Numerical and Experimental Modal Analyses of Inclined Supported Bridge Wing Structures

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(Received 25 January 2022; accepted 5 February 2022)

This paper studies ship bridge extensions called "bridge wings", which are one of the most critical areas of importance in ship vibration. These extensions, which have been modeled at the laboratory scale, have been analyzed using both experimental and finite element analysis methods. Bridge wing models have been constructed with varying inclination angles, and the natural frequencies and mode shapes obtained, both experimentally and numerically, have been compared. In this way, the effects of design properties on the results have been emphasized. Besides, the natural frequency discrepancies between the experimental and finite element analyses have been found to be very low. Another important result of this study has been obtained such that a certain support angle has been detected which the mode shape transition occurs in resonance. Natural frequency and mode shape analyses of the plates which have been supported with beams with varying inclination angles and achievement of mode shape transition phenomenon can be considered as the novelty about the contribution of this study to the state-of-the-art.

1. INTRODUCTION

With increasing ship size and speed, shipboard vibration has become a great concern in the design and construction of vessels. Excessive ship vibration is avoided for passenger comfort and crew habitability. In addition to undesired effects on humans, excessive ship vibration may result in fatigue failure of local structural members or malfunction of machinery and equipment. In this study, numerical and experimental modal analyses regarding ship bridge extensions called "bridge wings" were conducted, and the effect of the support angle on the wing models was emphasized.

Frame-type structures are frequently used in the design of bridges, towers, cranes, buildings, ships, aerospace structures and electronic equipment. Several investigators have studied the natural vibrations of frames. The dynamic behavior of frame structures can be predicted using certain analytical and numerical methods. The finite element method (FEM) has been very commonly used in recent years in this field. A detailed literature review of frame structures such as bridge wings is presented below.

Basci et al.[1] improved the accuracy of procedures for generating consistent mass matrices for structural elements, which may then be integrated into a standard finite element program. They achieved this by using exact displacement functions for the elements rather than approximate ones, and they derived these displacement functions by solving the differential equations regulating the free vibration behavior of structural components.

The study conducted by Albarracin and Grossi[2] is about determining the eigenfrequencies of a frame composed of a beam supported by a column and subjected to intermediate elastic constraints. Elastic restraints are used to secure the frame's ends against rotation and translation. The transverse and axial vibration theory of an Euler–Bernoulli beam is considered to regulate the individual members of the frame.

According to Ansell,[3] the dynamic element method (DEM) produces more accurate results than the standard finite element method (FEM) when the same number of degrees of freedom is utilized. This is because polynomial shape functions were introduced, incorporating frequency dependency into mass matrix expressions. This paper illustrates how this impacts free vibration analysis, as well as solutions to nonlinear eigenvalue problems. In addition, various numerical techniques are discussed for resolving these polynomial problems. The polynomial matrix formulations for stiffness and mass matrices are presented for a beam, a bar, and a cable element. Numerical examples show how DEM can be employed in frame-type structures with a comparison of efficiency with standard FEM.

Antes et al.[4] indicate that, since the theory of Tymoshenko, especially on higher frequencies, yields more reliable results than the theory of Euler-Bernoulli, systems of beams, such as frames, should be examined according to this refined theory under arbitrary dynamic excitation. Following the derivation of the basic fundamental solutions for a lateral unit point force and a single unit moment, the deflection and rotation of the beam cross-section are provided as integral equation forms of the governing second-order differential equation system in the Laplace and frequency domains in their study. The study included two examples to illustrate the accuracy of the method compared to standard finite element findings.

Bokaian[5] demonstrated the impact of a constant axial tensile strain on the natural frequencies and mode shapes of a uniform single-span beam subjected to various end circumstances. Numerical measurements in this study reveal that the fluctuation in the normalized natural frequency parameter with the normalized tension parameter is nearly identical for clamped-pinned and pinned-free beams and similarly comparable for clamped-clamped and clamped-sliding beams; only a slight difference exists in the variance in the sliding-free beam between the subsequent pair and free beam. This variation may be accurately described for pinned-pinned, pinned-sliding, and sliding-sliding beams as follows: When the beam vibrates in a third mode or higher, this formula may be utilized for beams with other types of end constraints.

Géradin and Chen[6] introduced a precise and direct technique for modeling beam structures. This technique is based on the use of transfer and dynamic stiffness matrices in conjunction. This technique initially divides the entire structure into substructures depending on the required master degrees of freedom. The global dynamic stiffness matrix (DSM) for each substructure is derived directly by rearranging the appropriate global transfer matrix. Thus, the internal degrees of freedom of each substructure are omitted from the model. The results are compared to those produced using alternative model reduction techniques, such as static and dynamic model reduction, as well as experimental results.

Heppler et al.[7] investigated the dynamics of a two-member open frame structure that traveled in and out-of-plane. The frames are modeled according to the theory of Euler-Bernoulli beams and are generalized by allowing an arrangeable angle between the beams and a payload attachment at the end of the second beam. The motion equations are derived from the Hamilton principle, and the criteria of orthogonality are shown. It has been proven to be possible to decouple the in- and out-of-plane motions by inserting axial deformation elements in the supposed displacement areas.

Howson and Williams[8] introduced a convenient, dynamic stiffness matrix method for determining the natural frequency of a plane frame with certainty in their study. The mass distribution of the constituent members was assumed to be uniform, as were the effects of axial load, rotatory inertia, and shear deflection. The first four in-plane, natural flexural frequencies of an H-shaped frame were investigated experimentally and theoretically for a variety of column lengths and axial loads. Theory and experiment were demonstrated to be reasonably consistent.

Lin and Ro[9] presented a hybrid analytical/numerical method for efficiently analyzing the dynamics of planar serial frame structures, which employs a numerical implementation of the solution to the motion equation via a transfer matrix. Through analyzing the transverse and longitudinal motions of each segment concurrently and taking into account the compatibility requirements for each frame angle, the total number of unspecified variables in the whole frame structure system can be reduced to six, which can then be specified using boundary conditions. The present study aimed to decrease the matrix dimensions associated with the finite element methods along with certain other analytical methods.

