Size-Dependent Stability of a Cantilevered Piezoelectrically Actuated Micropipe Conveying Fluid

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(Received 20 April 2020; accepted 17 October 2020)

Size-dependent effects of a cantilevered piezoelectrically actuated micropipe conveying fluid are investigated. Based on the modified strain gradient beam theory, the model of system is obtained using Hamilton’s principle. The motion equation is discretized into ordinary differential equations by Generalized Differential Quadrature Method (GDQM). A stability analysis of the system is completed through eigenvalue analysis. Numerical results show the effect of geometrical shape size, and length scale parameters on critical flow velocity, and critical voltage. Results prove that the modified strain gradient theory (MSGT) has a higher critical flow velocity and critical voltage than predicted by modified couple stress theory (MCST) and classical theory (CT).

1. INTRODUCTION

Pipe conveying fluid is widely used in important components of some engineering machinery. Since the middle of the last century, various vibration analyses of pipe conveying fluid have been performed by many scholars. Interested readers can refer to monographs and the review’s paper. Compared with the macro-scale pipes conveying fluid, micro-scale pipes conveying fluid are often used to make microreactors, current-carrying devices, and microsensors. It is of great practical significance to study the vibration characteristics of micro-scale pipes conveying fluid.

Scholars have shown in the past decades that the micro-scale structures have size-dependent phenomena. Lam et al. developed a modified set of second-order deformation metrics, the dilatation gradient vector, the deviatoric stretch gradient tensor, and the rotation gradient tensor, and derived the corresponding work-conjugate stress metrics as the basic strain and stress measures for a strain gradient theory for elasticity, i.e., the modified strain gradient theory. Tang and Alici presented a method to obtain the length-scale factors for silicon cantilevers by using the experimental data provided by the micro-and nanoindentation measurements. The natural frequency and deflection models for micro-and nano-sized silicon cantilevers are provided to incorporate the effect of the length-scale factors. Dehrouyeh-Semnani investigated the static bending of thin plane-strain microbeam based on modified couple stress and strain gradient elasticity constitutive beam models. The modified couple stress and strain gradient elasticity of the Euler–Bernoulli beam models overestimate the bending rigidity of the micro-cantilever when the size effect is significant. The size effect in the structural behavior of systems can also be found in some literature based on the modified strain gradient theory.

A few research results have been obtained for the micro-scale pipes conveying fluid on the microscale theory of some of the above documents. Several achievements have been made in the research of micro-scale pipe conveying fluid in the last 10 years. Rinaldi et al. presented the first set of results on the effects of miniaturization on stability, damping, and frequency shifts in straight micromachined pipes containing an internal flow. Wang established a theoretical model for fluid-conveying microtubes using the modified couple stress theory and explored the effects of the internal material length scale parameter on the natural frequencies and the critical flow velocities. Xia and Wang developed a microstructure–dependent Timoshenko model for the microscale pipes containing internal fluid using modified couple stress theory. Yin et al. proposed a microstructure–dependent Bernoulli–Euler model for microscale pipes containing internal fluid by using the strain gradient and analyzed the vibration and stability of microscale simply supported pipe. Yang et al. demonstrated the microstructure–dependent size effect. The post–buckling configurations, nonlinear frequency, and response were presented.

