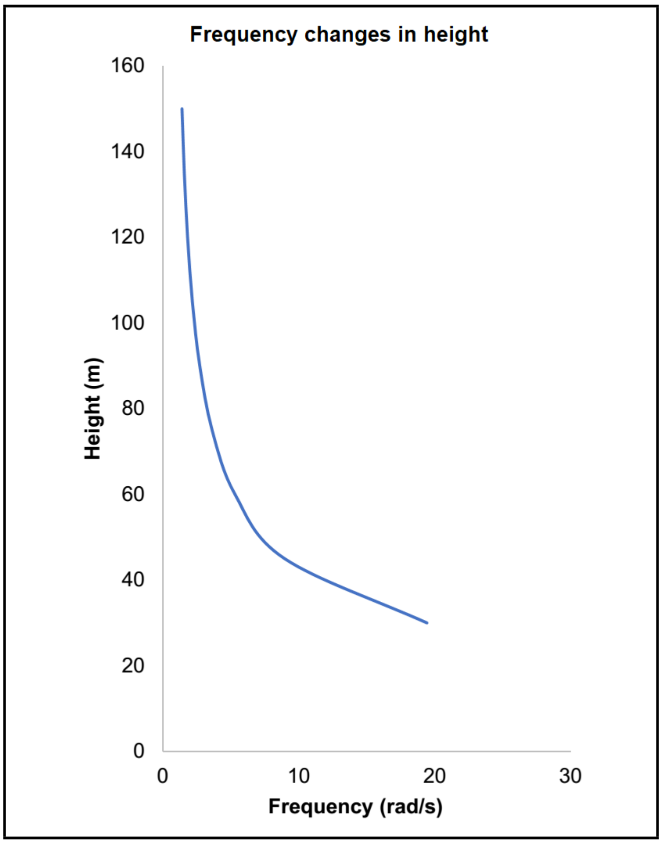

Figure 8. The changes of vibrational frequency of the system with increasing the height.

quency. The effectiveness of the method was presented by comparing the mathematical results and finite element, and the available results for tubes in tube-in-tube and single tubes were compared and confirmed only through the previous papers. Mathematical examples showed that the approximate values of the natural frequency of a tall building with stepped tube-in-tube structures are acceptable with more accurate results of the finite element method in the range by using the proposed method, and the use of the obtained natural frequency in this method was approved. The proposed method in this study was generally simple, accurate, economical, reliable and particularly suitable in approximation analysis for use in the preliminary design stages. Further, tube-in-tube structural systems were more compatible with the proposed analytical method. The proposed analytical method was most compatible with a higher number of stories. In other words, the proposed method was more accurate as the number of stories increased. In the proposed analytical method for tubular systems, the addition of an external tube increased the stiffness of the whole system.

Further, in the conditions where the stiffness of the structure remained constant, the higher height of the external tube led to a lower natural frequency.

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