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# Resonance with Respect to Angular Positions of an Unbalance of a Cracked Rotor in a Nonlinear Rotor System

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Cracked rotors have nonlinear spring characteristics of a piecewise linear type due to an open-closed mechanism of cracks. There have been many studies on the dynamic behaviour of cracked rotors in order to develop fault diagnosis systems for detecting cracks. However, most of these studies concern the change in resonance phenomena mainly due to cracks in rotor systems with linear supports. In many practical rotor systems, various kinds of nonlinear spring characteristics may exist due to mechanical elements, such as nonlinear bearing supports, and the rotor systems become nonlinear. Therefore, existing fault diagnosis systems are unable to detect cracks in such nonlinear rotor systems which are supported by nonlinear bearings. In this paper, we study the vibrational behaviour of a cracked rotor with nonlinear bearing supports and focus on the effect of combined resonances caused by a crack and nonlinear bearing supports. In particular, we investigate in detail the resonance phenomena of a harmonic resonance and a 1/2-order subharmonic resonance by numerical simulations using a PWL model and theoretical solutions using a PS model. The results show that the dynamic behaviours of a cracked rotor in a nonlinear rotor system are obviously different from those in a rotor system with linear supports, and the change in vibrational behaviour of a harmonic resonance and a 1/2-order subharmonic resonance are significant due to changes of angular positions of an unbalance.

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## 1. INTRODUCTION

Transverse cracks due to fatigue in rotor shafts are one of the most serious causes of accidents in rotating machinery. A detailed investigation into the vibrations of cracked rotors is very important for developing a fault diagnosis system and detecting rotor cracks. In order to find a method for detecting cracks in rotating shafts, the dynamic behaviour of cracked rotors has been studied since the middle of the 1970s. Most of the early research results were well summarised in a book by Dimarogonas and Pafelias<sup>1</sup> and more literature on the dynamics of cracked rotor systems was presented in a review paper by Wauer<sup>2</sup>, and more recently, a survey on simple cracked rotors and a review on general cracked structures were given by Gasch<sup>3</sup> and Dimarogonas<sup>4</sup>, respectively.

There are two major research fields in the study of dynamics of cracked rotors. One is the modelling of cracked rotors and the other is the detecting of rotor cracks. In order to predict the change in vibrational behaviour due to rotor cracks, a lot of crack models have been developed. The typical model is the breathing crack model developed by Gasch<sup>5</sup>. Gasch<sup>5,6</sup> and Henry and Okah-Avae<sup>7</sup> firstly considered the nonlinear mechanism of a crack with different stiffnesses for the open and closed crack in a body-fixed rotating coordinate system. They presented the spring characteristics of a cracked rotor as a piecewise linear type. They made computer simulations using equations of motion with nonlinear spring characteristics of a piecewise linear type and found that the following resonances occurred in the cracked rotor: 1) a harmonic resonance at the major critical speed, 2) a super-harmonic resonance at the secondary critical speed, 3) super-har-

monic resonances of orders higher than the second ( $n = 3, 4$ ), 4) a 3/2-order super-subharmonic resonance and 5) a 1/2-order subharmonic resonance. The harmonic resonance and the 1/2-order subharmonic resonance which we discuss in this paper were also found in experiments. Mayes and Davies<sup>8</sup> used the Green function to calculate the compliance of a cracked rotor in a space-fixed coordinate system and correlated some experimental results with their theoretical background. In later papers, Mayes and Davies<sup>9,10</sup> presented an approximate method to model a cracked rotor in which the crack was simulated by a reduced diameter section and the cracked rotor was analysed in a multi-rotor-bearing system. They found that the vibrational behaviour was similar to that of a slotted shaft with additional excitation due to the opening and closing of a crack. Grabowski<sup>11</sup> assumed that whether the cracks are fully open, partially open or fully closed depends on the crack location in relation to the horizontal diameter of the shaft. Grabowski<sup>12</sup> investigated the dynamic behaviour of a realistic cracked rotor system using a model formulation and found the 1/2-order subharmonic resonance from experiments. Dimarogonas and Papadopoulos<sup>13,14</sup> pointed out that the crack position has a large influence on resonances, and the information of subharmonic resonances and frequency shifting are very important for crack identification. Papadopoulos and Dimarogonas<sup>15</sup>, and Sekhar and Balaji<sup>16</sup> used the first four terms of a Fourier cosine series to express the stiffness variation of the cracked rotor. Gasch<sup>17</sup> and Tamura<sup>18</sup> investigated the dynamic behaviour of a cracked rotor and solved the equations of motion of a parametrically-excited system. The solution was obtained by linearising the spring characteristics of the cracked rotor. In this study, they could not obtain the solution for the steady-state oscillations