Hosseini et al. investigated the size–dependent stability analysis of the cantilever micropipes conveying fluid and examined the effect of length scale parameter, outside diameter, aspect ratio, and the type of microstructure model on the mechanical behaviors of micropipes. Deng et al. examined the free vibration and stability of multi–span FGM micropipe conveying fluid. A hybrid method, which com-
bines reverberation-ray matrix and wave propagation, is developed in consideration of the natural frequencies of the system. Ghazavi et al.\textsuperscript{23} analyzed the in-plane and out-of-plane vibrations and stability of curved microtubes. The size effects were also studied on the basis of strain gradient theory, and the results were compared with those obtained on the basis of couple stress and classical elasticity theory. On the other hand, the application of piezoelectric materials in engineering structures has drawn increasing attention. Rezazadeh et al.\textsuperscript{24} investigated the effect of applying piezoelectric layers on the stability of the fluid conveying micropipes. The effect of piezoelectric voltage on frequencies and critical flow velocities was thoroughly discussed using eigenfrequency branches. Hosseini et al.\textsuperscript{25} studied the forced vibrations of double piezoelectric functionally graded material micropipes conveying fluid carrying a moving load based on the flexoelectricity theory and modified couple stress theory. Hosseini et al.\textsuperscript{25} also demonstrated the divergence and flutter instability of a cantilever piezoelectric carbon nanotube and discussed the effects of different parameters such as piezoelectric voltage, surface residual stress, surface elasticity constant, and surface piezoelectric constant, on the critical flutter velocity of the CNT.

The classical continuum theory of elasticity shows the uncertainty in describing the mechanical behavior of the system considering the study of microscale pipe conveying fluid. Theories of higher order, such as, modified couple stress theory and strain gradient theory, can effectively describe the mechanical behavior of micro-scale structures, as presented by the aforementioned literature.\textsuperscript{17,18,20} However, the discussion of the piezoelectrically actuated microbeams\textsuperscript{26} and micropipe conveying fluid\textsuperscript{25} did not involve the influence of the size effect and only established a model using classical theory. The authors believe that the dynamic model established by this method is insufficient to reflect the mechanical behavior of piezoelectrically actuated micropipe conveying pipe.

The goal of this paper is to investigate the size-dependent effect of piezoelectrically actuated micropipe conveying fluid by using modified strain gradient theory. The governing equation of motion of the system is obtained using strain energy together with Hamilton’s principle. The Generalized Differential Quadrature Method (GDQM) is used to calculate the natural frequency and flutter critical flow velocity and voltage of micro-scale pipe conveying fluid, and the effect of geometrical parameters is discussed.

2. MATHEMATICAL MODELLING

The modified strain gradient elasticity theory for micropipe conveying fluid is presented in the literature.\textsuperscript{19} Interested readers can refer to the literature\textsuperscript{19} for additional details on this theory. The basic equations associated with the modified strain gradient and the couple stress theories will be omitted. The size effect is considered for the micropipe and the piezoelectric layer. The Knudsen number (the ratio of the molecular mean free path length of fluid to the inner diameter of micropipe) in the current problem is sufficiently small due to the influence of fluid flow based on the classical equations of fluid dynamics. The assumptions made in this paper are as follows: (1) The fluid in the micropipes is ideal, that is, the inviscid and incompressible fluid. (2) The fluid velocity in the pipe is constant. (3) The length of the pipe is far greater than the diameter of the pipe, and the materials of the pipes are the same and uniform. (4) Only the in-plane vibration of the pipe is considered. (5) The Poisson’s effect was neglected.

A cantilevered piezoelectrically actuated micropipe conveying fluid is shown in Fig. 1. A pipe of length $L$, pipe density $\rho_p$, outer layer width $w_{\text{out}}$, and inner layer width $w_{\text{in}}$, outer layer height $h_{\text{out}}$, inner layer height $h_{\text{in}}$ are considered isotropic with a pair of piezoelectric layers bonded on its surfaces, each piezoelectric layer has a thickness $h_p$, and the width $w_p$ of each piezoelectric layer is equal to the outer layer width $w_{\text{out}}$, that is, $w_p = w_{\text{out}}$. The fluid in the pipe is incompressible fluid with density $\rho_f$ and axial flow velocity $U$. The piezoelectric layered cantilever pipe is considered to be an Euler–Bernoulli type. $x$ and $y$ represent the axial and transverse directions, respectively; the transverse one, in the y-direction, is represented by $w(x,t)$.