Mead[10] investigated natural frequencies and transverse vibration modes for two basic redundant systems with straight uniformed Euler–Bernoulli beams in which internal axial loads are self-balanced (e.g., loads due to non-uniform thermal strains). Computed vibration modes demonstrated that when the axial modes reach their critical values, the buckled beam(s) distort with massive amplitudes, but the unbuckled beam(s) move either as rigid bodies or with bending that deteriorates swiftly from the ends to a near-rigid-body movement over the center part of the beam. These findings explain several computed results relating to the flexural modes and frequencies of flat plates with non-uniform thermal stress distributions.

Utilizing the dynamic ratio coefficient method, Moon and Choi[11] established a free and forced vibration analysis algorithm for frame structures. The principle behind this method was the transfer of the coefficient of dynamic stiffness, which was linked with the force and the displacement vector in each node from the left to the right end of the structure. The numerical findings of the transfer dynamic stiffness coefficient method were compared with the finite element method and experiment results for a space frame structure. The validity and practicality of the dynamic stiffness transmission were validated accurately in the resolution of dynamic problems.

According to Ohga et al.,[12] the combined method for finite element-transfer matrix has the advantage of decreasing the matrix size to less than that achieved through the standard method of finite element. Their study outlined the analytical procedure for bending and buckling problems and suggested techniques for handling the structure with intermediate conditions. Different numerical examples of these problems proved the effectiveness and precision of this method. These examples are consistent with the method of the finite element and others.

Rafezy et al.[13] introduced a global analysis approach to calculate the naturally occurring frequency of asymmetrical, three-dimensional frame structures in which mainframes operate in two orthogonal directions and whose properties are variable at one or more story levels over the height of the structure gradually. A continuum approach was used to formulate the governing differential equations for the substitution system in the form of a basic dynamic members stiffness matrix. In conjunction with the Wittrick–Williams method, the required natural frequencies were finally identified using a gradual cantilever to ensure that no natural frequency was overlooked. The solution can be easily obtained by hand when the structure can be realistically depicted using a uniform cantilever. In order to compare the precision of the current approach with that of complete finite element analysis, a parametric study comprising four asymmetrical three-dimensional frame structures was performed.

A new differential quadrature element method (DQEM) to measure the weighting coefficients of a higher order of derivatives was addressed by Wang et al.[14] Additionally, the application of the DQEM was discussed in the analysis of problems regarding free vibration for beam and beam structures. Based on the innovative approach that applies boundary conditions without using the delta method, the DQEM calculates weighting coefficients. This study demonstrates how the DQEM combined the differential quadrature method's (DQM) rapid convergence and high accuracy with the generality of the finite element method (FEM) for structural analysis.

According to Yuan et al.,[15] when accurate dynamic stiffness matrices are used to compute natural frequencies and vibration modes for skeletal structures and some other structures, a complex transcendental eigenvalue problem arises. This study proposes a novel, mathematically elegant, and computationally efficient method for computing natural frequencies and vibration modes with high accuracy and reliability. First of all, the transcendental problem is lowered to a generalized linear problem using Newton's method at the exact natural frequency determined with the algorithm of Wittrick–Williams. Then, using a conventional inverse iteration or subspace iteration method, the generalized linear eigenvalue problem is effectively solved. Numeric examples, including certain challenging circumstances (e.g., with coincident, natural frequencies, rigid body motions, and large structures), reveal the outstanding performance of this method.

In order to eliminate detrimental vibrations generated by the main engine or propellers, long bridge wings that extend over deckhouses are typically reinforced with bracing connected to the upper decks, among others. Noguchi et al.[16] optimized the arrangement of the bracing structurally using a genetic algorithm by examining the natural frequencies of the wing structures. This structure maintains the optimal weight distribution and serves as a guide for anti-vibration structural design. The structural arrangement is explored in comparison to the planned structures that were previously described in this study.

It is critical to understand the dynamic properties of beam structures in order to design them properly. Ratazzi et al.[17] investigated the free in-plane vibrations of a system composed of two orthogonal beam members connected via an internal elastic hinge. At one end, this system is clamped, while the other is elastically connected. The vibrations of the elastically connected end were investigated under various boundary conditions, including classical conditions such as clamped, simply supported, and free. According to the Bernoulli-Euler theory, the beam system was assumed to behave. Hamilton's principle is used to determine the governing equations of motion for this structural system incorporating free bending vibration. Natural frequencies are accurately defined utilizing the calculus of variations technique and the method of separation of variable s. Special consideration was made during the frequency analysis of the elastic hinge's flexibility and placement. The results were remarkably close to those produced using the finite element method, employing published values for specific cases in the model and measurements in an experimental device.

Grossi and Albarracin[18] discussed the application of the calculus of variations to derive the boundary and eigenvalue problems that characterize the dynamic behavior of frame structures with elastically restricted ends and intermediate points and precisely specified eigenfrequencies. Additionally, the study introduced appropriate functionals along with a thorough description of the related domains.

Siddika et al.[19] used ANSYS software and shaking tables to conduct an analysis of free vibration and investigate the behavior of framed structures at various vibrational frequencies. In the laboratory, a small-scale uniaxial shaking table was developed that produced low to moderate vibrations in terms of frequency and velocity. Moment resisting framed structures were tested on a shaking table and analyzed using the ANSYS program. They were constructed with connecting beams and column elements of mild steel wire in various dimensions. Additionally, the effect of the mass and stiffness of the structures on their natural frequency and deflection in response to various ground vibrations were researched and analyzed. The results of the tests revealed that this shaking table complied with the overall principle of free vibration. For structures with the same lateral stiffness, the height had an inverse effect on the natural frequency. After a few shaking cycles, the structure's inherent frequency began to diminish as the stiffness decreased. As a result, it was determined that the constructed shaking table was suitable for free vibration analysis.

According to Daneshmand et al.,[20] vibration analysis of complex structures or problems requiring a large number of modes requires either fine meshing or the use of higher-order polynomials as shape functions in conventional finite element analysis. Due to the difficulty of predicting the vibration mode of a complex structure a priori, a uniform fine mesh is frequently used, wasting several degrees of freedom investigating various local modes. The proposed wavelet element approach allows for structural vibration analysis using a coarse mesh first and then adaptively improving the results through multi-level refinement of the model's necessary sections. This approach yields accurate data while requiring fewer degrees of freedom and computation. For frame structure vibration analysis, the B-spline wavelet scaling functions on the interval (BSWI) were used as trial functions. These functions combine the versatility of the finite element method with the accuracy of the B-spline function approximation, as well as the multiresolution wavelet strategy. Rather than using typical polynomial interpolation, shape functions and wavelet-based elements were formed using scaling functions at a certain scale. The vibrations of cantilever beams and plane structures were investigated in this paper to validate the proposed method. The analyses and findings illustrated both the multi-level procedure and local wavelet enhancement. Additionally, the results were consistent with those obtained using the standard finite element method and analytical solutions.

