Sound Propagation in Swirling Flows[1]

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The propagation of small-amplitude disturbances in an annular duct with mean swirling flow is studied. A normal mode analysis leads to an eigenvalue problem which is *not* of the Sturm-Liouville type; the eigenmodes are not orthogonal and do not form a complete set. The analysis reveals the existence of nearly-sonic and nearly-convected modes. Acoustic radiation in the duct is dominated by the nearly-sonic modes and changes significantly in presence of the mean swirl which introduces a swirl-induced Doppler shift of acoustic cut-on frequencies, refraction of propagating pressure modes due to the mean flow non-uniformity, and acoustic-vorticity coupling produced by Coriolis forces in a vortical swirling flow. The nearly-convected eigenvalues are vorticity-dominated, and accumulate at the borders of the convected critical layer which forms for a vortical mean swirl. Two numerical methods are used to solve the eigenvalue problem. One is a finite-difference method, the other is a pseudo-spectral collocation method. The accuracy of the finite-difference approximation is not good enough near the critical layer, but the pseudo-spectral method gives significantly more accurate results.

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1. INTRODUCTION

The propagation of acoustic waves in axisymmetric flows has been extensively studied as a result of the emergence of engine noise as a major technological problem. In the 70s, the emphasis was on the effect of shear layers next to the walls, and acoustic properties of duct lining materials. A detailed review of the subject was given by Nayfeh et al.[1] These studies point out the importance of the mean flow gradients and its refraction effects on the sound waves. In addition, the eigenvalue problems associated with these waves are often not of the Sturm-Liouville type. Thus the eigenfunctions may not be orthogonal and may not form a complete system.

Swirling non-axisymmetric flows appear naturally in a number of applications. External geophysical flows are typically characterized by small swirl velocities and Rossby numbers, and have been extensively studied in the past (e.g.[2]). On the contrary, internal ducted flows with swirl, such as encountered in turbomachinery and jet flow applications, have not yet gained sufficient attention. Such flows may possess mean swirl components of the same order as axial velocities.

Kerrebrock [3] was the first to carry out a normal mode analysis and to examine the effects of a mean flow swirl on the propagation of wave disturbances in a duct. In his analysis, the mean flow was represented as a combination of a uniform axial flow, a solid body rotation, and a free vortex. This mean flow model, which is also used in the present work, gives a good approximation of turbomachinery flows, and also allows a parametric study of the effect of different kinds of swirl on the propagation of wave disturbances.

Kerrebrock showed that vorticity, pressure, and entropy disturbances are *not independent* in swirling flows. Because of the mean flow swirl, a radial force imbalance due to the presence of Coriolis and centrifugal forces results in a coupling between the various flow perturbation modes. Moreover, shear disturbances are not purely convected by the mean flow but rather exhibit an oscillatory behavior. Such *nearly-convected* disturbances carry a weak pressure component with them. It also follows from Kerrebrock's analysis that modes from a *nearly-sonic* part of eigenspectrum are *not* strongly influenced by a swirling motion. We shall see, however, that the acoustical cutoff conditions appear to be very sensitive to the amount and nature of the swirl. Due to the Doppler effect in a swirling mean motion, the nearly-sonic eigenvalues split in two branches corresponding to the modes rotating in the direction or opposite to the direction of the mean swirl. More propagating acoustic modes spin opposite to the mean swirl direction. In the present work, we also give a detailed illustration of the effects of refraction and acoustic-vorticity coupling on the propagation of pressure-dominated nearly-sonic waves in a duct with mean swirl.

For the purpose of analysis, the total unsteady velocity field is split into a nearly-convected part and a nearly-sonic part, obeying weakly coupled equations. This splitting has the advantage of elucidating the physical phenomena which involve two types of waves, vorticity-dominated and pressure-dominated, and thus paves the way for generalizing the classical definition of a gust in vortical swirling flow. The velocity decomposition will also indicate the degree of interaction between the various modes.

The mathematical formulation leads to a system of equations for the normal modes. The resulting eigenvalue problem is, however, not of the Sturm-Liouville type. This is similar to the case of axisymmetric sheared flows studied first by Pridmore-Brown[4]. Using asymptotic expansions, Nayfeh and Telionis[5] showed that algebraically growing modes may exist in *critical layers* when the Reynolds number $Re \to \infty$. The critical layers are formed in the flow regions where the wave speed equals the fluid speed. Within these regions, the inviscid initial value problem may have algebraically growing travelling waves, and the corresponding normal mode formulation may have a singular solution. Similarly, in the present problem we expect singular critical layers to form due to the presence of vortical mean swirl. A special numerical technique should then be used to properly