On the basis of the modified strain gradient theory,\textsuperscript{29} the potential energy of pipe includes the following terms\textsuperscript{21}.

$$U_b = \frac{1}{2} \int_0^L \left( S_b \cdot \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + K_b \cdot \left( \frac{\partial^3 w(x,t)}{\partial x^3} \right)^2 \right) dx; \quad (1)$$

where

$$K_b = I_6 (2G_b l_0^2 + \frac{1}{2} G_b l_1^2); \quad S_b = E_b I_6 + 2G_b A_6 l_b^2 + \frac{8}{25} G_b A_6 l_1^2 + G_b A_6 l_2^2; \quad (2)$$

$$A_b = h_{\text{out}} w_{\text{out}} - h_{\text{in}} w_{\text{in}}, \quad I_b = \frac{w_{\text{out}} h_{\text{out}}^3}{12} - \frac{w_{\text{in}} h_{\text{in}}^3}{12}; \quad (3)$$

in which, $E_b$ and $G_b$ are the elastic and shear modulus of the pipe in the classical elasticity theory, respectively. $l_0$, $l_1$, and $l_2$ denote the three independent material length scale parameters related to the constitutive equations of the higher-order stresses. In the reference,\textsuperscript{21} material scale parameters $l_0$, $l_1$, and $l_2$ are independent material length scale parameters respectively associated with dilatation, deviatoric stretch, and rotation gradients. The length scale parameters in this paper do not denote real material length scale parameters. So, the value of the material length scale parameters is just an arbitrary assumption.

Applying a voltage $V_p$ on the piezoelectric layers located on the micro-pipe surfaces, the piezoelectric voltage will lead to a
follower force as provided below:\(^{24}\)

\[ F_p = -2w_{out}\bar{e}_{31}V_p; \]  
(4)

where \(\bar{e}_{31}\) the equivalent piezoelectric coefficient.

Considering the piezoelectrically exciting force as the follower force, the potential energy related to the piezoelectric layers can be written as:\(^{25}\)

\[ \Pi_{Fx} = \frac{1}{2}F_p \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx; \]  
(5)

and

\[ \delta \Pi_{Fx} = F_p \frac{\partial w}{\partial x} \delta w \bigg|_{x=L}; \]  
(6)

where \(\Pi_{Fx}\) and \(\Pi_{Fy}\) denote the work respectively conducted by the \(x\)- and \(y\)-components of the follower force \(F_p\). Current authors indicate that equation (6) does not appear in the literature. Some literature works reveal that actuated force \(F_p\) is not included in the equations of the boundary conditions. However, the authors use the same method to re-deduce the governing equations and the boundary conditions. The process of derivation shows that Eq. (6) cannot be omitted.

The potential energy of the piezoelectric layer includes the following terms:\(^{21}\)

\[ U_p = \frac{1}{2} \int_0^L \left( S_p \cdot \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + K_p \left( \frac{\partial^3 w(x,t)}{\partial x^3} \right)^2 \right) dx; \]  
(7)

where

\[ K_p = I_p (2G_p \rho_2^2 + \frac{4}{5}G_p \rho_1^2); \]

\[ S_p = E_p I_p + 2G_p A_p \rho_2^2 + \frac{8}{15} G_p A_p \rho_1^2 + G_p A_p \rho_2^2; \]

\[ A_p = 2w_{out} p_o, I_p = \frac{2w_{out}}{3} \left( \frac{3\delta_{out}}{4} + h_p^2 + \frac{3}{2} h_0 w_{out} h_p \right); \]  
(9)

where \(E_p\) and \(G_p\) are the elastic and shear modulus of piezoelectric layer in the classical elasticity theory, respectively.

The kinetic energy includes the following terms

\[ T(t) = T_b + T_p + T_f = \frac{1}{2\rho_0} \int_0^L \left( \frac{\partial \delta w(t,x)}{\partial x} \right)^2 dx + \frac{1}{2\rho_f} \int_0^L \left( \frac{\partial \delta w(t,x)}{\partial t} \right)^2 dx; \]

(10)

where \(A_f = h_{in} w_{in}\).