Mehmood[21] sought to comprehend the dynamic behavior of frames subjected to moving point loads. The vibration analysis was carried out using the finite element method and numerical time integration method (new mark method). The effect of the velocity of the moving load on the dynamic magnification factor was described as the ratio of the highest dynamic displacement at the corresponding node over time to the static displacement at the structure's midpoint. Additionally, the impact of the spring stiffness attached to the frame at the beam and column intersection points was examined.

Rezaiee-Pajand et al.[22] used the differential transform method to perform free in-plane vibration analysis on a frame with four arbitrarily inclined members. The related structural eigenvalue problem was analytically formulated using four dif-

Figure 1. A crude oil tanker with large bridge wings.

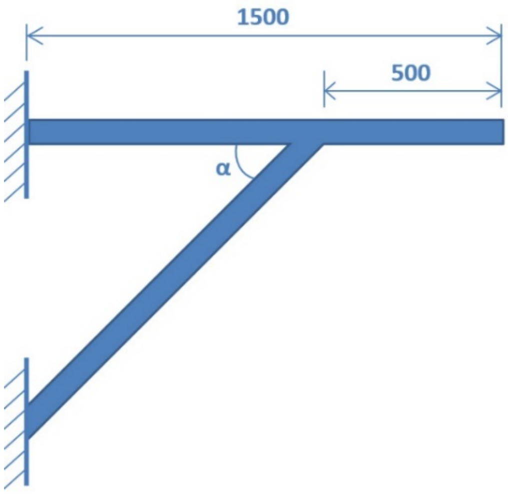
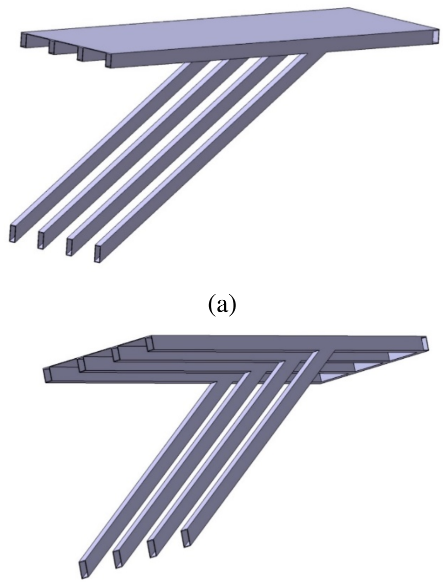


Figure 2. Bridge wing model (Dimensions in millimeters).



(a)

(b)

Figure 3. Solid model of the 35-degree supported bridge wing.

ferential equations and sixteen boundary and compatibility criteria. The frequency parameters and mode forms of the frame were estimated for a range of structural variables, including joint angles, spring stiffness, and member flexural rigidity. Finally, the authors' finite element program was used to validate the solution acquired by the suggested method.

According to Inoue and Sueoka,[23] the transfer influence coefficient method is a beneficial tool for analyzing the dynamic response of a system with various degrees of freedom. The transfer influence coefficient method addressed several of the transfer matrix method's drawbacks and displayed certain advantages in terms of computational correctness and speed. However, this method could not be used to handle truss and rahmen structures since the algorithm is incapable of modeling the branches and links that make up a closed-loop in truss and rahmen structures. This study presents a novel algorithm using the transfer influence coefficient method to perform an in-plane free vibration analysis of frame structures with closed loops. The new algorithm maintains the original algorithm's computational accuracy and speed advantages. The advantages are highlighted through a comparison of a free vibration analysis to a standard procedure.

In the study of Civalek and Avcar,[24] the free vibration and buckling analyses have been conducted for functionally graded carbon nanotube-reinforced (FG-CNTR) laminated non-rectangular plates, i.e., quadrilateral and skew plates, us-

Table 1. Natural frequencies of bridge wing models obtained using FEM.

Mode Type	Natural Frequency (Hz)								
	20^0	25^0	30^0	35^0	40^0	45^0	50^0	55^0	60^0
First Flexural	93.90	97.92	99.32	99.40	98.66	96.66	93.06	85.50	71.36
First Torsional	113.22	117.61	117.04	112.12	102.90	88.60	74.04	59.78	45.96
Second Flexural	173.49	183.41	187.69	185.24	174.38	156.04	135.91	118.51	107.41
Second Torsional	199.08	220.58	223.73	215.78	204.98	192.54	184.16	178.13	170.51

Table 2. Regression constants for natural frequency functions.

Mode Type	a	b	c	d	e
First Flexural	-0.000053	0.007674	-0.429505	10.893597	-5.081212
First Torsional	0.000028	-0.002910	0.012348	3.501334	56.980303
Second Flexural	0.000113	-0.014997	0.583110	-6.567252	173.341818
Second Torsional	-0.000150	0.028118	-1.926864	55.103077	-333.343636

ing a four-nodded straight-sided transformation method. At first, the related equations of motion and buckling of quadrilateral plate have been given, and then, these equations are transformed from the irregular physical domain into a square computational domain using the geometric transformation formulation via discrete singular convolution (DSC). Detailed numerical solutions are performed, and obtained parametric results are presented to show the effects of carbon nanotube (CNT) volume fraction, CNT distribution pattern, geometry of skew and quadrilateral plate, lamination layup, skew and corner angle, thickness-to-length ratio on the vibration, and buckling analyses of FG-CNTR-laminated composite non-rectangular plates with different boundary conditions. Some detailed results related to critical buckling and frequency of FG-CNTR non-rectangular plates have been reported which can serve as benchmark solutions for future investigations.

In the study by Hadji et al.,[25] an analytical solution for the free vibration of nanoplates made of functionally graded materials (FGMs) under various boundary conditions is provided. In this context, a new refined plate theory with four variables based on the theory of non-local elasticity including the small-scale influences is adopted. Using the rule of mixture, the material properties of nanoplates are supposed to vary continuously across the thickness direction. Based on Eringen's non-local elasticity theory, the equations of motion of functionally graded (FG) nanoplate are derived using Hamilton's principle, and the obtained equations are solved analytically.

