Force Measurement in Vibration Testing — Applications to Modal Identification

Ulrich Füllekrug and Michael Sinapius

German Aerospace Center, Institute of Aeroelasticity, Bunsenstrasse 10, Göttingen, Germany

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This paper first concentrates on the fundamentals of the vibration behaviour of a structure under base excitation. The equations for the structural accelerations and the dynamic forces at the interface are presented and discussed. The principles of modal identification for structures under base excitation including the determination of the effective and modal masses are explained. Then the practical measurement of the interface forces, consisting of the resultant six-degree-of-freedom components, is described and the utilised force measurement devices are presented. The application of the modal identification method is demonstrated on a laboratory test structure as well as on a more complex structure. Examples for typical measurement data and the extracted modal parameters are shown.

1. INTRODUCTION

The measurement of forces in vibration testing at the interface between the shaking table and the structure offers a very promising possibility for the application of new methods. On the one hand, the availability of interface forces enables the application of new test specifications which lead to more realistic simulations of the dynamic environment.[1,2] On the other hand, the modal identification of structures undergoing base excitation can be enhanced. A knowledge of the modal properties is essential for the validation of analytical dynamic models and can clearly improve the appropriate design of the structures investigated.

For the purpose of modal identification, the test structure has to be mounted on a multi-axial shaker table and accelerated in all spatial degrees of freedom. Both the multi-axial base acceleration and the structural responses evoked by the base excitation are the basis for the identification of the eigenfrequencies, modal damping values, and mode shapes. If the interface forces between the vibration table and the structure tested are measured in addition, it is possible to determine the effective masses and generalised masses of the modes identified.

2. THEORY

Figure 1 shows a base-accelerated structure S mounted on a force measurement device which is fixed on the vibration table of a multi-axial vibration simulator. In the figure, the deformed structure is shown as a solid line, and the undeformed structure is indicated by a dashed line. The upper part of the force measurement device forms the base B. The centre of the base B represents a point at which the table accelerations and the interface forces can be related.

In order to measure its accelerations the structure is equipped with a sufficient number of accelerometers. In general cases the kinematic relations are quite complex. However, for most practical cases, a linearised kinematic relation between the measured absolute accelerations $\{\ddot{u}\}$ and the relative accelerations $\{\ddot{v}\}$

$$\{\ddot{u}\} = \{\ddot{v}\} + [G]\{\ddot{u}_0\} \tag{1}$$

can be used.[3] The columns of the matrix $[G]$ are the rigid body movements of the structure related to the respective translational and rotational accelerations $\{\ddot{u}_0\}$ of the base. Following the detailed derivation of reference[3], a relation between the structural responses $\{\ddot{v}\}$ and the base acceleration $\ddot{u}_{ok}$ of the axis k can be formulated in the frequency domain as

$$\{\ddot{v}(\omega)\} = \sum_{r=1}^{n} \{\psi\}_r \frac{\omega^2}{\omega_r^2 - \omega^2 + 2\zeta_r \omega_r \omega i} \sqrt{\frac{\mu_{rk}}{m_r}} \cdot \ddot{u}_{ok}(\omega). \tag{2}$$

Thus the structural responses are determined by the modal quantities, i.e., mode shape $\{\psi\}_r$, eigenfrequency ω_r, damping value ζ_r, modal mass m_r and effective mass μ_{rk}. An effective mass μ_{rk} exists for each base excitation axis k and each mode r. It can be interpreted as an equivalent physical mass of the r-th modal degree of freedom.[4] Eq. (2) also reveals the degree of controllability of the mode shape $\{\psi\}_r$ by means of the base acceleration. A large effective mass leads to a significant participation of the mode shape in the structural responses and usually enables good identification. However, the identification of modes with small effective masses may become difficult.

The input-output relationship of Eq. (2) can be rewritten in the more general formulation

$$\{\ddot{v}(\omega)\} = [H(\omega)] \cdot \{\ddot{u}_0(\omega)\}, \tag{3}$$

where the rectangular frequency response function (FRF) matrix $[H(\omega)]$ is composed of modal parameters.

2.1. Dynamic Forces at the Interface

The resulting six dynamic forces and moments at the interface $\{f_0\}$ can be related to the six translational and rotational accelerations of the base $\{\ddot{u}_0\}$. In the derivation of the equations it can be assumed that the upper part of the force measurement device (i.e. the base B, see Fig. 1) is rigid. Following the derivation of reference[5] and some assumptions used there, the following equation can be formulated

$$f_{0k}(\omega) = \left(\sum_{r=1}^{n} \frac{\omega^2}{\omega_r^2 - \omega^2 + 2\zeta_r \omega_r \omega\, i} \cdot \mu_{rk} + M_{Sk} + M_{Bk} \right) \cdot \ddot{u}_{ok}(\omega). \tag{4}$$