The governing equation of the transverse motion can be obtained by the minimization of the Lagrangian using the variational principle:

\[ \int_0^t (\delta T - \delta \Pi) dt = \int_0^t \Pi MU(\delta w_L + U w_{out}) \delta w_L dt; \]  
(11)

where

\[ \delta \Pi = \delta \Pi_b + \delta \Pi_{Fx} + \delta \Pi_{Fy} + \delta \Pi_p; \]  
(12)

Substituting expressions (1), (4), (5), (6), (7), and (10) into Eq. (11) and using the integral transformation

\[ \delta \int_0^t (T - U) dt - \int_0^t MU(\delta w_L + U w_{out}) \delta w_L dt = -\int_0^t (\rho_0 A_p + \rho_p A_p + m_f) \frac{\partial w}{\partial t} \delta \omega \int_0^L dx + \int_0^L \int_{t_f}^{l_0} w'' \delta w \int_0^L dx + \int_0^L \int_{t_f}^{l_0} w' \delta w \int_0^L dx + \int_0^L \int_{t_f}^{l_0} w \delta w \int_0^L dx - \int_0^L \int_{t_f}^{l_0} w'' \delta w \int_0^L dx + \int_0^L \int_{t_f}^{l_0} w' \delta w \int_0^L dx + \int_0^L \int_{t_f}^{l_0} w \delta w \int_0^L dx - \int_0^L \int_{t_f}^{l_0} w'' \delta w \int_0^L dx + \int_0^L \int_{t_f}^{l_0} w' \delta w \int_0^L dx + \int_0^L \int_{t_f}^{l_0} w \delta w \int_0^L dx ; \]

(13)

The governing equation of motion is derived as follows:

\[ (\rho_p A_p + \rho_p A_p + m_f) \frac{\partial^2 w}{\partial t^2} + (S_p + S_b) \frac{\partial^4 w}{\partial x^4} - (K_p + K_b) \frac{\partial^6 w}{\partial x^6} - F_p \frac{\partial^2 w}{\partial x^2} + m_f U_0 \frac{\partial^2 w}{\partial x^2} + 2m_f U_0 \frac{\partial^2 w}{\partial x \partial t} = 0. \]  
(14)

The corresponding boundary conditions from Hamilton’s method for cantilevered micro-pipe conveying fluid are written as follows:

\[ x = 0: [(S_b + S_p) w'' - (K_p + K_b) w'''] = 0; \]

\[ [-(S_b + S_p) w' + (K_p + K_b) w'''] = 0; \]

\[ K_p + K_b w''w'' = 0; \]  
(15)

\[ x = L: [-(S_b + S_p) w' - (K_p + K_b) w'''] = 0; \]

\[ [(S_b + S_p) w' - (K_p + K_b) w'''] = 0; \]

\[ K_p + K_b w''w'' = 0. \]  
(16)

For a pipe with a cantilever, the classical boundary conditions at ends of pipe are as follows:

\[ x = 0: w = 0, w' = 0; \]

\[ x = L: (S_b + S_p) w'' - (K_p + K_b) w''' = 0; \]

\[ -(S_b + S_p) w'' + (K_p + K_b) w'' = 0. \]  
(17)

Two possible boundary conditions at both ends are available for non-classical boundary conditions. One of the higher-order boundary conditions can be written as:

\[ x = 0: (K_p + K_b) w'' = 0 \text{ or } w'' = 0; \]

\[ x = L: (K_p + K_b) w''' = 0 \text{ or } w'' = 0. \]  
(18)

The boundary conditions in Eq. (17)–(18) can be reduced to the following four boundary conditions:

**BC1:**

\[ x = 0: w = 0, w' = 0, w'' = 0; \]

\[ x = L: -(S_b + S_p) w'' + (K_p + K_b) w''' = 0; \]

\[ -(S_b + S_p) w'' + (K_p + K_b) w''' = 0, (K_p + K_b) w'' = 0. \]  
(19)
In dimensionless terms, disregarding the sign "*", substituting (26) into Eq. (23), the dimensionless governing equation can be shown as follows:

\[
\begin{align*}
\frac{\partial^2 w}{\partial t^2} + \tilde{S} \frac{\partial w}{\partial t} - \tilde{F}_p \frac{\partial w}{\partial x} - \mu U^2 \frac{\partial^2 w}{\partial x^2} \\
+ 2\sqrt{\beta U} \frac{\partial^2 w}{\partial x \partial t} = 0.
\end{align*}
\]