This study which focuses on ship bridge extensions called "bridge wings", which are one of the most critical areas of importance in ship vibration, have been modeled at the laboratory scale and been analyzed using both experimental and finite element analysis methods. Bridge wing models have been constructed with varying inclination angles, and the natural frequencies and mode shapes have been obtained and compared. The effects of design properties on the results have been examined, as well. Another important aim of the study can be stated such that there can be found a certain support angle which the mode shape transition occurs. Natural frequency and mode shape analyses of the plates which have been supported with beams with varying inclination angles and achievement of mode shape transition phenomenon can be considered as the novelty of this study.

2. BRIDGE WING MODELS

Bridge extensions called "bridge wings" lie on both sides of ship bridges when the deck breadths are very high. This kind of bridge wing can commonly be found on crude oil tankers and large container ships, as shown in Fig. 1. These structures allow ship personnel to see the sides of the ship during maneuvers and to locate special navigation equipment called repeaters. Large bridge wing structures are one of most critical parts of a ship when considering vibration.

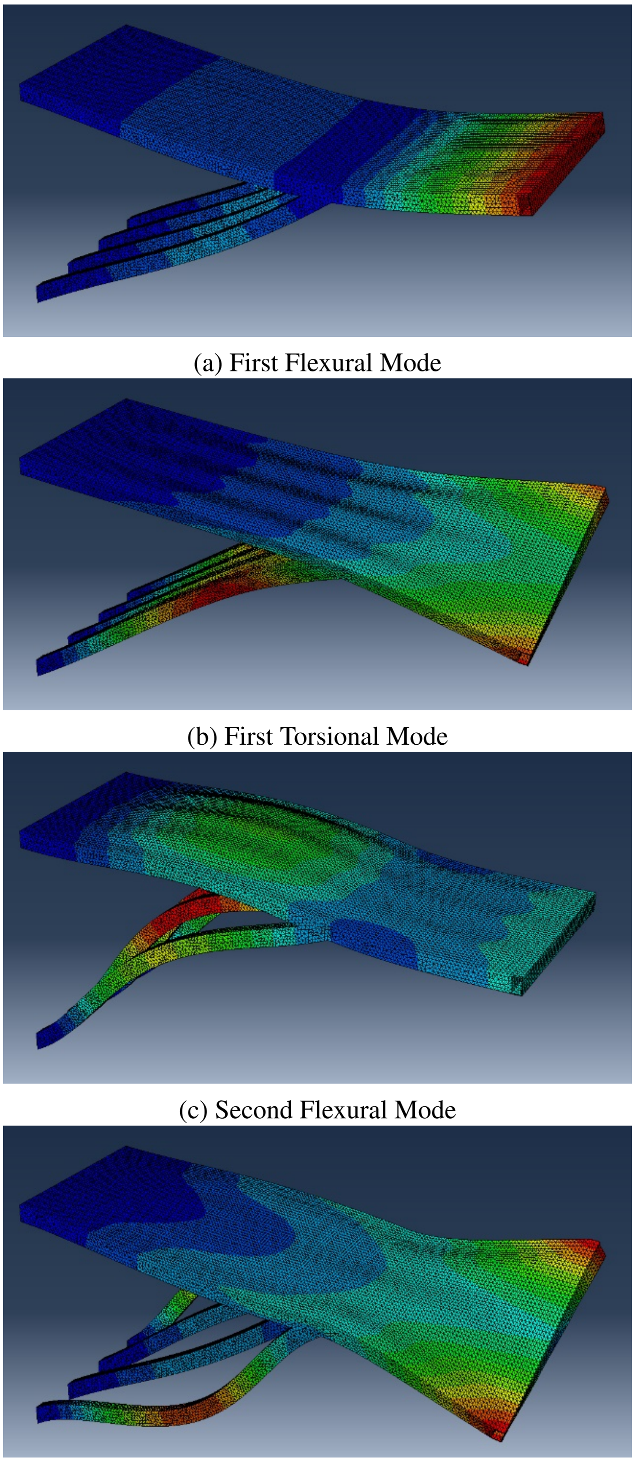
Since the length of the bridge wing primarily depends on the deck and bridge breadths, there is no flexibility in the design of the length of the bridge wing. To achieve stability with regard to vibration, these structures use transversal and perpendicular supports. In this study, laboratory-scale bridge wings were modeled as a composition of frame and plane elements with a certain constant length and various support angles. The side view of the model is shown in Fig. 2.

The three-dimensional solid models were created using the CATIA modeling software, which is a commercial design and modeling software. Nine different models were created with support angles (α) of 20, 25, 30, 35, 40, 45, 50, 55 and 60 degrees. The realistic three-dimensional solid model of the 35-degree supported bridge wing model is shown in Fig. 3.

Four horizontal (top) and 4 support girders were used to construct the model. The breadth of the model is 500 mm, and the girders are equally spaced across that breadth. The girders are 50x30x2 rectangular profiles. The top horizontal girders are covered with a plate with a thickness of 2 mm. There is an extra transversal beam that connects the top horizontal girders at the free end of the model.

3. FINITE ELEMENT ANALYSES

In this section, the finite element method (FEM) is used as the numerical modal analysis method for inclined supported frame structures. The commercial finite element analysis software called ABAQUS was used for meshing, natural frequency calculations (eigenvalue extraction) and mode shape determi-



(a) First Flexural Mode

(b) First Torsional Mode

(c) Second Flexural Mode

(d) Second Torsional Mode

Figure 4. Mode shapes of the 35-degree supported bridge wing model (FEM).

nations. Fig. 5 shows the mesh model of the 35-degree supported bridge wing frame structure.

In the mesh models for all 9 structures, both tetrahedral and hexahedral three-dimensional elements with an average size of 10 millimeters were used. The mapped meshing algorithm was applied as much as possible. The density, modulus of elasticity and Poisson ratio were calculated to be 7850 kg/m^3, 200000 MPa and 0.3, respectively. The first four natural frequencies with respect to mode type for all models are shown in Table 1. Additionally, the corresponding mode shapes are shown in

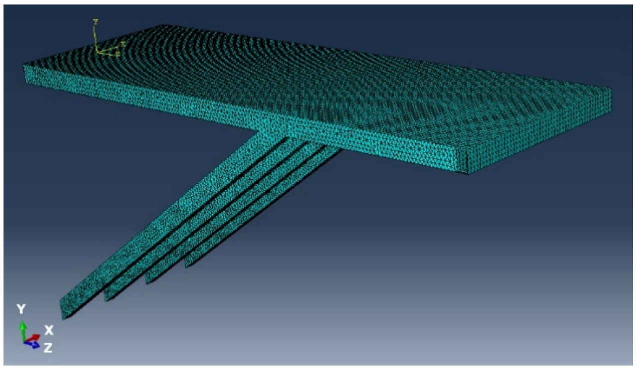


Figure 5. Mesh model of the 35-degree supported bridge wing frame structure.

