# Order and Realisability of Impulse Response Filters for Accurate Identification of Acoustical Multi-ports from Transient CFD

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So-called network models are popular tools for the analysis of acoustic phenomena, e.g. in mufflers, in ventilation or pipeline systems, and in combustors (thermo-acoustic instabilities). The building blocks of such models are multi-ports, represented mathematically by their respective transfer matrices. Within the limitations of linear acoustics, transfer matrices provide a complete description of the dynamic characteristics of the individual multi-ports. They may be determined experimentally or in an approximate manner by analytical means. Alternatively, transfer matrices may be reconstructed from transient CFD simulation data with the help of system identification tools. Specifically, it is possible to determine the unit impulse responses of a multi-port with correlation analysis and then obtain transfer matrix coefficients via the *z*-transform. The present study is concerned with the optimal choice of parameters for accurate transfer matrix identification. Recommendations for the optimal choice of acoustic variables, sample increment, and sample length, as well as filter order, are formulated. Remarkably, it is found that in many cases the use of formally non-causal filters is advantageous. It is argued that this is a consequence of the fact that causal interrelationships imposed by the underlying laws of fluid mechanics are not always represented properly with the standard acoustic variables.

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## Nomenclature

$c$ – Speed of sound
$\mathbf{c}$ – Cross-correlation vector
$dt$ – Time step of CFD calculation
$f$ – Riemann invariant for downstream travelling wave, also frequency (Hz)
$g$ – Riemann invariant for upstream travelling wave
$\mathbf{h}$ – Unit impulse response vector (UIR)
$\tilde{\mathbf{h}}$ – Approximation of UIR (Vector).
$k$ – Wave number $= \omega / c$.
$l_{eff}$ – Effective length
$l_{red}$ – Reduced length
$n$ – Interaction index of $n - \tau$ model.
$p'$ – Acoustic pressure
$\mathbf{r}$ – Response vector
$\mathbf{s}$ – Signal vector
$u'$ – Acoustic velocity
$F(\omega)$ – Frequency response
$L$ – Filter order and length of UIR
$M$ – Mach number $= U/c$
$N$ – Number of data points considered for identification
$\dot{Q}$ – Heat release rate
$T$ – Transfer matrix
$U$ – Mean flow velocity

### Greek Letters

$\alpha$ – Area ratio $= A_d / A_u$
$\Delta f$ – Sampling frequency
$\Delta t$ – Time step of system identification process
$\Delta x$ – Distance between locations "$u$" (upstream) and "$d$" (downstream)
$\Gamma$ – Auto-correlation (matrix)
$\rho$ – Density
$\tau$ – Time lag
$\omega$ – Angular frequency
$\zeta$ – Acoustic loss coefficient
$\xi$ – Random Variable

### Sub- and Superscripts

$d$ – Downstream
$u$ – Upstream
$\tilde{\ldots}$ – Approximation of …
$\ldots'$ – Fluctuation of …

## 1. INTRODUCTION

For the purpose of analysis or control of acoustic phenomena, e.g. in mufflers, in ventilation or pipeline systems, or in combustors ("thermo-acoustic instabilities"), so-called network models are popular.[1-5] With this approach, individual system elements are represented as multi-ports, and described mathematically by their respective transfer matrices. In its fundamental form, a transfer matrix furnishes linear relationships between acoustic variables – e.g. fluctuations of pressure $p'$ and velocity $u'$ – at the different ports of an element. Traditionally, the transfer matrices of individual elements are determined in an approximate analytical manner, or experimentally.[2,6] It has been proposed recently to use advanced tools from system identification to estimate or reconstruct transfer matrices from transient CFD data in order to combine the power of computational fluid dynamics (CFD) with the efficiency of network models.[7-12] More specifically, time series of fluctuating flow variables at the system element under investigation are generated by transient CFD calculations with the low-amplitude forcing of flow variables at