(27)

The corresponding dimensionless boundaries include the following:

BC1:

\[
\begin{align*}
\xi = 0 : w &= 0, w' = 0, w'' = 0; \\
\xi = L : -\tilde{S} w'' + \tilde{K} w''' &= 0, w''' = 0, w'' = 0.
\end{align*}
\]

(28)

BC2:

\[
\begin{align*}
\xi = 0 : w &= 0, w' = 0, w'' = 0; \\
\xi = L : -\tilde{S} w'' + \tilde{K} w''' &= 0, w''' = 0, w'' = 0.
\end{align*}
\]

(29)

BC3:

\[
\begin{align*}
\xi = 0 : w &= 0, w' = 0, w'' = 0; \\
\xi = L : -\tilde{S} w'' + \tilde{K} w''' &= 0, w''' = 0, w'' = 0.
\end{align*}
\]

(30)

The GDQM method is used to discretize the governing equation and boundary conditions to obtain the eigenvalues of micropipe conveying fluid. The GDQM can also be employed to solve the various vibration problem (33,34).

The mesh is divided as follows

\[
x_i = \frac{1 - \cos((i - 1)\pi/(N - 1))}{2}, \quad (i = 1, 2, 3, \ldots, N - 1, N).
\]

(32)

On the basis of the algorithm of GDQM, the independent variable vector is

\[
\{\mathbf{W}\} = \{W_1, W_2, W_3, W_4, \cdots, W_{N-1}, W_N,
\]

\[
W_{N+1}, W_{N+2} \}^T
\]

\[
= \{w_1, w_1', w_1'', w_2, w_3, \cdots, w_{N-1}, w_{N-1}', w_{N-1}''\}^T.
\]

(33)

Setting \( \mathbf{w}_b = \{w_1, w_1', w_1'', w_2', w_3', \cdots, w_{N-1}, w_{N-1}'\}^T \) and \( \mathbf{w}_d = \{w_2, w_3, w_4, \cdots, w_{N-1}, w_{N-1}, w_{N+1}\}^T \), and discretizing Eqs. (27)–(31) by the GDQM, one obtains:

\[
\begin{align*}
\{\mathbf{W}\} &= \{W_1, W_2, W_3, W_4, \cdots, W_{N-1}, W_N,
\]

\[
W_{N+1}, W_{N+2} \}^T
\]

\[
= \{w_1, w_1', w_1'', w_2', w_3', \cdots, w_{N-1}, w_{N-1}'\}^T.
\]

(33)

\[
\begin{align*}
\sum_{j=1}^{N+4} E_{ij}^4 W_j - \tilde{K} \sum_{j=1}^{N+4} E_{ij}^6 W_j + \\
\tilde{F}_p + \mu U^2 \sum_{j=1}^{N+4} E_{ij}^2 W_j + 2\sqrt{\beta U} \sum_{j=1}^{N+4} E_{ij}^1 W_j = 0,
\end{align*}
\]

\[
i = 2, 3, \cdots, N - 2.
\]

(34)
BC1:

\[ W_1 = 0, \quad \sum_{j=1}^{N+4} E_{1j}^{(1)} W_j = 0, \quad \sum_{j=1}^{N+4} E_{1j}^{(2)} W_j = 0; \]
\[ -\bar{S} \sum_{j=1}^{N+4} E_{Nj}^{(2)} W_j + \bar{K} \sum_{j=1}^{N+4} E_{Nj}^{(4)} W_j = 0; \]
\[ \sum_{j=1}^{N+4} E_{Nj}^{(5)} W_j = 0, \quad \sum_{j=1}^{N+4} E_{Nj}^{(3)} W_j = 0. \]  