Figure 6. A photograph showing the modal testing on the structure.

Fig. 4.

By analyzing the data in Table 1, which lists the first four natural frequencies obtained by the FEM and with respect to the bridge wing models' mode type, we determined a 4th degree polynomial relation between the support angles and the natural frequencies. Then, by conducting regressions over the first four modes, we derived Eq. (1) in which f stands for natural frequency (Hz) and α stands for support angle (degrees). The regression constants for the natural frequency function are given in Table 2.

$$f = a.\alpha^4 + b.\alpha^3 + c.\alpha^2 + d.\alpha + e \quad (1)$$

The plots of these regression functions are shown in Fig. 7.

The finite element method was applied to all types of bridge wing models, and the same mode shapes with respect to mode type were obtained in the same order, except for the order change between the first flexural and first torsional modes after the 40-degree supported model, which can be clearly seen in Fig. 7. Until the 45-degree supported model was simulated, the first flexural modes were always observed before the first torsional modes. However, starting with the 45-degree supported model, the order of these modes switched such that the first torsional modes were observed before the first flexural modes. The 42-degree support angle was detected as the support angle at which the mode shape transition begins.

4. EXPERIMENTAL MODAL ANALYSES

Experimental Modal Analysis (EMA) is an effective instrument for describing, understanding and modeling a structure's

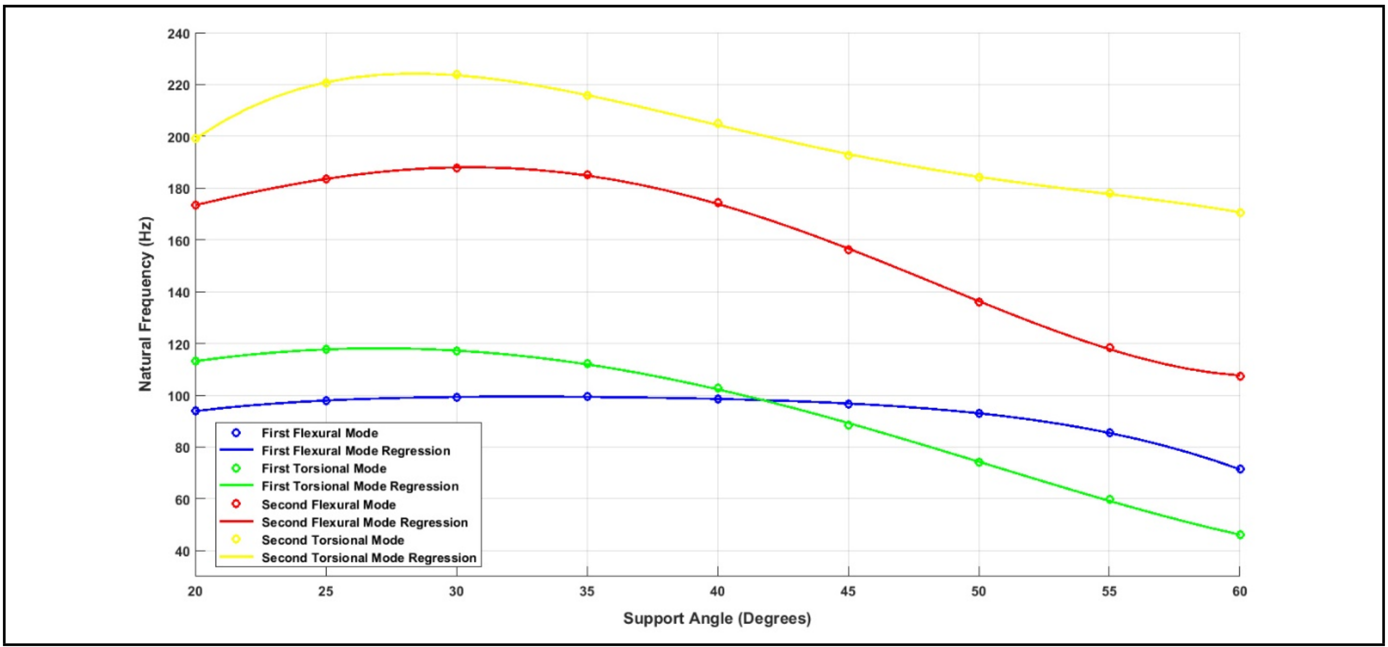


Figure 7. Natural frequency regression plots.

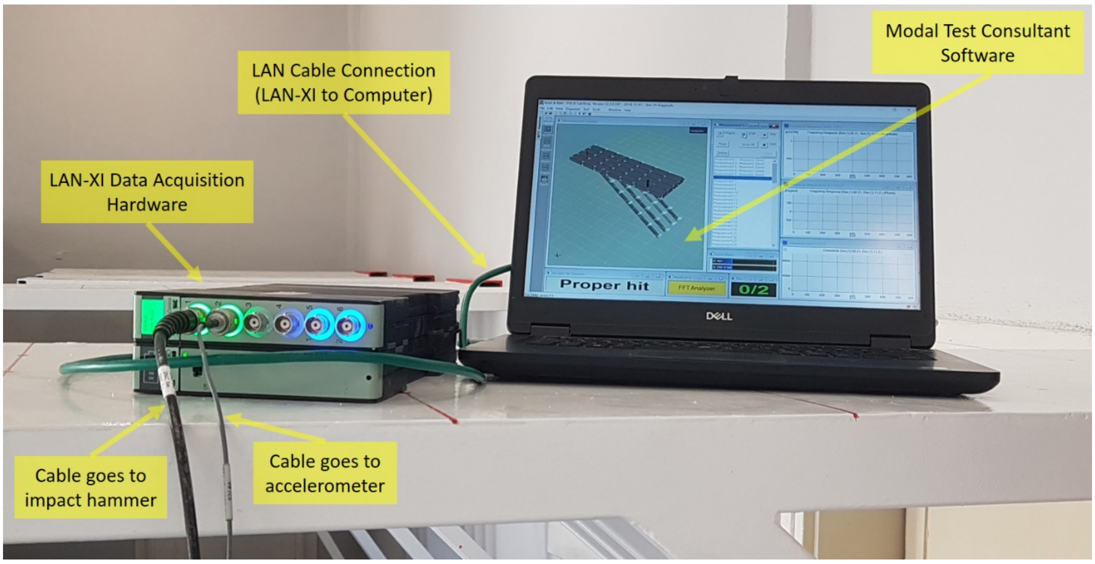


Figure 8. Data acquisition system.