(35)

BC2:

\[ W_1 = 0, \quad \sum_{j=1}^{N+4} E_{1j}^{(1)} W_j = 0, \quad \sum_{j=1}^{N+4} E_{1j}^{(2)} W_j = 0; \]
\[ \sum_{j=1}^{N+4} E_{Nj}^{(4)} W_j = 0, \quad \sum_{j=1}^{N+4} E_{Nj}^{(2)} W_j = 0; \]
\[ -\bar{S} \sum_{j=1}^{N+4} E_{Nj}^{(3)} W_j + \bar{K} \sum_{j=1}^{N+4} E_{Nj}^{(5)} W_j = 0. \]  

(36)

BC3:

\[ W_1 = 0, \quad \sum_{j=1}^{N+4} E_{1j}^{(1)} W_j = 0, \quad \sum_{j=1}^{N+4} E_{1j}^{(2)} W_j = 0; \]
\[ -\bar{S} \sum_{j=1}^{N+4} E_{Nj}^{(2)} W_j + \bar{K} \sum_{j=1}^{N+4} E_{Nj}^{(4)} W_j = 0; \]
\[ \sum_{j=1}^{N+4} E_{Nj}^{(5)} W_j = 0, \quad \sum_{j=1}^{N+4} E_{Nj}^{(3)} W_j = 0. \]  

(37)

BC4:

\[ W_1 = 0, \quad \sum_{j=1}^{N+4} E_{1j}^{(1)} W_j = 0, \quad \sum_{j=1}^{N+4} E_{1j}^{(2)} W_j = 0; \]
\[ \sum_{j=1}^{N+4} E_{Nj}^{(4)} W_j = 0, \quad \sum_{j=1}^{N+4} E_{Nj}^{(2)} W_j = 0; \]
\[ -\bar{S} \sum_{j=1}^{N+4} E_{Nj}^{(3)} W_j + \bar{K} \sum_{j=1}^{N+4} E_{Nj}^{(5)} W_j = 0. \]  

(38)

In Eqs. (34)–(38), the coefficients \( E_{ij}^{(r)} \) \((r = 1, 2, \ldots, 6)\) are the weighting coefficient of the r-th-order derivative at a grid point \( \xi \) can be obtained from the literature. According to the literature, Eq.(34) and one of the boundary (35)-(38) can be rewritten in matrix form:

\[
\left[ \begin{array}{cc} [K_{bb}] & [K_{bd}] \\ [K_{db}] & [K_{dd}] \end{array} \right] \left\{ \begin{array}{c} \{ w_b \} \\ \{ w_d \} \end{array} \right\} + \left[ \begin{array}{cc} [0] & [0] \\ [G_{db}] & [G_{dd}] \end{array} \right] \left\{ \begin{array}{c} \{ \dot{w}_b \} \\ \{ \dot{w}_d \} \end{array} \right\} + \left[ \begin{array}{cc} [0] & [0] \\ [M_{db}] & [M_{dd}] \end{array} \right] \left\{ \begin{array}{c} \{ \ddot{w}_b \} \\ \{ \ddot{w}_d \} \end{array} \right\} = \left[ \begin{array}{c} [0] \\ [0] \end{array} \right].
\]

(39)

The first equation of Eq. (39) can be solved as follows:

\[
\{ w_b \} = -[K_{bb}]^{-1} [K_{bd}] \{ w_d \} .
\]

(40)

Table 1. The material contents of the microcantilever piezoelectrically actuated micro—pipe conveying fluid