dynamic behavior. It is able to a structure's natural frequencies as well as their mode shapes and can also verify accuracy and calibrate the finite element model. Scaled modal models require a precise force measurement. This can be achieved with electrodynamic and servohydraulic exciters controlled by a signal generator via a power amplifier. A more convenient and economical excitation method is a hammer fitted with a high-quality piezoelectric force transducer. In applications where a high crest factor and a limited ability to shape the input force spectrum are of no concern, impact hammer testing is an ideal source of excitation. Impact hammers are highly portable for field work and provide no unwanted mass loading to the structure during testing.

The roving hammer impact test method was selected as the experimental modal analysis technique. To implement this technique, the accelerometer was fixed to a point over the mesh on the top plate of the structure, and the impact hammer was applied to each mesh point (nodal point) on the structure's elements. The impact forces from the hammer and the responses from the sensor (accelerometer) were collected by a data recorder and analyzer (B&K PULSE LAN-XI Data Acquisition Hardware and Modal Test Consultant Software). An image of the running data recorder and analyzer system connected to the computer captured during the experiment is shown in Fig. 8. Then, using these data, the Frequency Response Functions (FRFs) were obtained. Using modal analysis software (B&K Connect) and FRFs obtained from the recorder, we calculated the structures' natural frequencies and mode shapes. The impact hammer (Endevco 2302-10) had a sensitivity of 2.27 mV/N with a head mass of 100 grams. The impact tip was an aluminum tip with a diameter of 6.4 millimeters. A miniature piezoelectric uniaxial CCLD accelerometer

Table 3. Natural frequencies of bridge wing models obtained with EMA.

Mode Type	Natural Frequency (Hz)								
	20^0	25^0	30^0	35^0	40^0	45^0	50^0	55^0	60^0
First Flexural	87.28	89.10	92.34	92.63	91.84	86.43	88.96	78.97	63.79
First Torsional	108.39	111.34	111.32	105.72	95.54	82.91	69.48	55.83	43.04
Second Flexural	160.40	165.08	170.07	170.63	156.09	140.31	127.83	109.38	96.69
Second Torsional	184.35	204.41	212.91	197.80	188.01	181.54	166.62	170.18	155.68

Table 4. Natural frequencies of bridge wing models obtained with FEM and EMA with discrepancies.

Mode Type	Natural Frequency (Hz)								
	20^0	25^0	30^0	35^0	40^0	45^0	50^0	55^0	60^0
First Flexural (FEM)	93.90	97.92	99.32	99.40	98.66	96.66	93.06	85.50	71.36
First Flexural (EMA)	87.28	89.10	92.34	92.63	91.84	86.43	88.96	78.97	63.79
Discrepancy (%)	7.58	9.90	7.56	7.31	7.43	11.84	4.61	8.27	11.87
First Torsional (FEM)	113.22	117.61	117.04	112.12	102.90	88.60	74.04	59.78	45.96
First Torsional (EMA)	108.39	111.34	111.32	105.72	95.54	82.91	69.48	55.83	43.04
Discrepancy (%)	4.46	5.63	5.14	6.05	7.70	6.86	6.56	7.08	6.78
Second Flexural (FEM)	173.49	183.41	187.69	185.24	174.38	156.04	135.91	118.51	107.41
Second Flexural (EMA)	160.40	165.08	170.07	170.63	156.09	140.31	127.83	109.38	96.69
Discrepancy (%)	8.16	11.10	10.36	8.56	11.72	11.21	6.32	8.35	11.09
Second Torsional (FEM)	199.08	220.58	223.73	215.78	204.98	192.54	184.16	178.13	170.51
Second Torsional (EMA)	184.35	204.41	212.91	197.80	188.01	181.54	166.62	170.18	155.68
Discrepancy (%)	7.99	7.91	5.08	9.09	9.03	6.06	10.53	4.67	9.53