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Microcantilever pipe</th>
<th>Piezoelectric layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (µm)</td>
<td>800.0</td>
<td>800.0</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>169.0</td>
<td>78.6</td>
</tr>
<tr>
<td>Mass density (kg/m³)</td>
<td>2331</td>
<td>7500</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.06</td>
<td>0.3</td>
</tr>
<tr>
<td>( \epsilon_{41} )</td>
<td>-9.29</td>
<td>-9.29</td>
</tr>
</tbody>
</table>
the same, that is, $\ell_0 = \ell_1 = \ell_2 = \ell = 17.6 \mu m$. The influence of various boundary conditions on the vibration behavior of the system was initially studied. In Fig. 3, the real part of the third-order modal versus fluid velocity $U$ is shown under the following parameters $V = 20.0 \text{ vol}$, $h_{\text{out}} = 40.0 \mu m$, and $h_{\text{in}} = 38.0 \mu m$, $w_{\text{out}} = 50.0 \mu m$. Figure 3 shows that, the unstable critical flow velocity is $U_{cr} = 23.18$ in the present parameters, and the critical flow velocity under the four boundary conditions shows almost no difference. Therefore, the authors only considered using one of the four boundary conditions, that is, the first boundary condition, in the following analysis.

Figure 4 shows that considering the size effect of micro-flow and micro-structure, the coefficient $\mu$, which represents the effect of the flow velocity profile, is equal to 1.3746 with the same parameters in Fig. 3. The choice of $\mu=1.3746$ decreases the critical flow velocity and the unstable critical flow velocity is $U_{cr} = 14.25$.

The system degenerates to a cantilever beam when the fluid velocity is zero. The critical dimensionless load (20.05) of the Beck problem for the cantilever beam is denoted as. Figure 5 shows the first three order imaginary and real parts of the eigenvalue versus the applied DC voltage in these parameters $U = 0.0, h_{\text{out}} = 30.0 \mu m, h_{\text{in}} = 20.0 \mu m, w_{\text{in}} = 48.0 \mu m, w_{\text{out}} = 50.0 \mu m$. Figures 5(a) and (b) respectively indicate the flutter instability is $V_{cr} = 57.29$ and frequency $\omega_{cr} = 11.02$. The corresponding dimensionless piezoelectric force at this point is 20.07, which is in good agreement with the foregoing description. However, the system does not undergo divergence instability, which is not in agreement with. The present authors believes that the possible reason that the Galerkin method’s modal number is adopted by two order modal in. Figure 5 shows that the second flutter instability is obtained at $V_{cr} = 365.16$ by increasing the piezoelectric voltage.

Figure 6 shows the evolution of the four lowest complex eigenfrequencies of the system with increasing flow velocity under the piezoelectric voltage $V = 40.0 \text{ vol}$, $h_{\text{out}} = 30.0 \mu m$, $h_{\text{in}} = 20.0 \mu m, w_{\text{in}} = 48.0 \mu m, w_{\text{out}} = 50.0 \mu m$. In classical theory, Fig. 6(a) shows that the flutter unstable point of the system appears at the fluid velocity $U_{cr} = 7.139$ and the unstable critical frequency $\omega_{cr} = 26.452$ of the third-order mode. Fig. 6(b) reveals that the flutter unstable point of the system in the modified coupled stress theory appears at the fluid velocity $U_{cr} = 13.728$ and the unstable critical frequency $\omega_{cr} = 45.721$ of the third-order mode. In the modified strain gradient theory, as seen in Fig. 5(c), the flutter unstable point of the system appears at the fluid velocity $U_{cr} = 22.156$ and third-order mode unstable critical frequency $\omega_{cr}=71.714$ of
the third-order mode. The value of the micro-scale parameters has a significant influence on the stability of micro pipe conveying fluid.

Figure 7 depicts the variation of flutter critical flow velocity and micro-pipe voltage with the parameter $\alpha$, which is equal to the ratio of parameter $h_{in}/h_{out}$ and the other parameter $h_{in} = 20\, \mu m$, $w_{out} = 50.0\, \mu m$, $\gamma = 0.90$, $V = 20.0$ vol. Figure 7(a) shows that the critical flow velocity $U_{cr}$ increases in the modified couple stress and strain gradient theory by raising the value $\alpha$, that is, the external geometric height parameter $h_{out}$. Figure 7(b) reveals the critical voltage $V_{cr}$ decreases under the special parameter $U = 4.0$ by increasing the value of $\alpha$. In Figs. 7 and 8 the black, red, and blue lines denote the classical, modified couple stress, and modified strain gradient theories, respectively.