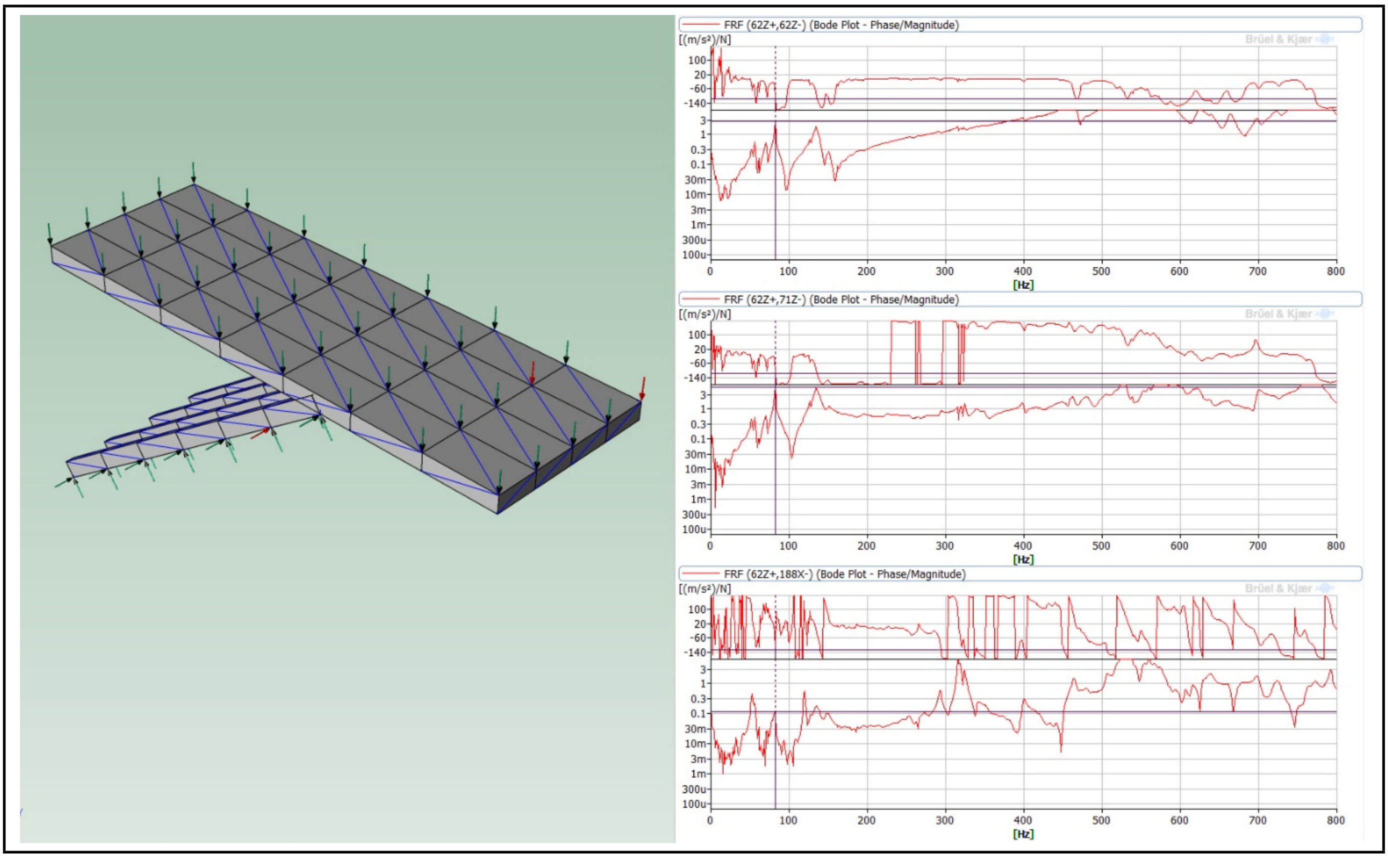


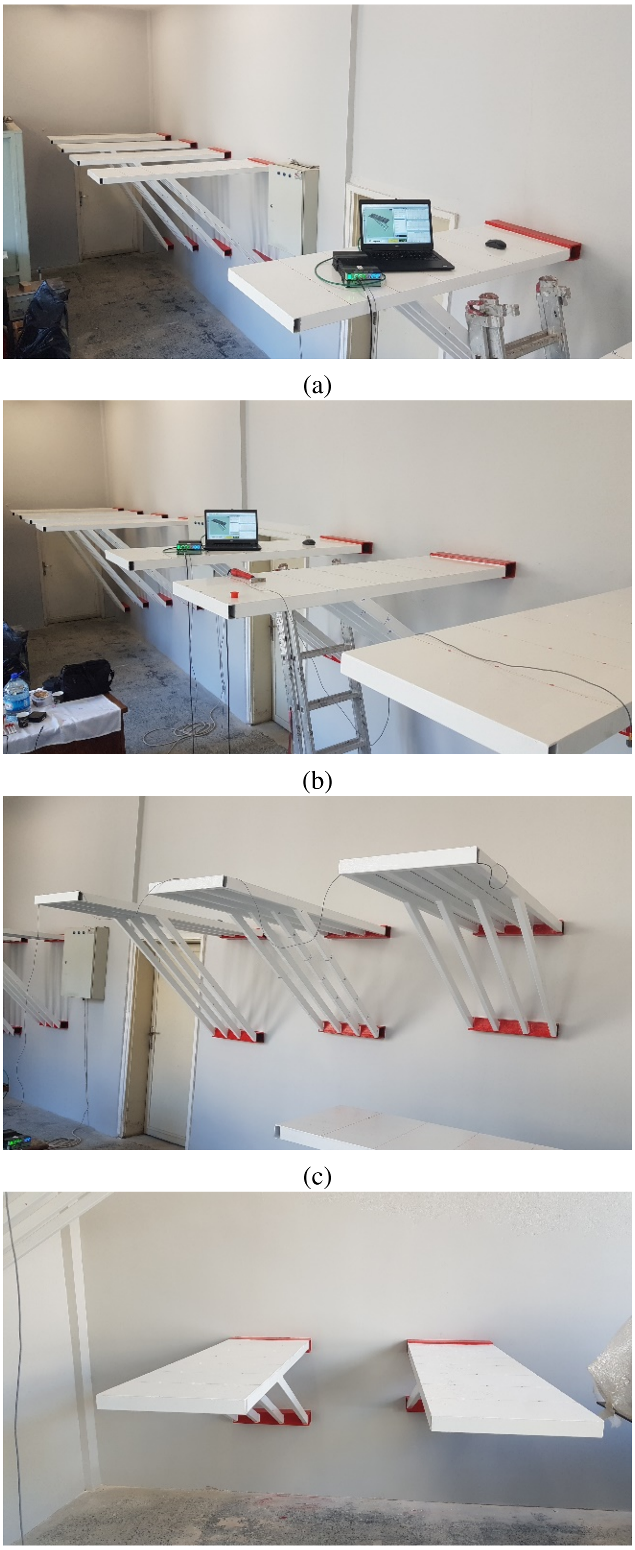
Figure 9. Data for the selected nodal points.

(B&K 4507) with a sensitivity of 100 mV/g was used to collect the time data. The accelerometer had a frequency range of 0.3-6000 Hz and a weight of 4.8 grams. The system's sampling frequency was 2048 Hz.

All 9 bridge wing models examined in the finite element analyses section were constructed at the laboratory scale with the same dimensions and clamped to the wall of our laboratory side by side (the top plates of the bridge wings were on the same level), as shown in Fig. 10.

The mesh, with hammer hitting and sensor nodal points, is shown in Fig. 11. The sensor is located on nodal point 62 (in the red circle).

A screen from the B&K Modal Test Consultant software taken while conducting the experiments is shown in Fig. 9. The



(a)

(b)

(c)

(d)

Figure 10. All models constructed for experimental modal analyses.

phase and FRFs of the selected (red nodal hit arrows) nodes are also shown in Fig. 9.

Table 3 presents the first four natural frequencies obtained with EMA with respect to mode type for all models. The corresponding mode shapes are also shown in Fig. 12.