Figure 8 depicts the variation of the critical fluid velocities and critical voltage of micro-pipes concerning the parameters $\gamma$, which is equal to the parameter ratio $w_{in}/w_{out}$ and the other parameters $w_{in} = 30.0\, \mu m$, $h_{in} = 20\, \mu m$, $\alpha = 0.95$. In Fig. 8(a), the critical flow velocity with voltage $V = 20.0$ has a hysteretic phenomenon, as indicated in the study of Paidoussis (1). In Fig. 8(b) the critical voltage $V_{cr}$ decreases with the increasing $\gamma$ parameter under flow velocity $U = 3.50$.

Figure 9 shows the variation of critical fluid velocity and voltage of micro-pipes considering size-dependent effect $\ell$ and the other parameters $\alpha = 0.95$, $\gamma = 0.4$ under modified strain gradient theory. Fig. 9(a) shows the effect of the size-dependent effect $\ell$ on the flutter critical flow velocity with the parameter $V = 5.0$. In Fig. 9(b) with the special parameter $U = 3.50$ when the size-dependent effect $\ell$ increasing, the flutter critical voltage $V_{cr}$ also increases.

The instability region in the $(U, V)$ plane for a cantilevered piezoelectrically actuated micropipe is shown in Fig. 10 considering the following parameters: $\alpha = 0.95$, $\gamma = 0.4$, $w_{in} = 30.0\, \mu m$, and $h_{in} = 20\, \mu m$. Three different theories are applied. This figure shows the parameter region in the $(U, V)$ plane is divided into two sub-regions according to different behaviors. The sta-
5. CONCLUSION

This paper introduced a cantilevered piezoelectrically actuated micropipe conveying fluid to enhance the analysis of the mechanical behavior. The modified strain gradient theory is used instead of the classical theory to build the equation of system motion and the corresponding boundary. The size effect on the stability of the system was illustrated. By employing the GDQM, the various numerical results revealed the effect of geometrical shape size, length scale parameter on critical flow velocity, and critical voltage. The main results are included as follows:

1. The influence of four boundary conditions derived from the principle on system’s stability is the same.
2. The critical flow and critical voltage obtained by the modified couple stress and strain gradient theory were higher than those obtained by classical theory under the same geometric and material parameters.
3. Various geometric and scale parameters had an important influence on the critical flow velocity and voltage of the system. The height $h_{\text{out}}$ of the outer rectangle section changes with the parameter $\alpha$ when the $h_{\text{in}}$ of the inner rectangle section is unchangeable. The critical flow velocity $U_{cr}$ increases with the parameter $\alpha$. However, the critical voltage $V_{cr}$ decreased. Similarly, the parameter $\gamma$ is the ratio of the inner rectangle section width $w_{\text{in}}$ and the outer rectangle section width $w_{\text{out}}$, the effect of which is the same with the parameter $\alpha$. The critical flow velocity and voltage also increased with the size-dependent effect $\ell$.

The authors need point out that the current dynamic model was created without considering geometric nonlinearity. However, this paper focused on the stable analysis of the system, and it was reasonable to adopt the current model. A further discussion about the nonlinear post-instability behavior of the cantilever can be found in the author’s follow-up articles.

ACKNOWLEDGEMENT

The research was partially supported by the National Natural Science Foundation of China (GrantNumbers11872043), and the Opening Project of Sichuan Province University Key Laboratory of Bridge Non-destruction Detecting and Engineering Computing (GrantNumbers2016QZJ03), and Fund Project of Sichuan University of Science and Engineering in hit-haunting...
for talents (GrantNumbers2016RCL31 and 2018RCL11), and Key projects of Department of Education of Sichuan Province (GrantNumbers18ZA0353), and Zigong Science and Technology Program(GrantNumbers2020YJGC03). The authors thank the anonymous reviewers for their helpful suggestions.

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