As seen in Fig. 12, the same mode shapes were obtained with experimental modal analysis for the selected model (35

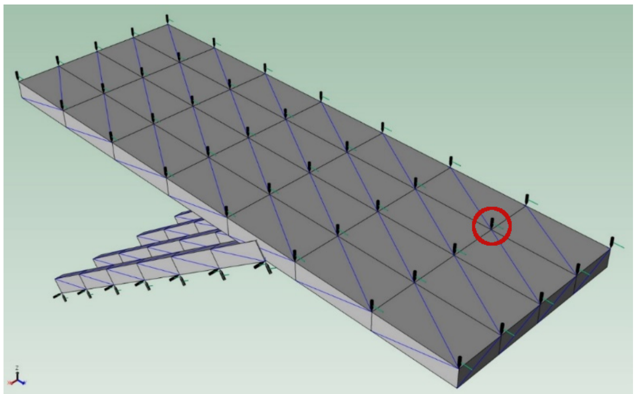


Figure 11. EMA mesh (hammer points) for the 35-degree supported bridge wing construction.

degrees) with respect to FEM. In Table 4, the overall frequency values obtained from finite element and experimental modal analyses for all models are listed, and the discrepancies are also included. The discrepancy formula is given by Eq. (2).

$$\text{Discrepancy }(\%) = \frac{FEM - EMA}{\text{EMA}} 100 \qquad (2)$$

5. CONCLUSION

Examining both the finite element and experimental modal analyses yields the following conclusions.

Table 4 presents the overall frequency values obtained by finite element and experimental modal analyses for all models, including discrepancies. It is noted that the experimental results are always smaller than those obtained with the finite element method. The maximum discrepancy between FEM and EMA is approximately 12%. There are several explanations for this. In EMA,

- The constructions did not have the same (ideal) material mechanical properties as in the FEM.
- The constructions were heavily coated and painted.
- The constructions were not theoretically clamped as in FEM.
- The weldings did not ideally mimic the surface contacts in the FEM.

In addition, as seen in Fig. 7, the natural frequencies decrease as the support angle increases. In the FEM and EMA, the same mode shapes were obtained in the same order except for the order change between the first flexural and first torsional modes after the 40-degree supported model, which can be clearly seen in Fig. 7. Until the 45-degree supported model was simulated, the first flexural modes were always observed before the first torsional modes. However, starting from the 45-degree supported model, the order of these modes switched such that the first torsional modes were observed before the first flexural modes. A mode shape transition phenomenon was also detected in the study.

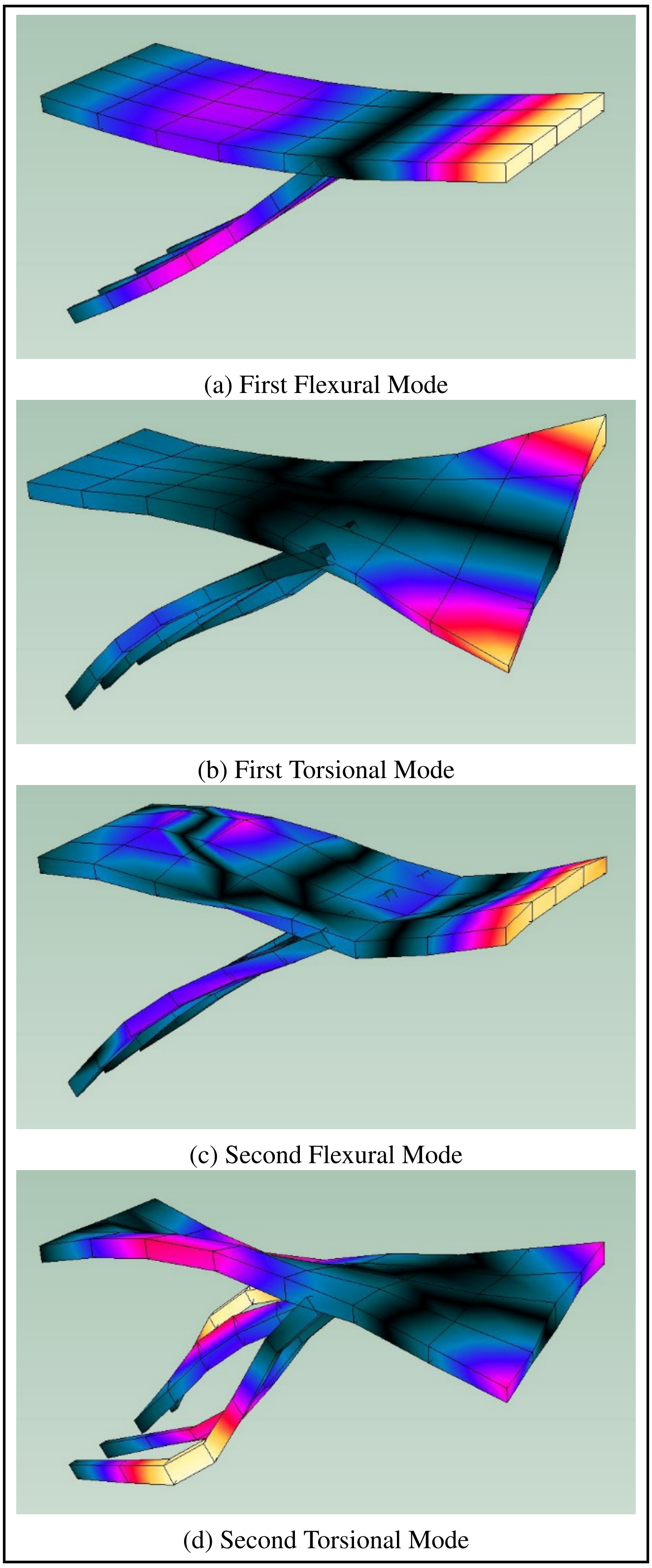


Figure 12. Mode shapes of the 35-degree supported bridge wing model (EMA).

As the results of the comparison between the finite element values and the experimental values demonstrate, the correlation between the FEM and EMA has been validated. In future work, the bridge wing width and length can be varied, and a single formula representing the natural frequency can be constructed to also consider the bridge wing width and length.

6. ACKNOWLEDGMENTS

We would like to express our special thanks to Electronics Engineer M.Sc. Alper Akgül from B&K Turkey for his support during experimental analyses and device setup. We would also like to thank to Ocean Going Chief Engineer İlker Meşe and Ocean Going Master Engin Yücel for advisory support.

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